Capm in economics

Main Resources:

Ch.5 of Microeconomics by Pindyck&Rubinfeld

Ch.13 in Intermediate microeconomics by Varian

Outline

1. Describing Risk

2. Preferences Toward Risk

3. Reducing Risk

4. The Demand for Risky Assets

5. CAPM

1 DESCRIBING RISK

Probability

- **probability** Likelihood that a given outcome will occur.

Subjective probability is the perception that an outcome will occur.

Expected Value

- **expected value** Probability-weighted average of the payoffs associated with all possible outcomes.

- **payoff** Value associated with a possible outcome.

The expected value measures the *central tendency*—the payoff or value that we would expect on average.

Expected value = Pr(success)($40/share) + Pr(failure)($20/share)
= (1/4)($40/share) + (3/4)($20/share) = $25/share

$$E(X) = \Pr_1 X_1 + \Pr_2 X_2$$

$$E(X) = \Pr_1 X_1 + \Pr_2 X_2 + \ldots + \Pr_n X_n$$

Variability

- **variability** Extent to which possible outcomes of an uncertain event differ.

TABLE 5.1 Income from Sales Jobs

	OUTCOME 1		OUTCOME 2		
	Probability	Income ($)	Probability	Income ($)	Expected Income ($)
Job 1: Commission	.5	2000	.5	1000	1500
Job 2: Fixed Salary	.99	1510	.01	510	1500

- **deviation** Difference between expected payoff and actual payoff.

TABLE 5.2 Deviations from Expected Income ($)

	Outcome 1	Deviation	Outcome 2	Deviation
Job 1	2000	500	1000	-500
Job 2	1510	10	510	-990

Variability

Table 5.3 Calculating Variance ($)

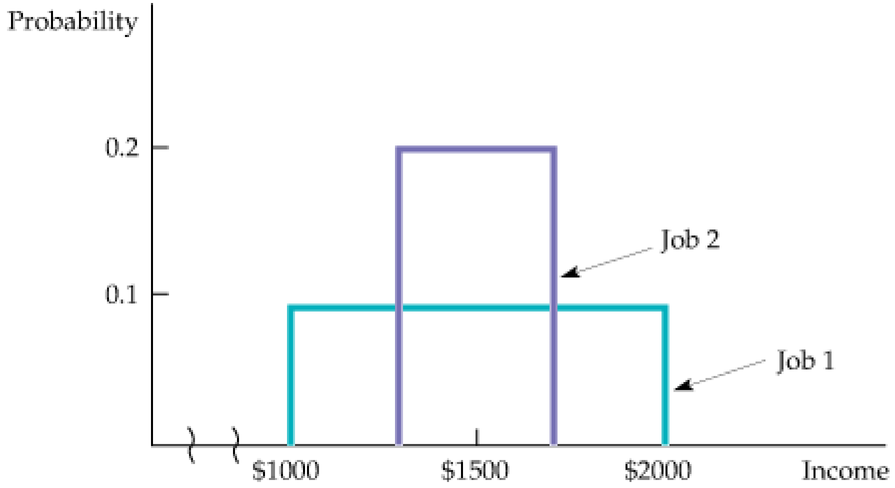
	Outcome 1	Deviation Squared	Outcome 2	Deviation Squared	Weighted Average Deviation Squared	Standard Deviation
Job 1	2000	250,000	1000	250,000	250,000	500
Job 2	1510	100	510	980,100	9900	99.5

Figure 5.1

Outcome Probabilities for Two Jobs

The distribution of payoffs associated with Job 1 has a greater spread and a greater standard deviation than the distribution of payoffs associated with Job 2.

Both distributions are flat because all outcomes are equally likely.



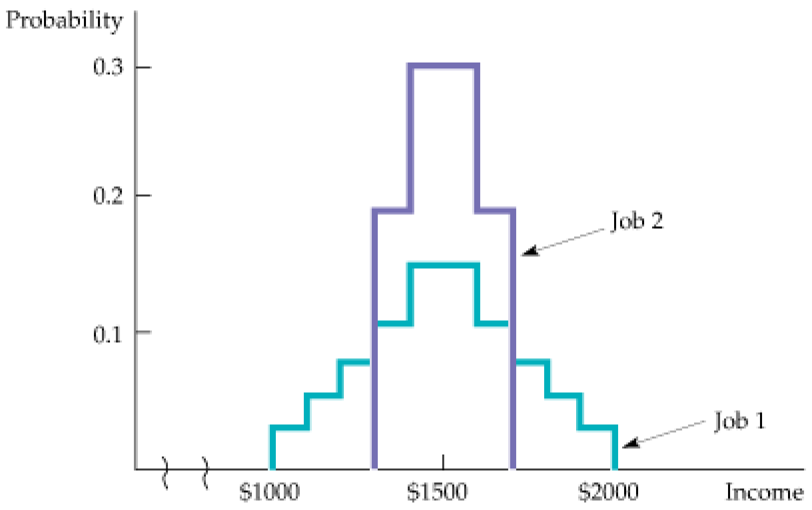
Variability

Figure 5.2

Unequal Probability Outcomes

The distribution of payoffs associated with Job 1 has a greater spread and a greater standard deviation than the distribution of payoffs associated with Job 2.

Both distributions are peaked because the extreme payoffs are less likely than those near the middle of the distribution.



Decision Making

Table 5.4 Incomes from Sales Jobs—Modified ($)

	Outcome 1	Deviation Squared	Outcome 2	Deviation Squared	Expected Income	Standard Deviation
Job 1	2100	250,000	1100	250,000	1600	500
Job 2	1510	100	510	980,100	1500	99.5

The **expected utility** of the risky option is the sum of the utilities associated with all her possible incomes weighted by the probability that each income will occur.

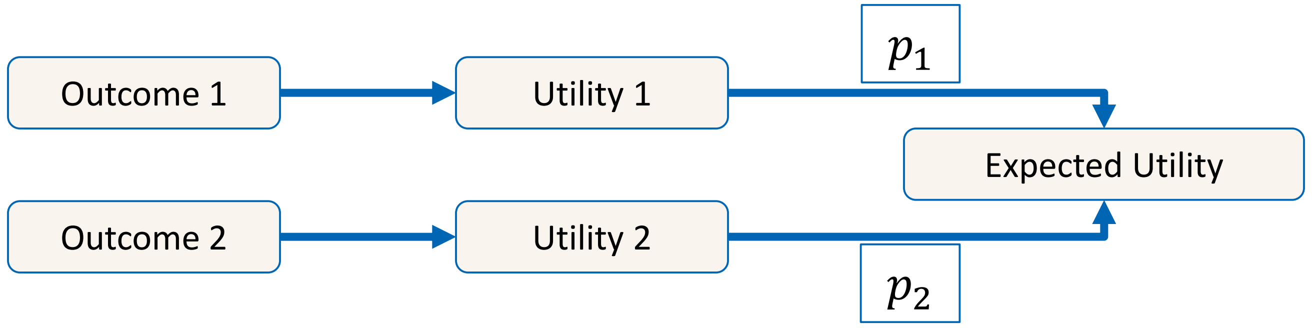
$$U(o_1, o_2, p_1, p_2) = p_1 u(o_1) + p_2 u(o_2)$$

o_1 is outcome 1

o_2 is outcome 2

p_1 is probability that o_1 occurs

p_2 is probability that o_2 occurs



Example: A person is earning $15,000.
She is considering a new, but risky job

0.50 chance of $5,000
0.50 chance of $25,000

Suppose she gets

12 units of utility from $5,000
21 units of utility from $15,000
24 units of utility from $25,000

Must compare utility from the risky job with utility from the current job.

For current job: $U_c = 1*21 = 21$
For risky job: $U_r = 0.5*12 + 0.5*24 = 18$
Therefore, the person prefers to stay in the current job.

Figure 5.3

Risk Averse, Risk Loving, and Risk Neutral

In (**a**), a consumer's marginal utility diminishes as income increases.

The consumer is risk averse because she would prefer a certain income of $20,000 (with a utility of 16) to a gamble with a .5 probability of $10,000 and a .5 probability of $30,000 (and expected utility of 14).

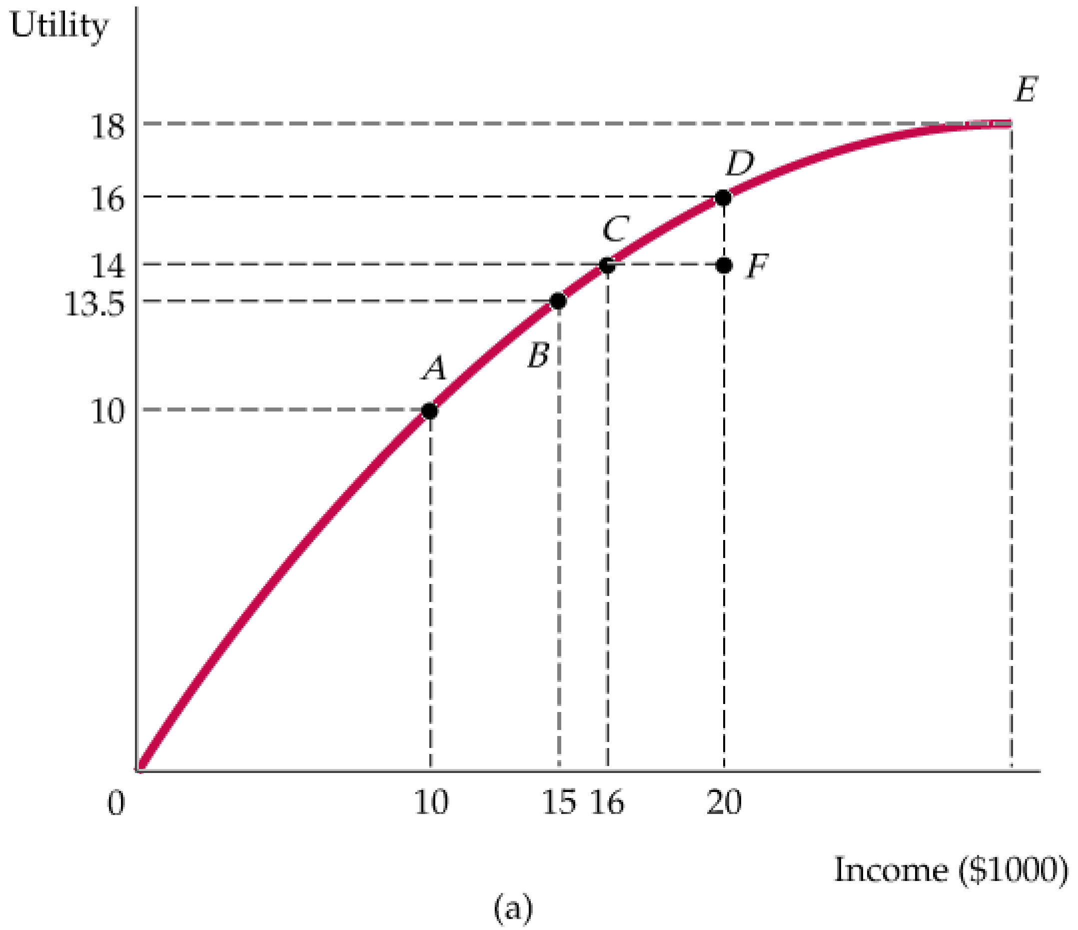


Figure 5.3

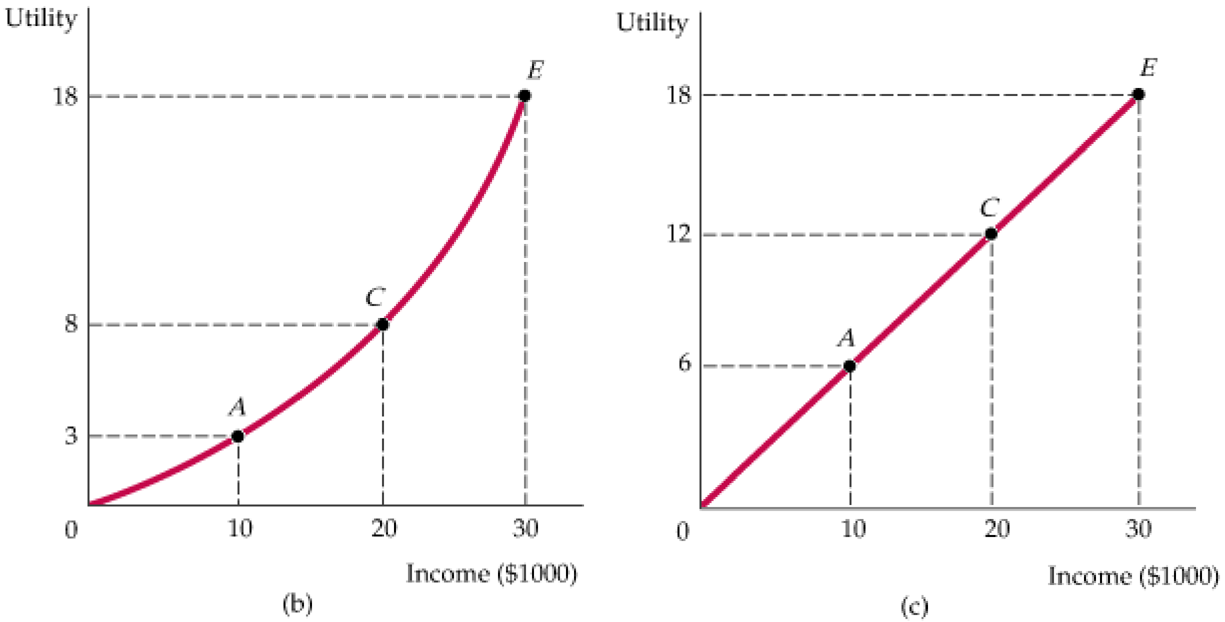
Risk Averse, Risk Loving, and Risk Neutral

In (**b**), the consumer is risk loving:

She would prefer the same gamble (with expected utility of 10.5) to the certain income (with a utility of 8).

Finally, the consumer in (**c**) is risk neutral,

and indifferent between certain and uncertain events with the same expected income.



- **expected utility** Sum of the utilities associated with all possible outcomes, weighted by the probability that each outcome will occur.

Different Preferences Toward Risk

- **risk averse** Condition of preferring a certain income to a risky income with the same expected value.
- **risk neutral** Condition of being indifferent between a certain income and an uncertain income with the same expected value.
- **risk loving** Condition of preferring a risky income to a certain income with the same expected value.

Different Preferences Toward Risk

Risk Premium

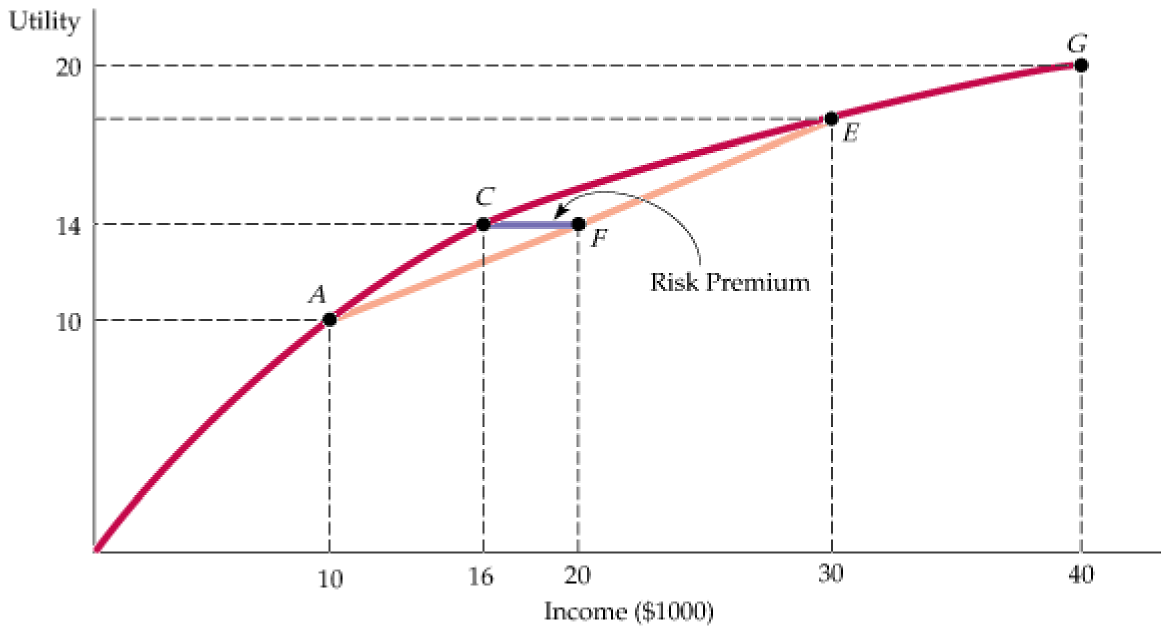
- **risk premium** Maximum amount of money that a risk-averse person will pay to avoid taking a risk.

Figure 5.4

Risk Premium

The risk premium, CF, measures the amount of income that an individual would give up to leave her indifferent between a risky choice and a certain one.

Here, the risk premium is $4000 because a certain income of $16,000 (at point C) gives her the same expected utility (14) as the uncertain income (a .5 probability of being at point A and a .5 probability of being at point E) that has an expected value of $20,000.



Different Preferences Toward Risk

Risk Aversion and Income

The extent of an individual’s risk aversion depends on the nature of the risk and on the person’s income.

Other things being equal, risk-averse people prefer a smaller variability of outcomes.

The greater the variability of income, the more the person would be willing to pay to avoid the risky situation.

Different Preferences Toward Risk

Risk Aversion and Indifference Curves

Figure 5.5

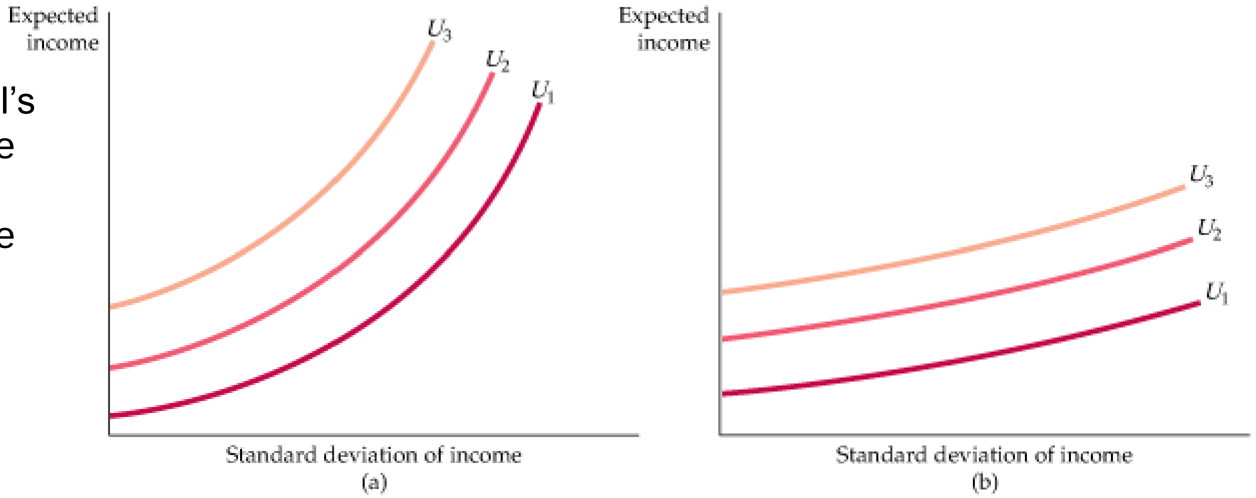
Risk Aversion and Indifference Curves

Part (**a**) applies to a person who is highly risk averse:

An increase in this individual's standard deviation of income requires a large increase in expected income if he or she is to remain equally well off.

Part (**b**) applies to a person who is only slightly risk averse:

An increase in the standard deviation of income requires only a small increase in expected income if he or she is to remain equally well off.



Diversification

- **diversification** Practice of reducing risk by allocating resources to a variety of activities whose outcomes are not closely related.

TABLE 5.5 Income from Sales of Appliances ($)

	Hot Weather	Cold Weather
Air conditioner sales	30,000	12,000
Heater sales	12,000	30,000

- **negatively correlated variables** Variables having a tendency to move in opposite directions.

The Stock Market

- **mutual fund** Organization that pools funds of individual investors to buy a large number of different stocks or other financial assets.

- **positively correlated variables** Variables having a tendency to move in the same direction.

Insurance

TABLE 5.6 The Decision to Insure ($)

Insurance	Burglary (Pr = .1)	No Burglary (Pr = .9)	Expected Wealth	Standard Deviation
No	40,000	50,000	49,000	3000
Yes	49,000	49,000	49,000	0

The Law of Large Numbers

The ability to avoid risk by operating on a large scale is based on the *law of large numbers*, which tells us that although single events may be random and largely unpredictable, the average outcome of many similar events can be predicted.

Actuarial Fairness

- **actuarially fair** Characterizing a situation in which an insurance premium is equal to the expected payout.

The Value of Information

- **value of complete information**
  Difference between the expected value of a choice when there is complete information and the expected value when information is incomplete.

Assets

- **asset** Something that provides a flow of money or services to its owner.

An increase in the value of an asset is a *capital gain*; a decrease is a *capital loss.*

Risky and Riskless Assets

- **risky asset** Asset that provides an uncertain flow of money or services to its owner.

- **riskless (or risk-free) asset** Asset that provides a flow of money or services that is known with certainty.

Asset Returns

- **return** Total monetary flow of an asset as a fraction of its price.
- **real return** Simple (or nominal) return on an asset, less the rate of inflation.

Expected versus Actual Returns

- **expected return** Return that an asset should earn on average.
- **actual return** Return that an asset earns.

TABLE 5.8 Investments—Risk and Return (1926–2006*)

	Average Rate of Return (%)	Average Real Rate of Return (%)	Rate Risk (Standard Deviation, %)
Common stocks (S&P 500)	12.3	9.2	20.1
Long-term corporate bonds	6.2	3.1	8.5
U.S. Treasury bills	3.8	0.7	3.1

*Source: *Stocks, Bonds, Bills, and Inflation: 2007 Yearbook,* Morningstar, Inc.

Suppose: R_f denotes the rate of return of the riskless asset
R_m denotes the expected return of the risky asset
b denotes the fraction in risky asset
O_s denotes the return of risky asset if state s occurs
p_s denotes the probability that state s will occur

The expected return of the portfolio is:

$$R_p = bR_m + (1-b)R_f.$$

The variance of the portfolio will be:

$$\sigma_p^2 = (bO_1 + (1-b)R_f - R_p)^2 p_1 + (bO_2 + (1-b)R_f - R_p)^2 p_2.$$

Substitute R_p from above, we have:

$$\sigma_p^2 = b^2\sigma_m^2.$$

Thus, the standard deviation is:

$$\sigma_p = b\sigma_m.$$

The Trade-Off Between Risk and Return

The Investment Portfolio

$$R_p = bR_m + (1-b)R_f \quad (5.1)$$

$$\sigma_p = b\sigma_m \quad (5.2)$$

The Investor's Choice Problem

$$R_p = R_f + b(R_m - R_f)$$

$$R_p = R_f + \frac{(R_m - R_f)}{\sigma_m}\sigma_p \quad (5.3)$$

- **Price of risk** Extra risk that an investor must incur to enjoy a higher expected return.

The Investor's Choice Problem

Risk and Indifference Curves

Figure 5.6

Choosing Between Risk and Return

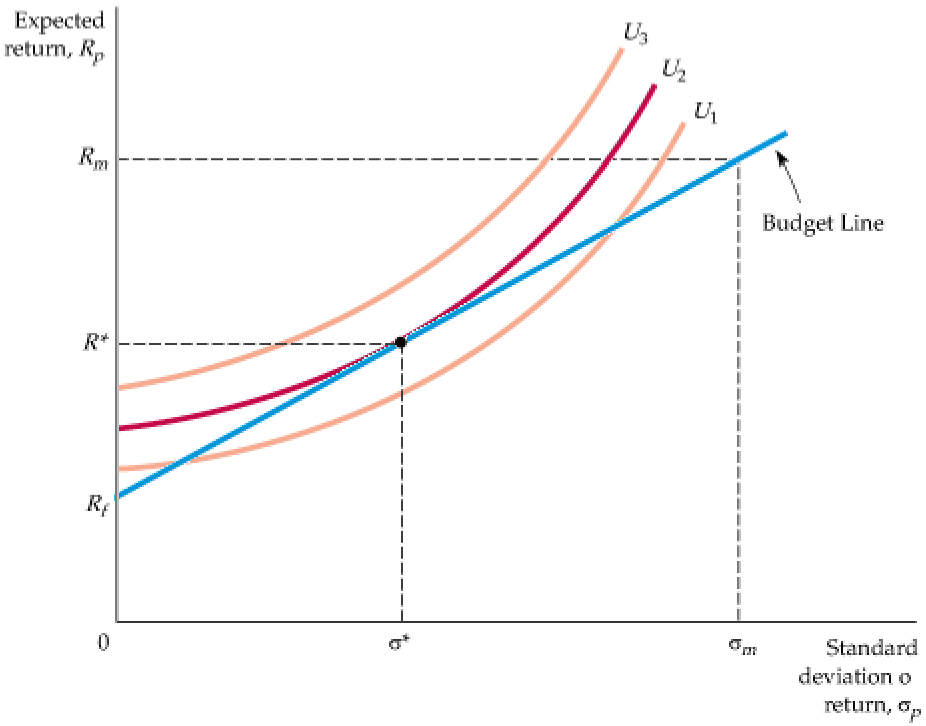
An investor is dividing her funds between two assets—Treasury bills, which are risk free, and stocks. The budget line describes the trade-off between the expected return and its riskiness, as measured by the standard deviation of the return.

The slope of the budget line is $(R_m - R_f)/\sigma_m$, which is the ***price of risk***.

Three indifference curves are drawn, each showing combinations of risk and return that leave an investor equally satisfied.

The curves are upward-sloping because a risk-averse investor will require a higher expected return if she is to bear a greater amount of risk.

The utility-maximizing investment portfolio is at the point where indifference curve U_2 is tangent to the budget line.



The Investor's Choice Problem

Risk and Indifference Curves

Figure 5.7

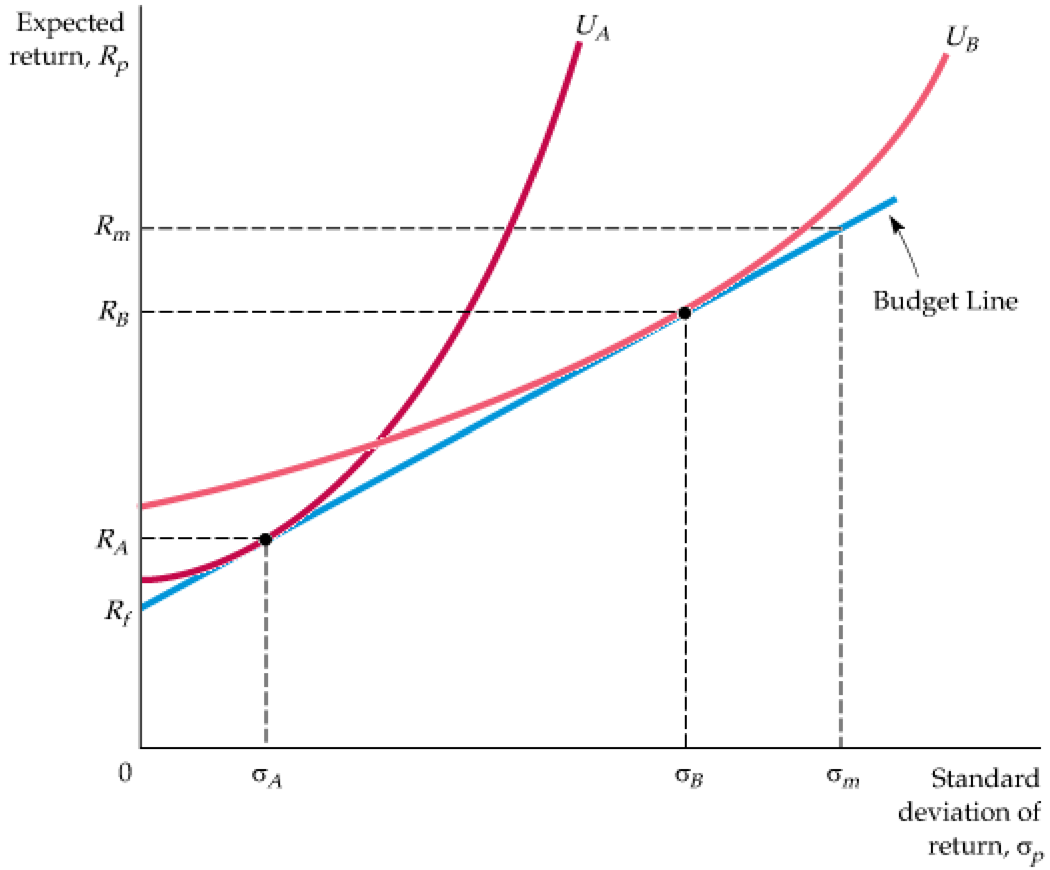
The Choices of Two Different Investors

Investor *A* is highly risk averse.

Because his portfolio will consist mostly of the risk-free asset, his expected return R_A will be only slightly greater than the risk-free return. His risk σ_A, however, will be small.

Investor *B* is less risk averse. She will invest a large fraction of her funds in stocks.

Although the expected return on her portfolio R_B will be larger, it will also be riskier.



The Investor's Choice Problem

Risk and Indifference Curves

Figure 5.8

Buying Stocks on Margin

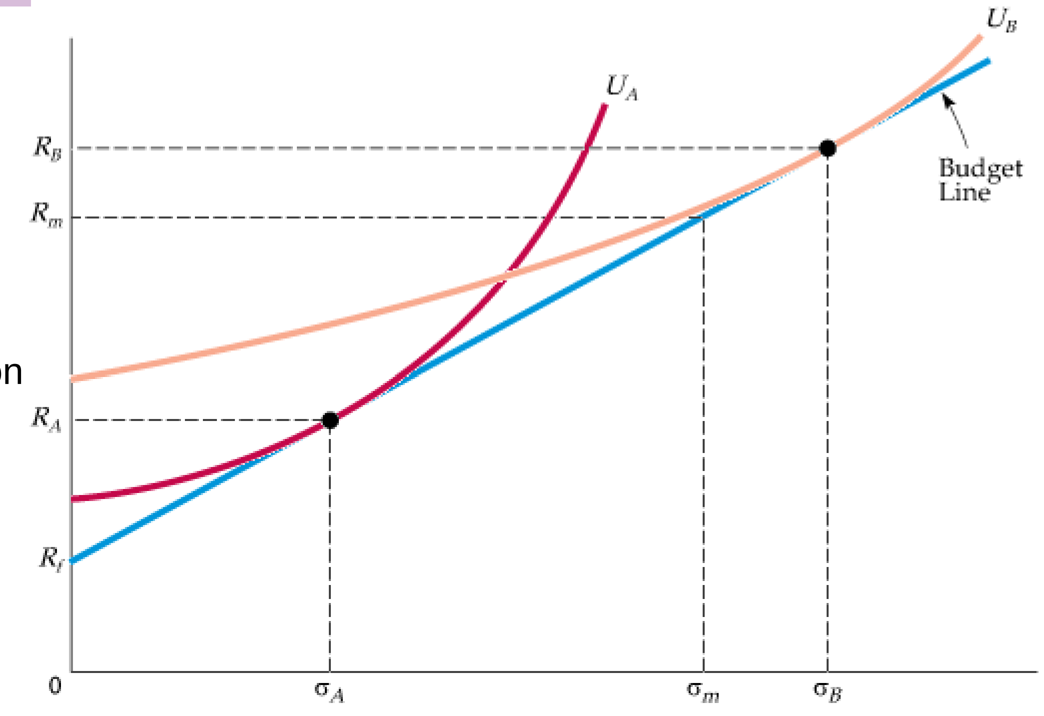
Because Investor *A* is risk averse, his portfolio contains a mixture of stocks and risk-free Treasury bills.

Investor *B*, however, has a very low degree of risk aversion.

Her indifference curve, U_B, is tangent to the budget line at a point where the expected return and standard deviation for her portfolio exceed those for the stock market overall.

This implies that she would like to invest *more* than 100 percent of her wealth in the stock market.

She does so by buying stocks *on margin*—i.e., by borrowing from a brokerage firm to help finance her investment.



*5 CAPM

Suppose there are many risky assets, the standard deviation is not an appropriate measure for the amount of risk in an asset.

This is because a consumer’s utility depends on the mean and variance of total wealth – not the mean and variance of any single asset that she might hold.

Assets that move in opposite directions are very valuable because they reduce overall risk. In general, the value of an asset tends to depend much more on the correlation of its return with other assets than with its own variation.

It is convenient to measure the risk in an asset relative to the risk in the stock market as a whole. We call the riskiness of a stock relative to the risk of the market the ***beta*** of a stock, denoted by β.

Roughly speaking:

$$\beta_i = \frac{how\ risky\ asset\ i\ is}{how\ risky\ the\ stock\ market\ is}.$$

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Roughly speaking:

$$\beta_i = \frac{how\ risky\ asset\ i\ is}{how\ risky\ the\ stock\ market\ is}.$$

*5 CAPM

We saw previously that the price of risk is given by $(R_m - R_f)/\sigma_m$. We also saw that the amount of risk in a given asset i relative to the total risk in the market is denoted by β_i. This means the total amount of risk in asset i is $\beta_i \sigma_m$.

The risk adjustment can be calculated by multiplying the total amount of risk to the price of risk, which is:

$$\text{Risk adjustment} = \beta_i \sigma_m \frac{R_m - R_f}{\sigma_m} = \beta_i (R_m - R_f).$$

In equilibrium, all assets should have the same risk-adjusted rate of return. If there are two assets i and j that have expected returns R_i and R_j, and betas β_i and β_j, we must have

$$R_i - \beta_i (R_m - R_f) = R_j - \beta_j (R_m - R_f).$$

Since the riskless asset has zero amount of risk, we must have:

$$R_i - \beta_i (R_m - R_f) = R_f$$

*5 CAPM

Since the risk-free asset has zero amount of risk, we must have:

$$R_i - \beta_i(R_m - R_f) = R_f.$$

Rearranging, this equation becomes:

$$R_i = R_f + \beta_i(R_m - R_f).$$

The expected return of any asset must be the risk-free return plus the risk adjustment. This equation is the main result of the ***Capital Asset Pricing Model (CAPM)***.