Wednesday, September 24, 2014
10:56 AM

DEMAND FOR RISKY ASSETS

AN INVESTOR FACES WITH AN ALLOCATION PROBLEM :

GIVEN HIS LIMITED RESOURCES, HOW DOES HE ALLOCATE HIS MONEY AMONG RISK-FREE ASSETS AND RISKY ASSETS.

FACT #1 HIGHER "MEAN RETURN" (OR EXPECTED RETURN) IS PREFERED. SO "MEAN RETURN" IS GOOD, i.e., MORE IS BETTER.

FACT #2 LESS VARIATION IN RETURN IS PREFERED (LESS RISK)

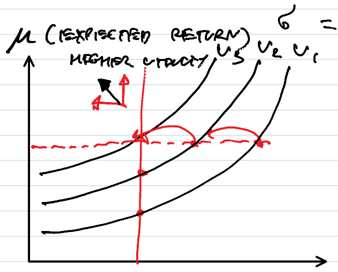
SO RISK IS BAD, i.e., LESS IS PREFERED TO MORE.

FACT #3 HIS PREFERENCE CAN BE DESCRIBED BY

$U(\mu, \sigma)$

WHERE μ = EXPECTED RETURN FROM PORTFOLIO,

σ = DEGREE OF RISK OF THE PORTFOLIO



HIGHER μ IS PREFERED
LOWER σ IS PREFERED

$$MRS = \frac{d\mu}{d\sigma} = \frac{\partial U / \partial \sigma}{\partial U / \partial \mu} \quad \left[\text{OR } \frac{dU/d\sigma}{dU/d\mu} \right]$$

PROOF : $U(\mu, \sigma)$

$$dU = \frac{dU}{d\mu} d\mu + \frac{dU}{d\sigma} d\sigma = 0$$

$$\frac{\partial U}{\partial \mu} d\mu = - \frac{\partial U}{\partial \sigma} d\sigma$$

$$\frac{d\mu}{d\sigma} = - \frac{\partial U / \partial \sigma}{\partial U / \partial \mu}$$

$\frac{\partial U}{\partial \sigma}$ ⇒ MARGINAL UTILITY OF σ

$\frac{\partial U}{\partial \mu}$ ⇒ MARGINAL UTILITY OF μ

BUDGET LINE OF AN INVESTOR

START W/ BASIC CONCEPT ABOUT A RANDOM VARIABLE

A RANDOM VARIABLE (R.V.) w TAKES VALUE $w_1, w_2, w_3, \ldots, w_S$ W/ PROBABILITIES $\pi_1, \pi_2, \pi_3, \ldots, \pi_S$.

NOTE : $\pi_1 + \pi_2 + \ldots + \pi_S = 1$,

- THE MEAN (EXPECTED VALUE) OF THE DISTRIBUTION IS

$$E[w] = \mu_w = \sum_{s=1}^{S} \pi_s \cdot w_s ,$$

- THE DISTRIBUTION'S VARIANCE IS THE R.V.'S AVERAGE SQUARED DEVIATION FROM ITS MEAN.

$$\mathrm{var}[w] = \sigma_w^2 = \sum_{s=1}^{S} (w_s - \mu_w)^2 \pi_s .$$

VARIANCE MEASURES THE R.V.'S VARIATION (= SPREAD)

$$S.D. = \sigma_w = \sqrt{\sum_{s=1}^{S} (w_s - \mu_w)^2 \cdot \pi_s}$$

WE CAN USE σ_w TO MEASURE THE R.V.'S VARIABILITY.

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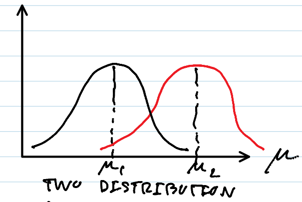
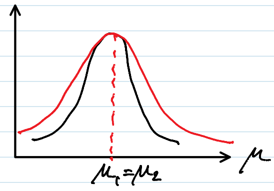


TWO DISTRIBUTION W/ THE SAME σ BUT DIFFERENT μ.



TWO DISTRIBUTION W/ THE SAME μ BUT DIFFERENT σ.

BUDGET CONSTRAINT

CONSIDER TWO ASSETS
- RISKY-FREE ASSETS (EX. GOVT. BONDS, TREASURY BILLS)
- RISKY ASSETS (EX. STOCKS)

RISK-FREE ASSET'S RETURN $= r_f$

RISKY STOCK'S RATE-OF-RETURN $= m_s$ IF STATE s OCCURS

W/ PROBABILITY $= \pi_s$

SO, RISKY STOCK'S MEAN RATE-OF-RETURN $= r_m = \sum_{s=1}^{S} \pi_s \cdot m_s$

A BUNDLE CONTAINING SOME OF RISK-FREE ASSET AND SOME OF RISKY ASSET IS "PORTFOLIO"

SUPPOSE x IS A FRACTION OF HIS WEALTH USED TO BUY THE RISKY ASSET (EX: STOCK)

$1 - x$ IS A FRACTION OF HIS WEALTH USED TO BUY THE RISK-FREE ASSET.

W/ A GIVEN x, THE AVERAGE RATE-OF-RETURN OF THE PORTFOLIO IS

$$\boxed{r_x = x \cdot r_m + (1-x) r_f} \quad \text{— (1)}$$

- IF $x = 0$, $r_x = r_f$ [WHEN PUTTING ALL MONEY INTO RISK-FREE ASSET]
- IF $x = 1$, $r_x = r_m$ [WHEN PUTTING ALL MONEY INTO RISKY ASSET]
- WHEN x IS HIGHER, r_x WILL BE HIGHER, VICE VERSA. IN OTHER WORDS, MORE RISKY STOCK GIVES HIGHER AVERAGE RATE-OF-RETURN OF THE PORTFOLIO.

HOW ABOUT RISKINESS OF THE PORTFOLIO?

$$\sigma_x^2 = \sum_{s=1}^{S} \left([x m_s + (1-x) r_f] - \underbrace{r_x}_{= x r_m + (1-x) r_f} \right)^2 \cdot \pi_s$$

$$\sigma_x^2 = \sum_{s=1}^{S} \left(x m_s + \cancel{(1-x) r_f} - x r_m - \cancel{(1-x) r_f} \right)^2 \cdot \pi_s$$

$$= \sum_{s=1}^{S} (x m_s - x r_m)^2 \cdot \pi_s$$

$$= x^2 \sum_{s=1}^{S} (m_s - r_m)^2 \cdot \pi_s$$

$$\sigma_x^2 = x^2 \cdot \sigma_m^2$$

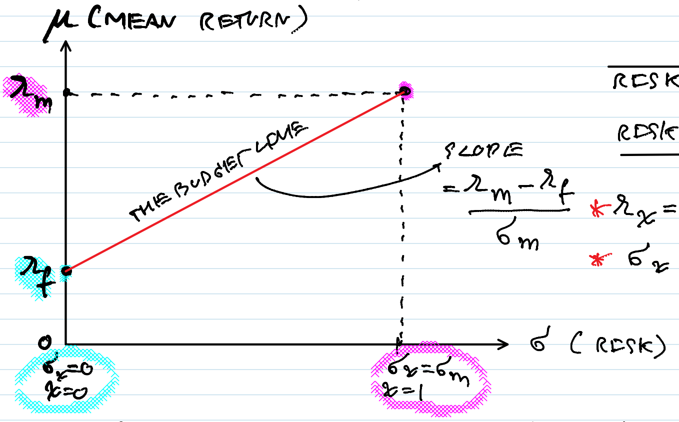
$$\text{VAR}(x) = \sum_{i=1}^{n} (x_i - \bar{x})^2 \cdot \pi_i$$

$$\sigma_x^2 = x^2 \cdot \sigma_m^2$$

$$\boxed{\sigma_x = x \cdot \sigma_m} \quad \text{— (2)}$$

σ_x → S.D.

- IF $x = 0$, $\sigma_x = 0$ [= THE PORTFOLIO CONTAINS NO RISK]
- IF $x = 1$, $\sigma_x = \sigma_m$ [i.e., THE RISKINESS OF THE PORTFOLIO IS DETERMINED BY RISKINESS OF THE RATE-OF-RETURN ON STOCK.]
- AGAIN, THE EQUATION (2) SAYS THAT WHEN $x \uparrow$, $\sigma_x \uparrow$.



	FRACTION	RETURN	RISK
RISKY	x	r_m	σ_m
RISK-FREE	$1-x$	r_f	$0 = \sigma_f$
		$\downarrow$	$\downarrow$
		$r_m > r_f$	$\sigma_m > \sigma_f = 0$

* $r_x = x r_m + (1-x) r_f$ (1)

* $\sigma_x = x\,\sigma_m$ (2)

BL:

$$r_x = x r_m + (1-x) r_f \quad \text{— (1)}$$

$$r_x = r_f + x (r_m - r_f)$$

$$r_x = r_f + \frac{\sigma_x}{\sigma_m} (r_m - r_f) \quad \text{SINCE } \sigma_x = x \cdot \sigma_m.$$

$$\boxed{r_x = \underbrace{r_f}_{\text{INTERCEPT Y-AXIS}} + \underbrace{\left(\frac{r_m - r_f}{\sigma_m}\right)}_{\text{SLOPE OF BL}} \cdot \sigma_x} \quad \text{(BL EQUATION)}$$

FACT #1 $\quad \frac{d r_x}{d \sigma_x} > 0 \iff$ HIGH RISK, HIGH RETURN

FACT #2 $\quad \frac{r_m - r_f}{\sigma_m}$ IS SO CALLED "PRICE OF RISK"

= EXTRA COMPENSATION PER UNIT OF RISK FOR TAKING ON THE RISKY ASSET

Ex:

ASSETS	EXP. RETURN	RISK (σ)
STOCK 1	30%	40%
STOCK 2	46.67%	73.33%

$$\text{PRICE OF RISK}_{\text{STOCK 1}} = \frac{30\% - 10\%}{40} = 0.5$$

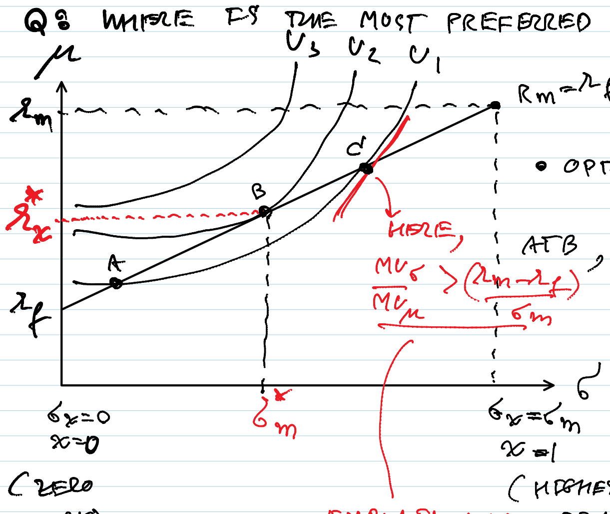
PRICE OF RISK

PRICE OF RISK$_{\text{STOCK 1}}$ = $\frac{[illegible] - [illegible]}{40}$ = 0.5

PRICE OF RISK$_{\text{STOCK 2}}$ = $\frac{46.67 - 10\%}{73.33}$ = 0.5

} PRICE OF RISK ARE THE SAME.

Q: WHERE IS THE MOST PREFERRED BUNDLE?



$R_m = r_f + \frac{(r_m - r_f)}{\sigma_m} \cdot \sigma_x$

- OPTIMAL CHOICE OCCURS AT B (σ_m^*, r_m^*)

AT B, SLOPE OF IC = SLOPE OF BL

$$\boxed{\underbrace{\frac{MU_\sigma}{MU_\mu}}_{MRS_{\sigma,\mu}} = \underbrace{\frac{r_m - r_f}{\sigma_m}}_{=\text{PRICE OF RISK}}}$$

EXPLAIN WHY C IS NOT AN OPTIMAL CHOICE?

EXERCISE

	FRACTION	E(RETURN)	RISK
RISKY ASSET	$x = 0.8$	$r_m = 0.2$ (=20%)	$\sigma_m = 3$
RISK-FREE ASSET	$1 - x = 0.2$	$r_f = 0.04$ (=4%)	$\sigma_f = 0$

FIND r_x AND σ_x.