BACHELOR of ECONOMICS
International Program

Thammasat University | Thailand

FN 312 Investments

Bond Price and Yields

Outline

- Bond characteristics
- Different kinds of bonds: by issuer
- Invoice price vs quoted price
- Innovations in bond markets
- Compounding interest
- Bond as a portfolio of cash flows
- Yield to Maturity (YTM) and holding period return (HPR)
- Bond prices over time

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Bond characteristics

- Maturity
- Face value or par
- Coupon rate and frequency of coupon payments
- Issuer
- Tax treatment
- Bond indenture

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Bond characteristics

1. Zero coupon bonds (par $1000)

cash flow of a 2-year zero coupon bond

				1000	
issue	0.5	1.0	1.5	2.0	years

2. Coupon bonds (par $1000): semi-annual coupon payments

cash flow of a 6% coupon 2-year bond

	30	30	30	1000+30	
issue	0.5	1.0	1.5	2.0	years

- How coupon gets paid

- Registered bonds

- Barer bonds

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Bonds by issuer: Treasury Bills

- Issued by US government and has no credit risk
- Pays par value at maturity and pays no coupons
- $10,000 par per T-bill
- Matures in less than one year
  - 91-day, 182-day maturities sold weekly
  - 52-week maturities sold monthly
  - Federal government holds auction

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Bonds by issuer: Treasury Bills

TREASURY BILLS

MATURITY	DAYS TO MAT.	BID	ASKED	CHG.	ASKED YLD.
Jul 26 '01	1	3.36	3.28	+ 0.10	3.33
Aug 02 '01	8	3.44	3.36	− 0.02	3.41
Aug 09 '01	15	3.51	3.43	− 0.03	3.48
Aug 16 '01	22	3.49	3.41	− 0.02	3.46
Aug 23 '01	29	3.48	3.40	− 0.03	3.46
Aug 30 '01	36	3.53	3.49	....	3.55

Bank discount yield convention for T-Bills

$$r_{BD} = \frac{10{,}000 - P}{10{,}000} * \frac{360}{n}$$

$$P = 10{,}000 * [1 - r_{BD} * (n/360)]$$

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Bonds by issuer: Treasury Notes and Bonds

MATURITY	DAYS TO MAT.	BID	ASKED	CHG.	ASKED YLD.
Jul 26 '01	1	3.36	3.28	+0.10	3.33
Aug 02 '01	8	3.44	3.36	-0.02	3.41
Aug 09 '01	15	3.51	3.43	-0.03	3.48
Aug 16 '01	22	3.49	3.41	-0.02	3.46
Aug 23 '01	29	3.48	3.40	-0.03	3.46
Aug 30 '01	**36**	**3.53**	**3.49**	**...**	**3.55**

$$P = 10{,}000 * [1 - r_{BD} * (n/360)]$$

$$P_{Bid} = 10{,}000 * [1 - .0353 * (36/360)]$$

$$= 9{,}964.70$$

$$P_{Ask} = 10{,}000 * [1 - .0349 * (36/360)]$$

$$= 9{,}965.10$$

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Bonds by issuer: Treasury Notes and Bonds

- Both pay semi-annual coupons
- Notes are issued with maturities of 2-10 years
- Bonds are issued with maturities longer than 10 years
- Some bonds issued before 1984 are <u>callable</u>
- As of 2002, Treasury no longer issues 30-year bonds
- Price quotes are in '$:32nds'

GOVT. BOND & NOTES

RATE	MATURITY MO/YR	BID	ASKED	CHG.	ASKED YLD.
5 1/2	Jul 01n	100:00	100:02	....	1.68
6 5/8	Jul 01n	100:00	100:02	– 1	2.76

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Notes and Bonds: Example (Today: July 24 '01)

Rate	Maturity	Bid	Ask	Ask yld.
10 ¾	May 03	111:21	111:25	3.93

- Cash flow $1000 face value

Jul 24 '01	Nov01	May02	Nov02	May03
	↓ 53.75	↓ 53.75	↓ 53.75	↓ 1000+53.75

- Ask price = (111+25/32)% *1000 = $1117.8125

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Bonds by issuer: Municipal Bonds

- Issued by state and local governments
- Interest is tax-free, capital gains are not
- To figure out equivalent taxable yield, set after-tax return equal to tax-free return:

$$r_{taxable}(1-t) = r_{Municipal}$$

$$r_{taxable} = \frac{r_{Municipal}}{(1-t)}$$

- Investors with tax rate above t_{crit} invest in municipal bonds

$$t_{crit} = 1 - \frac{r_{Municipal}}{r_{taxable}}$$

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Bonds by issuer: Municipal Bonds

- Suppose your tax bracket is 28%
- You have the choice of a municipal bond paying 4% or a Treasury Bond paying 6% interest

$$r_{taxable}(1-t) = 6\% * (1-.28) = 4.32\%$$

- So Treasury Bond offers a higher after-tax return

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Concept check question

Consider the choice of investing in municipal bond paying 4% or a Treasury Bond paying 6% interest, which of the following is/are true?

G) If your tax bracket is higher than 33.3% you should invest in the municipal bond

Y) If your tax bracket is higher than 33.3% you should invest in the Treasury bond

R) If your tax bracket is lower than 33.3% you should invest in the municipal bond

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Bonds by issuer: Corporate bonds

- Issued by corporations
- Most are semi-annual coupon bonds
- Have default risk
- Give investors a premium in terms of higher yield to compensate for the default risk

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Bonds by issuer: Corporate bonds

- Bond default risk, often called credit risk, is rated by Moody's Investor Service, Standard and Poor's Corporation, Duff and Phelp, and Fitch Investor Service.
- Bonds are categorized based on their credit ratings as

  - Investment-grade bonds (BBB or above)

  - Speculative-grade bonds (Junk) (below BBB)
- Some entities such as insurance can only invest in investment-grade bonds

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Bonds by issuer: Corporate bonds

Determinants of bond safety

- Coverage ratios: Ratios of company earnings to fixed costs
- Leverage ratio: Debt-to-equity ratio
- Liquidity ratios: Current ratio (current assets/current liabilities) and quick ratio (current assets excluding inventories/current liabilities)
- Profitability ratios: Measures of rates of return on assets or equity.
- Cash flow-to-debt-to-equity ratio: Total cash flow to debt

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Bond with call provisions and convertible bonds

- Straight bonds: pay the promised cash flow until maturity
- Callable bonds: the issuer has the right to buy back
- Putable bonds: the investor (buyer of the bond) has the right to sell the bond back
- Convertible bonds: give the bondholders an option to exchange bond for stock
- Example: A 6% coupon 5-year convertible bond issued by GE is convertible at a conversion ratio of 25:1 (i.e., one bond for 25 GE shares)

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Concept check

If you hold a putable bond that matures in 5 years and pays 10% coupon annually. You can put the bond back at par of 1000. When would you want to put the bond back? (assume there is one interest rate for all cash flows and the rates given are annual rates)

G) Interest rates are well above 10%

Y) Interest rates are well below 10%

R) Never want to exercise the put option

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Invoice price vs quoted price

- Invoice price is what you actually pay the dealer for the purchase of a bond.
- Invoice price = quoted price + accrued interest
- Accrued interest = (last coupon date up to but excluding settlement date/last coupon date up to but excluding next coupon date) * coupon
- Trade date: date on which the transaction is executed
- Settlement date: next business day after the trade date

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Invoice price (Trade date: July 24 ’01)

15 May 01	Jul 24 01	15 Nov 01
↓	↓	↓ 53.75

- Quoted price = 1117.8125
- Days since last coupon

15 May –June 30	17+30=47days
Jul 1-Jul 24	24 days
	=71 days

- Days in the coupon period = 15 May to 14 Nov = 184 days
- Invoice price = 53.75* (71/184) +1117.8125 = 1138.55

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Concept check (Trade date: July 13 '01)

15 May 01	July 13 01	15 Nov 01
↓	↓	↓ 53.75

Last coupon date is 15 May 01 and the next coupon date is 15 Nov 01. You bought this bond on July 13, which is a Friday

What is the accrued interest of this bond?

G) 18.111
Y) 18.99
R) 17.53

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Innovations in bond markets

- STRIPs: Treasury strips and sell each coupon that matures at a different date separately. Principle is also sold separately.

  STRIPs are claims to these separate cash flows

  STRIPs are basically zero-coupon bonds with maturity > 1 year
- Inflation index bonds (TIPS)

  The principal and and coupon are indexed to CPI data

  Cash flows grows/decrease with inflation/deflation
- Asset-backed bonds

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Compounding interest

- Bank I offers 10% interest rate compounded annually
- Bank II offers 10%/365 interest rate compounded daily
- Investment of $1000 today in 1 year will grow to:

  Bank I: after 1 year $1000(1+0.1)= 1100

  Bank II: $1000 → after 1 day 1000(1+0.1/365) → after 2 days 1000(1+0.1/365)^2 → after 1 year $1000(1+0.1/365)^365 = 1105.2

- As an investor, which do you prefer?

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Compounding interest

- Bank I must offer 10.52% compounded annually to attract investors.
- Investment of $1000 today in 1 year will grow to 1105.2
- In the market, interest rates must be comparable:

$$(1+r_{yearly})^T = (1+\frac{r_{daily}}{365})^{T*365} = (1+\frac{r_{semi-annual}}{2})^{T*2}$$

- It is customary to quote rates multiplied by its compounding period (annualized).

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Concept check

- You want to invest your money for one year. Which of the following interest rates do you prefer?

G) 4% annual percentage rate

Y) 3.98% semi-annual rate

R) 3.85% monthly rate

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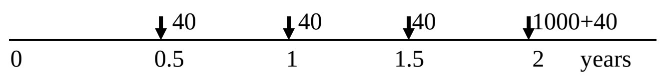
Bonds as portfolio of cash flows

- Example: 8% 2-year coupon bond face value =1000
- Cash flow



- Portfolio of cash flows or zero-coupon bonds

Mat./cash flow

0.5	40			
1		40		
1.5			40	
2				1040

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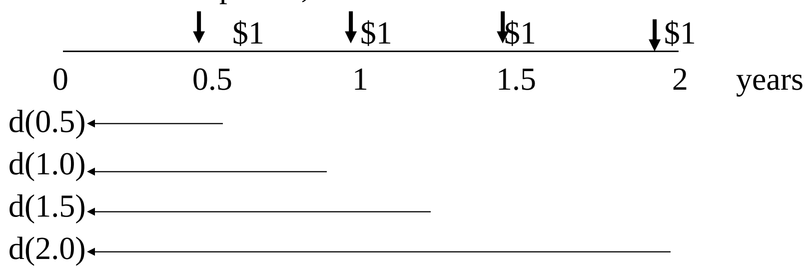
Bonds as portfolio of cash flows

- The discount factor, d(t): Present value of $1 to be received at the end of the period, t.

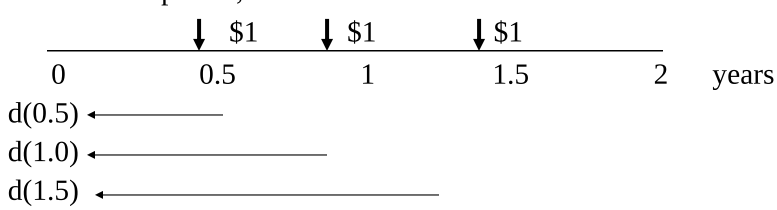


PV of 8% coupon 2-year bond using discount factors

= 40* d(0.5)+40* d(1.0)* 40* d(1.5)+ 1040* d (2.0)

Bonds as portfolio of cash flows

- The discount factor, d(t): Present value of $1 to be received at the end of the period, t.



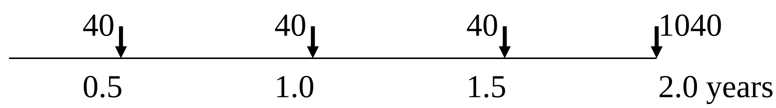
- The annual discount rate, r(t), associated with each discount factor is

  $d(t) = 1/(1+r(t)) \rightarrow r(t) = (1/d(t)^{1/t})-1$
- The semi annual discount rate, r(t), associated with each discount factor is

  $d(t) = 1/(1+r(t)/2)^2 \rightarrow r(t) = 2((1/d(t))^{1/t*2}-1)$

Bonds as portfolio of cash flows

- 2-year 8% semi-annual coupon bond face value =1000



PV using semi-annual interest rates

40/(1+r(0.5)/2)

40/(1+r(1.0)/2)^2

40/(1+r(1.5)/2)^3

1040/(1+r(2.0)/2)^4

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Bonds as portfolio of cash flows

- Bond price denoted P equals

$$P = \frac{\$40}{1+r(0.5)/2} + \frac{\$40}{(1+r(1.0)/2)^2} + \frac{\$40}{(1+r(1.5)/2)^3} + \frac{\$1040}{(1+r(2.0)/2)^4}$$

- If all the interest rates are the same, then

$$P = \frac{\$40}{1+r/2} + \frac{\$40}{(1+r/2)^2} + \frac{\$40}{(1+r/2)^3} + \frac{\$1040}{(1+r/2)^4}$$

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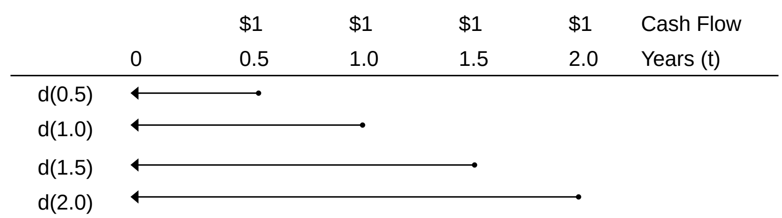
Spot Rate

- The discount factor, d(t)



- The 'market' discount factors from zeros

Mat.	Price	t	d(t) *
0.5	97.087	0.5	0.97087
1	93.897	1	0.93897
1.5	90.909	1.5	0.90909
2	87.336	2	0.87336

* d(t) = Price/Face Value

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Spot rate

- The 'market' discount factors can also be extracted from coupon bonds (face value 100)

B_t	Bond	Price	Discount Factor
$B_{0.5}$	0.5-year zero coupon bond	97.087	100*d(0.5)
$B_{1.0}$	1-year 10% coupon bond	103.4462	5* d(0.5) +105*d(1.0)
$B_{1.5}$	1.5-year 6% coupon bond	99.36579	3* d(0.5) +3*d(1.0)+103*d(1.5)

B_t	t	d(t)
$B_{0.5}$	0.5	0.97087
$B_{1.0}$	1.0	0.93897
$B_{1.5}$	1.5	0.90909

- Solving the three equations for the three discount prices, we get

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Spot rate

The discount factor d(2) = 0.87336 is equivalent to the following compounding rates

t=0	0.5	1.0	1.5	2.0
0.87336				1.0

Annual compounding at 7%

0.87336		0.935		1.0

Semi-annual compounding at 6.88%

0.87336	0.904	0.935	0.967	1.0

Rates of higher compounding intervals are lower because the interest payments get reinvested more often

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Spot rate

- **The spot rate, y_t, is the <u>yield</u> that is equivalent to the discount rate at time t.**
- The relation between the semi-annual spot rate and the discount rate $y_t = 2 * \left((1/d(t))^{1/year*2} - 1 \right)$

$$y_t = 2 * \left(\left(\frac{F}{B_t} \right)^{1/year*2} - 1 \right)$$

- Getting spot rates directly from <u>zero-coupon bonds</u>

where B_t=price of a zero coupon bond that matures at time t and F=face value

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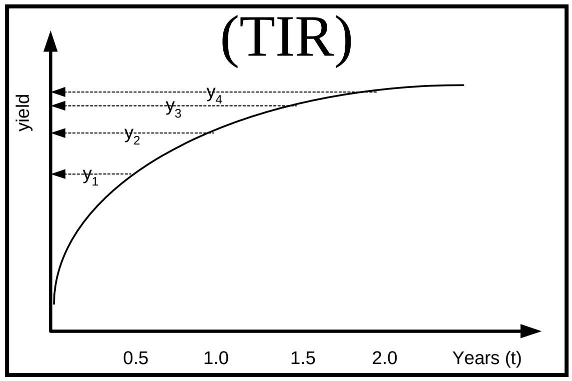
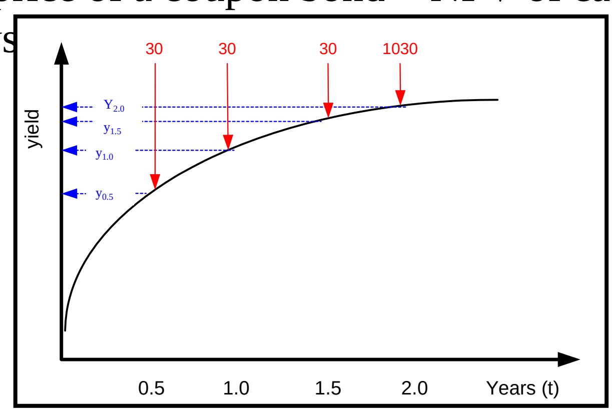


t	d(t)	$y_t = 2 * \left((1/d(t))^{1/t*2} - 1 \right)$
0.5	0.97087	6.0%
1.0	0.93897	6.4%
1.5	0.90909	6.46%
2.0	0.87336	6.88%

TIR: Pricing bonds

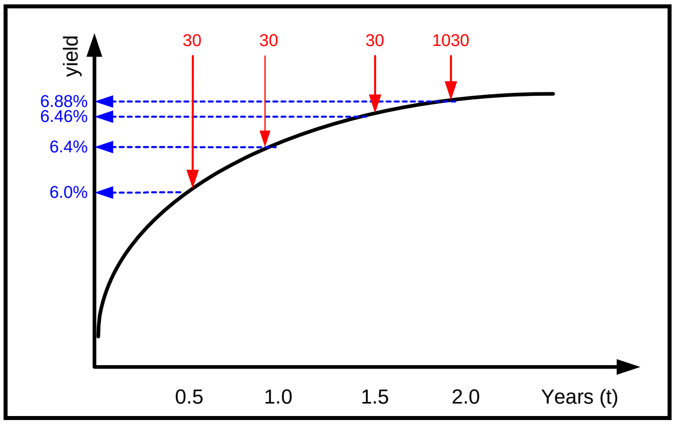
Bond price of a coupon bond = NPV of cash flows



$$\text{Price of a coupon bond} = \sum_t \frac{C_t}{(1+\frac{y_t}{2})^{t*2}}, \ t = \text{year}$$

TIR: Pricing bonds

A 2 year 6% coupon bond



$$\text{Price} = \frac{30}{(1.03)} + \frac{30}{(1.032)^2} + \frac{30}{(1.0323)^3} + \frac{1030}{(1.0344)^4} = 984.24$$

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Yield to Maturity (YTM) and HPR

- Yield To Maturity (YTM) is defined as the single rate that makes the present value of a bond's payment equal its price

$$P = \sum_{t=1}^{T} \frac{C_t}{(1 + YTM/2)^t}$$

C_t denotes the cash flow at time t

T denotes the number of periods. In the case of semi-annual rates, T=years*2.

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Yield to Maturity (YTM) and HPR

- The price of a 2-year 8% semi-annual coupon bond face value =1000 that has a YTM= 9% is

$$P = \frac{\$40}{1+0.09/2} + \frac{\$40}{(1+0.09/2)^2} + \frac{\$40}{(1+0.09/2)^3} + \frac{\$1040}{(1+0.09/2)^4}$$

= 982.06

- Using financial calculator functions:

PMT = 40; FV = 1000; number of periods = 4, I=9%/2 =0.045; PV= ?

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Concept check

What is the price of a 3-year 10% coupon bond that has a YTM = 9%?

G)1025.79

Y)1283.68

R) 820.56

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Yield to Maturity (YTM) and HPR

- What is the YTM of a 4-year zero-coupon bond that is selling at $750?

$$750 = \frac{\$1000}{(1+\text{YTM}/2)^8}$$

Solve for YTM

$$\text{YTM} = 2\left(\left[\frac{\$1000}{750}\right]^{1/8} - 1\right) = 0.073$$

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Yield to Maturity (YTM) and HPR

- What is the YTM of a 2-year 10% coupon bond that is selling at $1050?

$$1050 = \frac{\$50}{1+YTM/2} + \frac{\$50}{(1+YTM/2)^2} + \frac{\$50}{(1+YTM/2)^3} + \frac{\$1050}{(1+YTM/2)^4}$$

- Must solve for YTM using non-linear optimization method
- Using calculator functions:

  PV= -1050; PMT = 50; FV = 1000; number of periods = 4

- Solve for i: i = YTM/2 → YTM = i*2 = 3.634%*2=7.27%

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Yield to Maturity (YTM) and HPR

- An investor holding a bond from period 0 to period 1 obtains his return from the following sources:

  1. Capital gain/loss of the bond

  2. Coupon distributed during the period

  3. Interest from reinvesting the coupons (I)

- Holding period return (HPR) $= \dfrac{P_1 - P_0 + \text{coupon} + I}{P_0}$

where P_n is the bond price in period n

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Yield to Maturity (YTM) and HPR

- What is the 1-year HPR for holding a 2-year 8% semi annual coupon bond face value =1000 that has a YTM= 9% at time 0? YTM remains constant through out the year at 9%. We can reinvest coupon at 9% rate.

1. Price at time 0:

$$P_0 = \frac{\$40}{1+0.09/2} + \frac{\$40}{(1+0.09/2)^2} + \frac{\$40}{(1+0.09/2)^3} + \frac{\$1040}{(1+0.09/2)^4} = 982.06$$

2. After 6 months we get a $40 coupon which we reinvest at 9%

   I = $40* (0.09/2) = 1.8

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Yield to Maturity (YTM) and HPR

3. Price after 1 year

$$P_1 = \frac{\$40}{(1+0.09/2)} + \frac{\$1040}{(1+0.09/2)^2} = 990.64$$

4. Holding period return for one year

$$= \frac{P_1 - P_0 + \text{coupon} + I}{P_0} = \frac{990.64 - 982.06 + 80 + 1.8}{982.06} = 9.2\%$$

Converting to semi-annual rates

$$(1 + r_{annual})^{years} = (1 + \frac{r_{semi}}{2})^{years*2}$$

Semi-annual return =2* ((1+0.092)^0.5 –1) = 9%

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Yield to Maturity (YTM) and HPR

- Let’s consider the holding period return of the same bond, but assuming we can invest the coupon at 2% rather than at 9% .

1. Price at time 0:

$$P_0 = \frac{\$40}{1+0.09/2} + \frac{\$40}{(1+0.09/2)^2} + \frac{\$40}{(1+0.09/2)^3} + \frac{\$1040}{(1+0.09/2)^4} = 982.06$$

2. After 6 months we get a $40 coupon which we reinvest at 2%

   I = $40* (0.02/2) = 0.4

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Yield to Maturity (YTM) and HPR

3. Price after 1 year

$$P_1 = \frac{\$40}{(1+0.09/2)} + \frac{\$1040}{(1+0.09/2)^2} = 990.64$$

4. Holding period return for one year

$$= \frac{P_1 - P_0 + \text{coupon} + I}{P_0} = \frac{990.64 - 982.06 + 80 + 0.4}{982.06} = 9.06\%$$

Semi annual return = 2*((1+0.0906)^0.5 –1) = 8.86%

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Yield to Maturity (YTM) and HPR

- Let’s consider the holding period return of the same bond, but assuming we can invest the coupon at 9%, but the YTM of the bond after 1 year = 8%.

1. Price at time 0:

$$P_0 = \frac{\$40}{1+0.09/2} + \frac{\$40}{(1+0.09/2)^2} + \frac{\$40}{(1+0.09/2)^3} + \frac{\$1040}{(1+0.09/2)^4} = 982.06$$

2. After 6 months we get a $40 coupon which we reinvest at 9%

   I = $40* (0.09/2) = 1.8

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Yield to Maturity (YTM) and HPR

3. Price after 1 year

$$P_1 = \frac{\$40}{(1+0.08/2)} + \frac{\$1040}{(1+0.08/2)^2} = 1000$$

4. Holding period return for one year

$$= \frac{P_1 - P_0 + \text{coupon} + I}{P_0} = \frac{1000 - 982.06 + 80 + 1.8}{982.06} = 10.16\%$$

Semi annual return = 2*((1+0.1016)^0.5 –1)= 9.914%

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Yield to Maturity (YTM) and HPR

- The HPR equals the return an investor obtains by holding a bond for a period
- The HPR equals the YTM only when YTM remains that same and the coupons can be reinvested at the YTM rate

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Concept check

What is the 1-year HPR for holding a 2-year 10% semi annual coupon bond face value =1000 that has a YTM= 9% at time 0? We can reinvest coupon at 9% rate. YTM of this coupon at the end of one year is 10%.

G) HPR is higher than 9%

Y) HPR equals 9%

R) HPR is lower than 9%

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Bond prices over time

- Consider 3 bonds (par 1000)

1. A 1-year 4% coupon bond with YTM=4%. The price is 1000. This is a par bond
2. A 1-year 6% coupon bond with YTM = 4%. The price is 1019.42. This is a premium bond
3. A 1-year 2% coupon bond with YTM = 4%. The price is 980.58. This is a discount bond.

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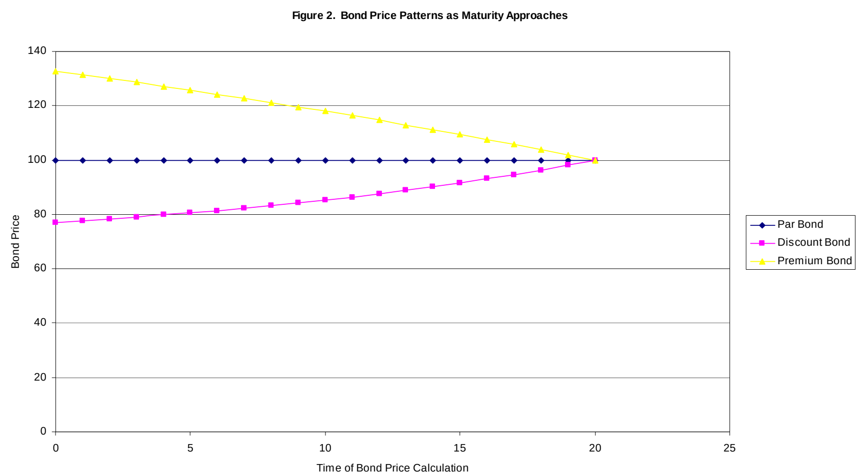
Bond prices over time



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Bond prices as bonds mature

- The price of a premium bond declines to par at maturity. An investor of this bond is compensated for the capital loss by a higher coupon rate that the require rate of return.

- The price of a discount bond increases to par at maturity. This capital gain is compensation to an investor due to the lower coupon rate than the require rate of return.

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Summary

- Compounding interest
- Bond as a portfolio of cash flows
- Yield to Maturity (YTM) and holding period return (HPR)
- Bond prices over time

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