

c) Now, reconsider your answer to (b). Find the structure of prices and quantities in each of the two blocks that maximizes profit. In other words, you no longer assume that the price and quantity that you determined in (a) is fixed. Instead, you must find the optimal price for both blocks.

$$Q_2 = \frac{Q_1 + 120}{2} \quad (\text{mid point})$$

$$PS = TR - VC$$

$$PS = (Q_1 \cdot P_1) + (P_2)(Q_2 - Q_1) - 10Q_2$$

find Q_1^* , Q_2^* , P_1 , P_2

$$\pi = (Q_1)(90 - \frac{Q_1}{2}) + (90 - \frac{Q_2}{2})(Q_2 - Q_1) - 10Q_2$$

$$\pi = 90Q_1 - \frac{Q_1^2}{2} + 90Q_2 - 90Q_1 - \frac{Q_2^2}{2} + \frac{Q_1Q_2}{2} - 10Q_2$$

$$\frac{\partial \pi}{\partial Q_1} = -Q_1 + \frac{1}{2}Q_2$$

$$\frac{\partial \pi}{\partial Q_1} = 0 \rightarrow 2Q_1 = Q_2$$

$$\frac{\partial \pi}{\partial Q_2} = 60 - Q_2 + \frac{1}{2}Q_1$$

$$\frac{\partial \pi}{\partial Q_2} = 0 \rightarrow Q_2 = \frac{1}{2}Q_1 + 60$$

Solving

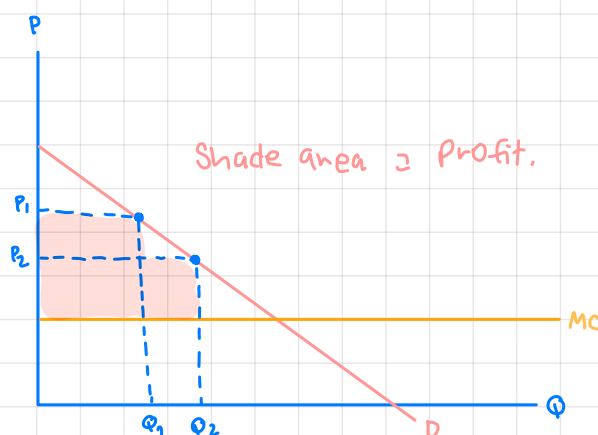
$$2Q_1 = \frac{1}{2}Q_1 + 60$$

$$Q_1 = 40$$

$$Q_2 = 2Q_1 = 80$$

$$P_1 = 50$$

$$P_2 = 30$$



12.** Consider a bar whose owner plans to set profit-maximizing two-part tariff (entry fee and per-drink price) on two types of customers. **The owner would like to welcome both types into his bar, meaning that he will not charge an entry fee that is too high.**

There are 20 people of the X-type whose individual demand is given by $P = 10 - Q_x$. There are 30 people of the Y-type whose individual demand is given by $P = 10 - 2Q_y$. The $MC = AC = \$2$ per drink. Find the optimal entry fee and per-drink price. Also, calculate the profit the bar can make from these 50 customers.

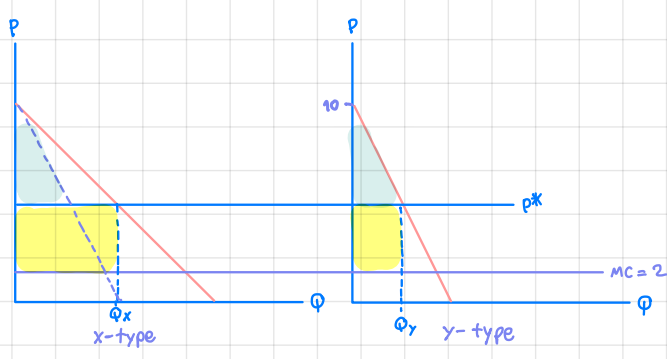
$$Q_y = \frac{10 - P}{2}$$

$$Q_x = 10 - P$$

X-type
 $P = 10 - Q_x$ 20 people

Y-type
 $P = 10 - 2Q_y$ 30 people

$MC = AC = \$2$



Total profit = TR - TC

$$\text{Total Revenue} = 50 \left(\frac{1}{2} (10 - P) Q_y \right) + 20 Q_x P^* + 30 Q_y P^*$$

$$= 50 \left(25 - 5P + \frac{P^2}{4} \right) + 200P - 20P^2 + 150P - 15P^2$$

$$= 12.5P^2 - 250P + 1250 + 200P - 20P^2 + 150P - 15P^2$$

$$\text{Total Cost} = 20(2 \cdot Q_x) + 30(2 \cdot Q_y)$$

$$= 40 \cdot (10 - P) + 60 \cdot \left(\frac{10 - P}{2} \right)$$

$$= 400 - 40P + 300 - 30P$$

$$= 700 - 70P$$

$$\text{Total profit} = 12.5P^2 - 250P + 1250 + 200P - 20P^2 + 150P - 15P^2 - 700 + 70P$$

$$\frac{d\pi}{dP} = 25P - 250 + 200 - 40P + 150 - 30P + 70 + 30 = 0$$

$P^* = 3.78$ - per drink price

Optimal entry fee

$$= (10 - 3.78) \left(\frac{10 - 3.78}{2} \right) \frac{1}{2}$$

$$= 9.67\$$$

$$\pi = (50) 9.675 + 1.78 (3.11)(30)$$

$$+ 1.78 (6.22)(20)$$

$$= 483.95 + 166.074 + 221.432$$

$$= 871.256$$



$$\text{Total } \pi = \pi_{\text{entry fee}} + \pi_{\text{drink sales}}$$

$$\pi_{\text{entry}} = \Delta \times 50$$

$$= 50 \times \frac{1}{2} (10 - p) (Q_y)$$

$$= 50 \times \frac{1}{2} (10 - p) \left(\frac{10 - p}{2} \right)$$

$$= \frac{50}{4} (10 - p)^2$$

$$\pi_{\text{drink sale}} = 20 \times \boxed{} + 30 \times \boxed{}$$

$$= (p - MC) Q_x (20) + (p - MV) Q_y (30)$$

$$= (p - MC) (20 Q_x + 30 Q_y)$$

$$= (p - 2) \left(20 (10 - p) + 30 \frac{(10 - p)}{2} \right)$$

$$= 35 (p - 2) (10 - p)$$

$$\pi_{\text{total}} = \frac{50}{4} (10 - p)^2 + 35 (p - 2) (10 - p)$$

$$\text{Find } p \text{ that max } \pi \Rightarrow \frac{\partial \pi_{\text{total}}}{\partial p} = 0$$

$$\frac{\partial \pi}{\partial p} = 25(10 - p)(-1) + 35((p - 2)(-1) + (10 - p)(1))$$

$2 - p + 10 - p$

$$\frac{\partial \pi}{\partial p} = 0 \Rightarrow 25(10 - p) = 35(12 - 2p)$$

$$250 - 25p = 420 - 70p$$

$$45p = 170$$

$$p = 3.777 \# \text{ - per drink price}$$

$$\text{entry fee per person} = \frac{1}{2} (10 - p) \left(\frac{10 - p}{2} \right)$$

$$= \frac{1}{2} (10 - 3.777) \left(\frac{10 - 3.777}{2} \right)$$

$$= 9.67 \#$$