B.E. International Program

Faculty of Economics, Thammasat University

Semester: 1/2015

Subject: EE 425 Econometrics 1

Homework 1

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1. From the model $Y_i = \beta_1 + \beta_2 X_i + u_i$ we can find the normal equations from the OLS estimation method

1.1 $$\sum_{i=1}^{n} Y_i = n\hat{\beta}_1 + \hat{\beta}_2 \sum_{i=1}^{n} X_i$$

1.2 $$\sum_{i=1}^{n} X_i Y_i = \hat{\beta}_1 \sum_{i=1}^{n} X_i + \hat{\beta}_2 \sum_{i=1}^{n} X_i^2$$

We can use 1.1 and 1.2 to solve for $\hat{\beta}_1$ and $\hat{\beta}_2$

(a) $\hat{\beta}_1 = \overline{Y} - \hat{\beta}_2 \overline{X}$

(b) $$\hat{\beta}_2 = \frac{n\sum X_i Y_i - \sum X_i \sum Y_i}{n\sum X_i^2 - \left(\sum X_i\right)^2}$$

Show that the expression

$$\frac{\sum x_i y_i}{\sum x_i^2} = \frac{\sum \left(X_i - \overline{X}\right)\left(Y_i - \overline{Y}\right)}{\sum \left(X_i - \overline{X}\right)^2}$$

is equivalent to the expression for $\hat{\beta}_2$ in (b).

2. Determine whether the following models are linear in parameters or linear in variables or neither. Provide your reason.

Model	Descriptive title
(a) $Y_i = \beta_1 + \beta_2\left(\frac{1}{X_i}\right) + u_i$	Reciprocal
(b) $Y_i = \beta_1 + \beta_2 \ln X_i + u_i$	Semi-logarithmic
(c) $\ln Y_i = \ln \beta_1 + \beta_2 \ln X_i + u_i$	Logarithmic or double logarithmic
(d) $\ln Y_i = \beta_1 + \beta_2\left(\frac{1}{X_i}\right) + u_i$	Logarithmic reciprocal
(e) $Y_i = \frac{1}{1+e^{\beta_1+\beta_2 X_i + u_i}}$	Logistic distribution

3. The National Economic and Social Development Board (NESDB) collects and publishes Thailand's national income and GDP data as follows.

Year	Real GDP (million baht) RGDP (at 2002 prices)	Nominal GDP (million baht) NGDP (at current prices)	Real Private Consumption (at 2002 prices)
2002	5,769,578	5,769,578	3,211,202
2003	6,184,372	6,317,302	3,447,107
2004	6,573,325	6,954,271	3,703,168
2005	6,848,605	7,614,409	3,858,733
2006	7,188,838	8,400,655	3,966,812
2007	7,579,558	9,076,307	4,014,082
2008	7,710,356	9,706,932	4,127,818
2009	7,653,432	9,654,016	4,076,030
2010	8,227,953	10,802,402	4,279,821
2011	8,296,548	11,300,485	4,355,384
2012	8,904,096	12,354,656	4,631,513
2013[p]	9,154,248	12,910,038	4,670,563
2014[p]	9,233,493	13,148,601	4,699,521

Note: P = preliminary

Source: www.nesdb.go.th

a) Divide the data by 1,000,000, plot RGDP and NGDP against time. In a separate diagram, plot real consumption expenditure against real GDP.

b) Let X = time and in 2002 = 1,…..2013 = 13, find the OLS estimators $\hat{a}$ and $\hat{b}$ from the following models.

$RGDP_t = a_1 + b_1X_t + u_t$

$NGDP_t = a_2 + b_2X_t + u_t$

Use the OLS formula $\hat{a} = \overline{Y} - \hat{b}\overline{X}$

$$\hat{b} = \frac{\sum (X_i - \overline{X})(Y_i - \overline{Y})}{\sum (X_i - \overline{X})^2} = \frac{\sum x_i y_i}{\sum x_i^2}$$

c) Interpret $\hat{b}_1$ and $\hat{b}_2$. Does the difference between $\hat{b}_1$ and $\hat{b}_2$ have any economic meaning?

d) Let Y = consumption expenditure in 2002 prices

X = real GDP in 2002 prices

Model : $Y_i = \beta_1 + \beta_2 X_i + u_i$

Use the data during 2002 – 2014 to find OLS estimators for $\hat{\beta}_1$ and $\hat{\beta}_2$. What is the value of the marginal propensity to consume (MPC) during this period? What is the value of a simple closed economy multiplier from this estimated equation?

4. Show the following:

(a) var $\hat{\alpha}$ $= E[\hat{\alpha} - E\hat{\alpha}]^2 = \sigma_u^2 \left(\frac{1}{n} + \frac{\bar{X}^2}{\sum x_i^2}\right)$

(b) $\sigma_u^2 \left(\frac{1}{n} + \frac{\bar{X}^2}{\sum x_i^2}\right) = \sigma_u^2 \left(\frac{\sum X_i^2}{n \sum x_i^2}\right)$

(c) cov $(\hat{\alpha}, \hat{\beta})$ $= E[(\hat{\alpha} - E(\hat{\alpha}))(\hat{\beta} - E(\hat{\beta}))]$

$= -\bar{X} \frac{\sigma_u^2}{\sum x_i^2}$

where $x_i = (X_i - \bar{X})$