

EE431 Economics of Financial Markets and Institutions

Problem Set 4

1. Use the following information to answer all parts. Assume that all assumptions of CAPM hold

Assets	Expected Return (%)	Standard Deviation (%)
Market Portfolio	7	10
Risk-free Asset	1	

- Market portfolio consists of 0.7 of risky asset X and 0.3 of risky asset Y.

- (a) What is the value of standard deviation of the risk-free asset? Does the question give you enough information?

Solution. In CAPM there are only 2 types of assets; risky and risk-free asset, and SD refers as level of risk of that asset. Which Risk-free asset mean an asset that provides a constant future return, SD of constant return is 0. So yes, there is enough information; since CAPM hold SD of Risk-free Asset will be 0

- (b) Suppose Stefan has 200US\$ and he wants 4% rate of return. How much should the investor invest his fund on risk free asset, asset X and asset Y ?

Solution.

$$\begin{aligned}
 ER &= w_f R^f + w_m E(R^m), \\
 &= w_f(0.01) + w_m(0.07). \\
 0.04 &= 0.01w_f + w_m 0.07.
 \end{aligned}$$

As weights sum up to 1,

$$\begin{aligned}
 w_f + w_m &= 1. \\
 w_m &= 1 - w_f.
 \end{aligned}$$

Substitute $w_m = 1 - w_f$ into the equation,

$$\begin{aligned}
 0.04 &= 0.01w_f + w_m 0.07. \\
 0.04 &= 0.01w_f + 0.07(1 - w_f) \\
 &= 0.01w_f + 0.07 - 0.07w_f \\
 0.06w_f &= 0.07 - 0.04 \\
 w_f &= \frac{0.03}{0.06} \\
 &= 0.5
 \end{aligned}$$

He will invests 0.5 of his wealth ($0.5(200) = \text{US } \$100$) in risk free asset and the rest 0.5 of his wealth ($0.5(200) = \text{US } \$100$) in the market portfolio.

Since the market portfolio consists of 0.7 of risky asset X and 0.3 of risky asset Y.

- Stefan will invest $0.7 \times 100 = 70$ dollars of his wealth in risky asset X and
- Stefan will invest $0.3 \times 100 = 30$ dollars of his wealth in risky asset Y.

- (c) What is the standard deviation of the portfolio in question (a)?

Solution.

Standard deviation of portfolio in question a $= w_m \sigma_m = 0.5 \times 10\% = 5\%$.

- (d) Suppose Stefanie has 100US\$ and she wants 10% rate of return. How much should she invest her fund on risk free asset, asset X and asset Y ?

$$\begin{aligned}
ER &= w_f R^f + w_m E(R^m), \\
&= w_f(0.01) + w_m(0.07). \\
0.10 &= 0.01w_f + w_m 0.07.
\end{aligned}$$

As weights sum up to 1,

$$\begin{aligned}
w_f + w_m &= 1. \\
w_m &= 1 - w_f.
\end{aligned}$$

Substitute $w_m = 1 - w_f$ into the equation,

$$\begin{aligned}
0.10 &= 0.01w_f + w_m 0.07. \\
0.10 &= 0.01w_f + 0.07(1 - w_f) \\
&= 0.01w_f + 0.07 - 0.07w_f \\
0.06w_f &= 0.07 - 0.10 \\
&= -0.03 \\
w_f &= \frac{-0.03}{0.06} \\
&= -0.5
\end{aligned}$$

Shee will invests -0.5 of his wealth $(-0.5(100) = -\text{US } \$50)$ in risk free asset. She will borrow $\text{US } \$50$ and invest $1.5 \times 100 = 150 \text{ US\$}$ in the market portfolio.

Since the market portfolio consists of 0.7 of risky asset X and 0.3 of risky asset Y.

- Stefan will invest $0.7 \times 150 = 105$ dollars of his wealth in risky asset X and
- Stefan will invest $0.3 \times 150 = 45$ dollars of his wealth in risky asset Y.

(e) What is the standard deviation of the portfolio in quation (c)?

Solution.

Standard deviation of portfolio in question a $= w_m \sigma_m = 1.5 \times 10\% = 15\%$.

(f) Write down CML equation.

$$CML : ER_p = R^f + \left[\frac{ER^m - R^f}{\sigma_m} \right] \sigma_p = 0.01 + \left[\frac{0.07 - 0.01}{0.10} \right] \sigma_p = 0.01 + 0.6\sigma_p$$

(g) Indicate Stefan's choice and Stefanie's choice, riskfree asset and market portfolio on CML. Grapically illustrate.

