Statistics for Business and Economics (13e)

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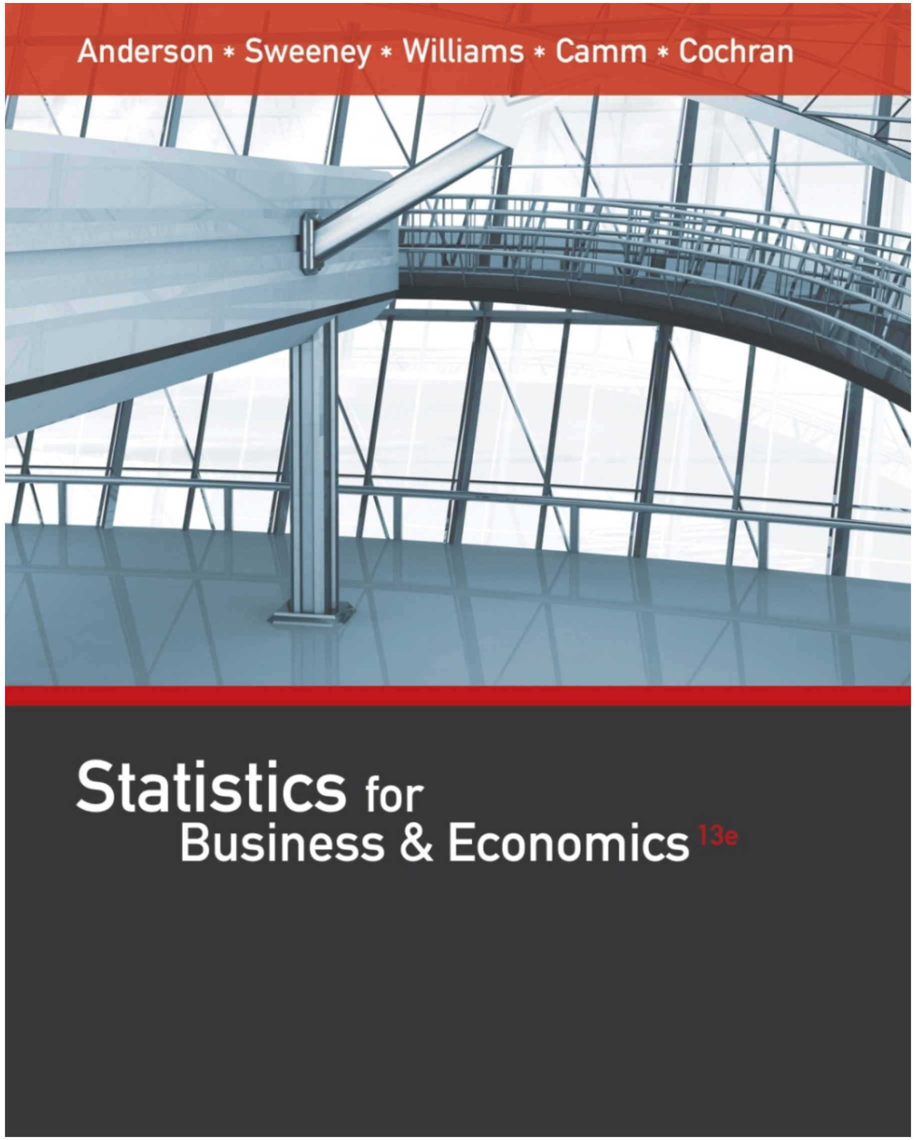
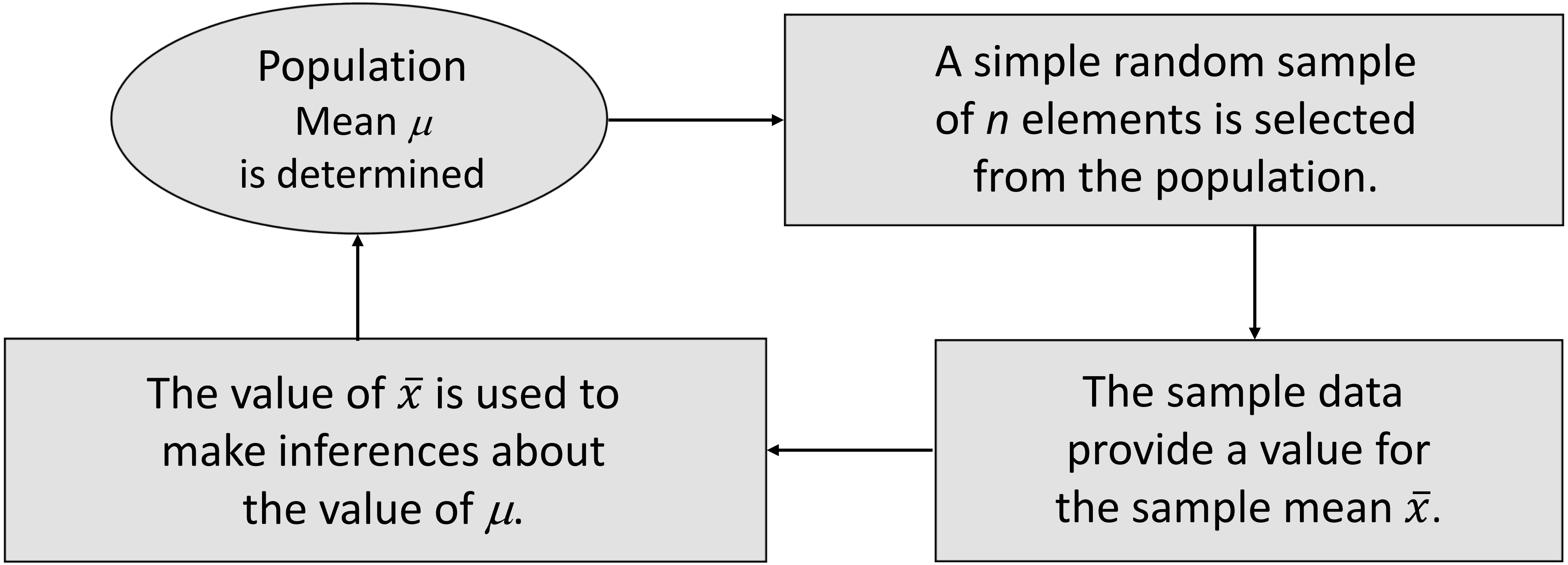


Sampling and Sampling Distributions

- Introduction to Sampling Distributions
- Sampling Distribution of $\bar{x}$

Sampling Distribution of $\bar{x}$

- Process of Statistical Inference



Sampling Distribution of $\bar{x}$

- The sampling distribution of $\bar{x}$ is the probability distribution of all possible values of the sample mean $\bar{x}$.
- Expected Value of $\bar{x}$

$$E(\bar{x}) = \mu$$

where: μ = the population mean

- When the expected value of the point estimator equals the population parameter, we say the point estimator is unbiased.

Sampling Distribution of $\bar{x}$

- We will use the following notation to define the standard deviation of the sampling distribution of $\bar{x}$.

$\sigma_{\bar{x}}$ = the standard deviation of $\bar{x}$

σ = the standard deviation of the population

n = the sample size

N = the population size

Sampling Distribution of $\bar{x}$

- Standard Deviation of $\bar{x}$

Finite Population

$$\sigma_{\bar{x}} = \sqrt{\frac{N-n}{N-1}}\left(\frac{\sigma}{\sqrt{n}}\right)$$

Infinite Population

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

- A finite population is treated as being infinite if $n/N \leq .05$.
- $\sqrt{(N-n)/(N-1)}$ is the finite population correction factor.
- $\sigma_{\bar{x}}$ is referred to as the standard error of the mean.

Sampling Distribution of $\bar{x}$

- When the population has a normal distribution, the sampling distribution of $\bar{x}$ is normally distributed for any sample size.
- In most applications, the sampling distribution of $\bar{x}$ can be approximated by a normal distribution whenever the sample is size 30 or more.

Sampling Distribution of $\bar{x}$

- The sampling distribution of $\bar{x}$ can be used to provide probability information about how close the sample mean $\bar{x}$ is to the population mean μ.

Central Limit Theorem

- When the population from which we are selecting a random sample does not have a normal distribution, the central limit theorem is helpful in identifying the shape of the sampling distribution of $\bar{x}$.

CENTRAL LIMIT THEOREM

In selecting random samples of size *n* from a population, the sampling distribution of the sample mean $\bar{x}$ can be approximated by a *normal distribution* as the sample size becomes large.

- Example: St. Andrew's College

St. Andrew's College received 900 applications for admission in the upcoming year from prospective students. The applicants were numbered, from 1 to 900, as their applications arrived. The Director of Admissions would like to select a simple random sample of 30 applicants.

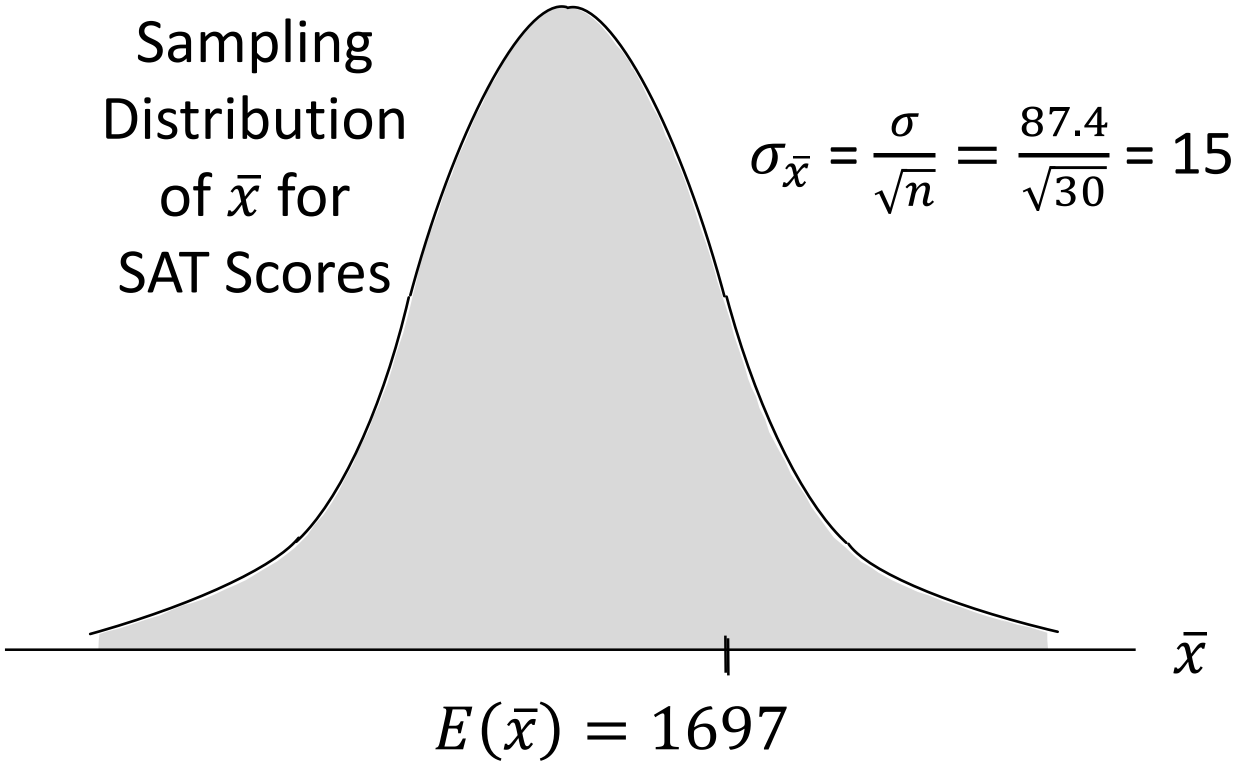
- Example: St. Andrew's College

Recall that St. Andrew's College received 900 applications from prospective students. The application form contains a variety of information including the individual's Scholastic Aptitude Test (SAT) score and whether or not the individual desires on-campus housing.

At a meeting in a few hours, the Director of Admissions would like to announce the average SAT score and the proportion of applicants that want to live on campus, for the population of 900 applicants.

Sampling Distribution of $\bar{x}$

- Example: St. Andrew's College



Sampling Distribution of $\bar{x}$

- Example: St. Andrew's College
  - What is the probability that a simple random sample of 30 applicants will provide an estimate of the population mean SAT score that is within +/-10 of the actual population mean μ ?
  - In other words, what is the probability that $\bar{x}$ will be between 1687 and 1707?