

EE320 (2/2012)

INTRODUCTORY MATHEMATICAL ECONOMICS

STATIC (or EQUILIBRIUM) ANALYSIS

(Part 1)

Topics

- Meaning of Equilibrium
- Partial Market Equilibrium: Linear Models
 - Linear models in economics
 - Break-even analysis
 - Individual and market demand
 - Individual and market supply
 - Partial market equilibrium
- Partial Market Equilibrium: Nonlinear Models
- General Market Equilibrium
- Applications of Market Equilibrium
 - Excise tax and market equilibrium
 - Elasticity concepts

Meaning of Equilibrium

- An equilibrium is *“a constellation of selected interrelated variables so adjusted to one another that no inherent tendency to change prevails in the model which they constitute.”*

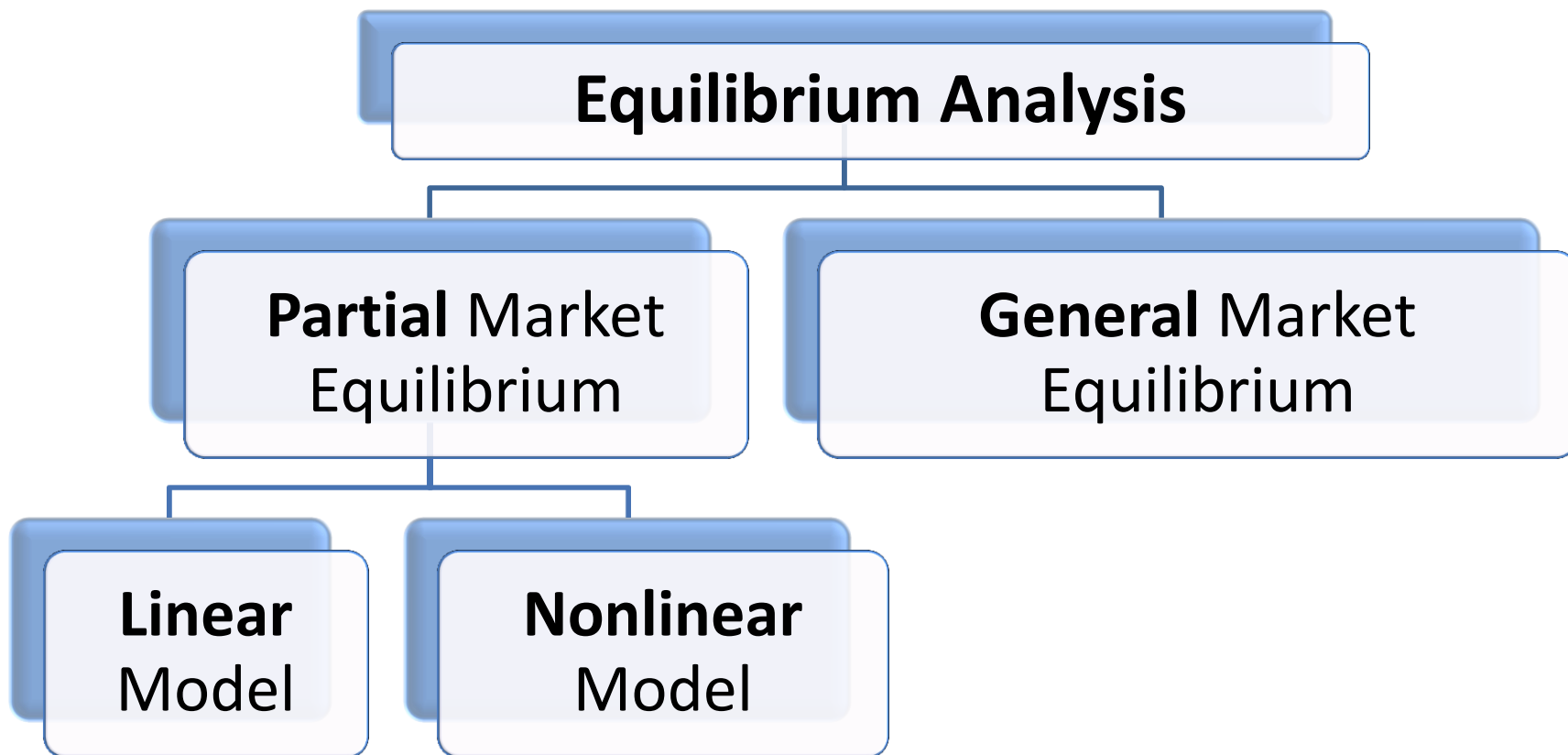
- Machlup, F. (1958).

- In essence, an **equilibrium** for a specified model is a situation characterized by **a lack of tendency to change**.
- Due to the lack of tendency to change, the **analysis of equilibrium** is referred to as **statics**.

Equilibrium Analysis

- However, an equilibrium does *not* necessarily constitute a desirable or an ideal state of affairs; *i.e.*, there are both good and bad equilibria.
- “Desirable” equilibrium is referred to as “goal” equilibrium (to be discussed in optimization problems).
- “Non-goal” equilibrium, which is discussed in this topic, is a result from a process of interaction and adjustment of economic forces.
 - Equilibrium in a market under given demand and supply (Lecture 3.1)
 - Equilibrium of national income given conditions of consumption and investment (Lecture 3.2)

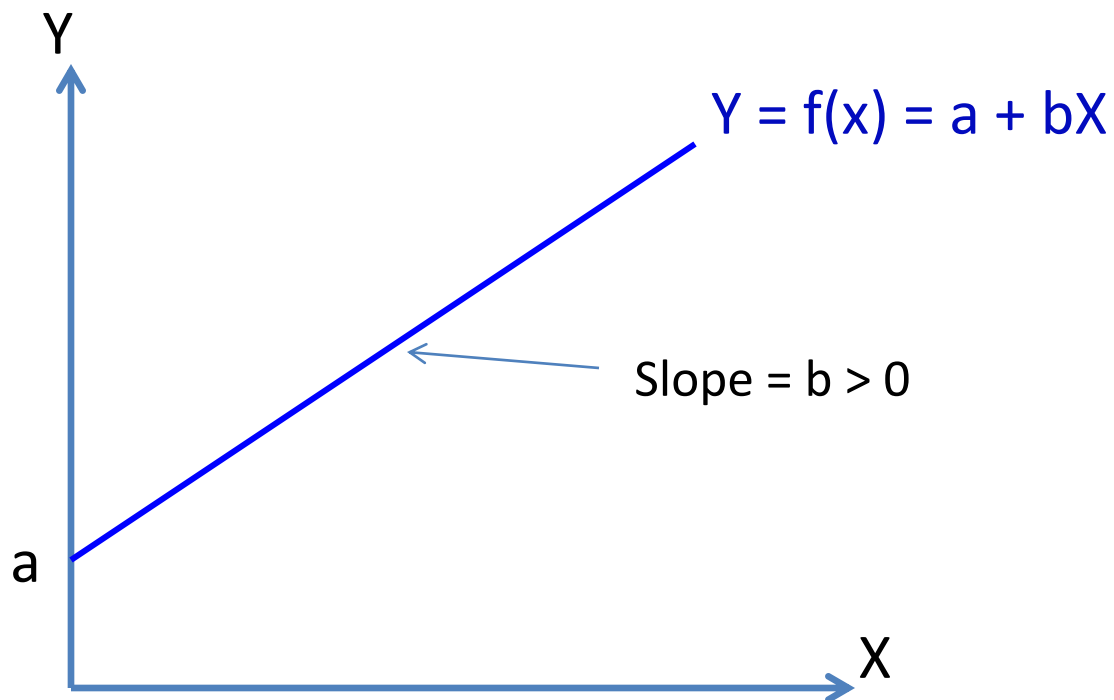
Types of Market Equilibrium Analysis



PARTIAL MARKET EQUILIBRIUM: LINEAR MODELS

Linear Models in Economics

- General form of a linear model: $Y = a + bX$



Example: Cost Function (1)

- Theory: Cost of a firm consists of fixed costs and variable costs.
- In a mathematical model:

$$TC = TFC + TVC$$

$$TC(Q) = TFC + TVC(Q) \rightarrow TC = f(Q)$$

- Assumption: Total cost function has a linear relationship.

$$TC(Q) = f(Q) = TFC + cQ,$$

where c is a constant.

- With this theoretical model and given data, we can approximate the cost function.
 - Example: Suppose $TC = \$5,000$ when $Q = 50$ units, and $TC = \$9,000$ when $Q = 100$ units. What is the cost function? What is the total cost if $Q = 120$ units?

Example: Cost Function (2)

- Answer: 1) $TC = 1000 + 80Q$.
2) When $Q = 120$, $TC = \$10,600$

Breakeven Analysis

- The **breakeven** level for a product is a level at which **total revenue (TR) equals total costs (TC)**; i.e., **normal profit (π) is 0**.

- Theoretical model:

$$\pi \equiv TR - TC = 0$$

$$TR = PQ$$

$$TC = a + cQ$$

- Empirical model:

Suppose the 2nd-year BE students want to raise a fund by selling T-shirts. The price of a T-shirt purchased from a factory is 100 baht, and the cost of travelling to the factory and other fixed cost is 2000 baht. The students can sell T-shirts at a price 120 baht per piece. How many T-shirts should these students sell in order to breakeven?

Illustration of a Breakeven Analysis

Individual and Market Demand

- Theory - Demand for a good is determined by many factors:

$$Q_d^x = f(P^x, P^y, I, \text{taste, etc.})$$

- **Individual demand** for a good as a function of its price:

$$Q_d = f(P), \quad \text{ceteris paribus}$$

$$Q_d = a - bP, \quad \text{where } b > 0.$$

- **Market demand:**

$$Q_{Md} = Q_{d1} + Q_{d2} + \dots + Q_{dn}$$

(Suppose that there are n buyers.)

- Example:

Suppose $Q_{d1} = 500 - 4P$ and $Q_{d2} = 1000 - 4P$. What does a market demand function look like?

Illustration of a Market Demand Function

Individual and Market Supply

- **Individual supply** of a good as a function of its price:

$$Q_s = f(P), \quad \text{ceteris paribus}$$

$$Q_s = c + dP, \quad \text{where } d > 0.$$

- **Market supply:**

$$Q_{Ms} = Q_{s1} + Q_{s2} + \dots + Q_{sq}$$

(Suppose that there are q sellers.)

- Example:

Suppose that there are 10 sellers, and each seller i has the same supply function: $Q_{si} = 23 + P$. What does the market supply function look like?

Illustration of a Market Supply Function

Some Notations:

System of Equations /Simultaneous Equations

- **Systems of equations** are two or more equations containing common multiple variables.

Example: $Q_d = a - bP$, where $b > 0$. - (1)

$Q_s = c + dP$, where $d > 0$. - (2)

➤ Note that any value of P makes this system correct.

- **Simultaneous equations** are two or more equations in which the common variables are jointly determined.

Example: $Q_d = a - bP$, where $b > 0$. - (1)

$Q_s = c + dP$, where $d > 0$. - (2)

$Q_d = Q_s$ - (3)

- This system contains simultaneous equations because there is a unique value of P^* that must be true in all equations.
- Equation (3) specifies the equilibrium condition in the market.

Partial Market Equilibrium

- Partial market equilibrium analysis deals with price determination in an isolated market. That is, only one commodity is being considered.
- All relevant factors except the price in question (e.g. prices of all substitutes and complements and income levels of consumers) are constant.
- This analysis can be used to determine changes in other exogenous parameters, e.g. the impact of a tax, without worrying about side-effects from other markets.
 - The analysis of a change in the equilibrium condition as a result of a change in the parameter in the model is called “comparative static analysis.”

Partial Market Equilibrium: A Linear Model

- **Equilibrium** occurs in the market *if and only* if the **excess demand is zero** (i.e. if and only if the market is clear).
- 1st step: Construct the model.
 - One conditional equation: $Q_d = Q_s$.
 - Two behavioral equations:
$$Q_d = a - bP, \quad (a, b > 0)$$
$$Q_s = -c + dP, \quad (c, d > 0)$$
 - a , b , c , and d are parameters, whereas Q_d , Q_s , and P are endogenous variables.
- 2nd step: Solve the model for the 3 endogenous variables (Q_d^* , Q_s^* , and P^*). These values are the equilibrium values of the model.

Solution by Elimination of Variables

- Show that $P^* = \frac{a+c}{b+d}$ and $Q^* = \frac{ad-bc}{b+d}$.

PARTIAL MARKET EQUILIBRIUM: NONLINEAR MODELS

Example: A Nonlinear Model

- Suppose that a market demand function is nonlinear. The model can be rewritten as:

$$Q_d = a - bP^2, \quad (a, b > 0)$$

$$Q_s = -c + dP, \quad (c, d > 0)$$

$$Q_d = Q_s.$$

$$\rightarrow -bP^2 + dP - a - c = 0$$

- Numerical example: $Q_d = 4 - P^2$ and $Q_s = 4P - 1$. What are the equilibrium price and quantity in this model?

Quadratic Formula

- Given a quadratic equation of the form

$$aX^2 + bX + c = 0,$$

there are 2 roots, which can be obtained from the quadratic formula:

$$X_1^*, X_2^* = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If $b^2 - 4ac > 0$, $X_1^* \neq X_2^*$.

If $b^2 - 4ac = 0$, $X_1^* = X_2^*$.

If $b^2 - 4ac < 0$, no real value root exists.

- Own-practice: Use the process called “completing the square” to show the 2 roots of a quadratic equation.
 - Hint: Start by dividing both sides by a .

Graphic Solution of a Nonlinear Model

- $D = \{(P, Q) \mid Q = 4 - P^2\}$
- $S = \{(P, Q) \mid Q = 4P - 1\}$

GENERAL MARKET EQUILIBRIUM

General Market Equilibrium

- General market equilibrium deals with simultaneous equilibria in a group of related markets, e.g. input and output markets.
 - Ex: A change in the price of an input good may affect the price of the output good, with a possible subsequent effect on the price of the input good.
- The equilibrium condition of an n -commodity market model involves n equations:

$$E_i \equiv Q_{di} - Q_{si} = 0 \quad (i = 1, 2, \dots, n)$$

$$Q_{di} = Q_{di}(P_1, P_2, \dots, P_n)$$

$$Q_{si} = Q_{si}(P_1, P_2, \dots, P_n)$$

- If a solution exists, there will be a set of price P_i^* and quantity Q_i^* such that all n equations in the equilibrium conditions are simultaneously satisfied.

2-Commodity Market Model

- Market for good 1:

$$Q_{d1} - Q_{s1} = 0$$

$$Q_{d1} = a_0 + a_1 P_1 + a_2 P_2$$

$$Q_{s1} = b_0 + b_1 P_1 + b_2 P_2$$

- Market for good 2:

$$Q_{d2} - Q_{s2} = 0$$

$$Q_{d2} = \alpha_0 + \alpha_1 P_1 + \alpha_2 P_2$$

$$Q_{s2} = \beta_0 + \beta_1 P_1 + \beta_2 P_2$$

- Let $c_i \equiv a_i - b_i$ and $\gamma_i \equiv \alpha_i - \beta_i$

➤ Show that $P_1^* = \frac{c_2 \gamma_0 - c_0 \gamma_2}{c_1 \gamma_2 - c_2 \gamma_1}$ and $P_2^* = \frac{c_0 \gamma_1 - c_1 \gamma_0}{c_1 \gamma_2 - c_2 \gamma_1}$

2-Commodity Market Model (cont'd)

Numerical Example (Own Practice!)

- Suppose that the demand and supply functions of a two-commodity market model are as follows:

$$Q_{d1} = 8 - 3P_1 + P_2 \quad \text{and} \quad Q_{s1} = 2 + 2P_1$$

$$Q_{d2} = 12 + 5P_1 - 2P_2 \quad \text{and} \quad Q_{s2} = 2 + 3P_2$$

Find P_i^* and Q_i^* ($i = 1, 2$).

Answer:

$$(Q_1^*, P_1^*) = (6, 2)$$

$$(Q_2^*, P_2^*) = (14, 4)$$

APPLICATIONS OF MARKET EQUILIBRIUM

Excise Tax and Market Equilibrium

- **Excise tax** (a.k.a. excise duty) is a type of **tax charged on goods produced within the country**, as opposed to custom duties which are charged on goods from outside the country.
- Two types of excise tax:
 1. **Specific tax** – tax imposed based on **physical units** (e.g. pieces, gallons, or individual items) of the taxed item good.
 - $P^t = P + T$, where P is price without tax.
 2. **Ad-valorem tax** – tax imposed based on the **monetary value** of the taxed item. Suppose that a $t\%$ ad-valorem tax is imposed.
 - $P^t = (1+t)P$, where P is price without tax.

Specific Tax (imposed on consumers)

- Suppose the government imposes a T baht specific tax on a good. Find the P^* , Q^* , P^t (market price), and tax revenue.

➤ Without tax: $Q_D = a - bP$ and $Q_S = -c + dP$

➤ With tax: $P_D' = P + T = P^t$ and $P_S = P$,

At the new equilibrium, $Q_D' = Q_S$.

$$a - b(P + T) = -c + dP$$

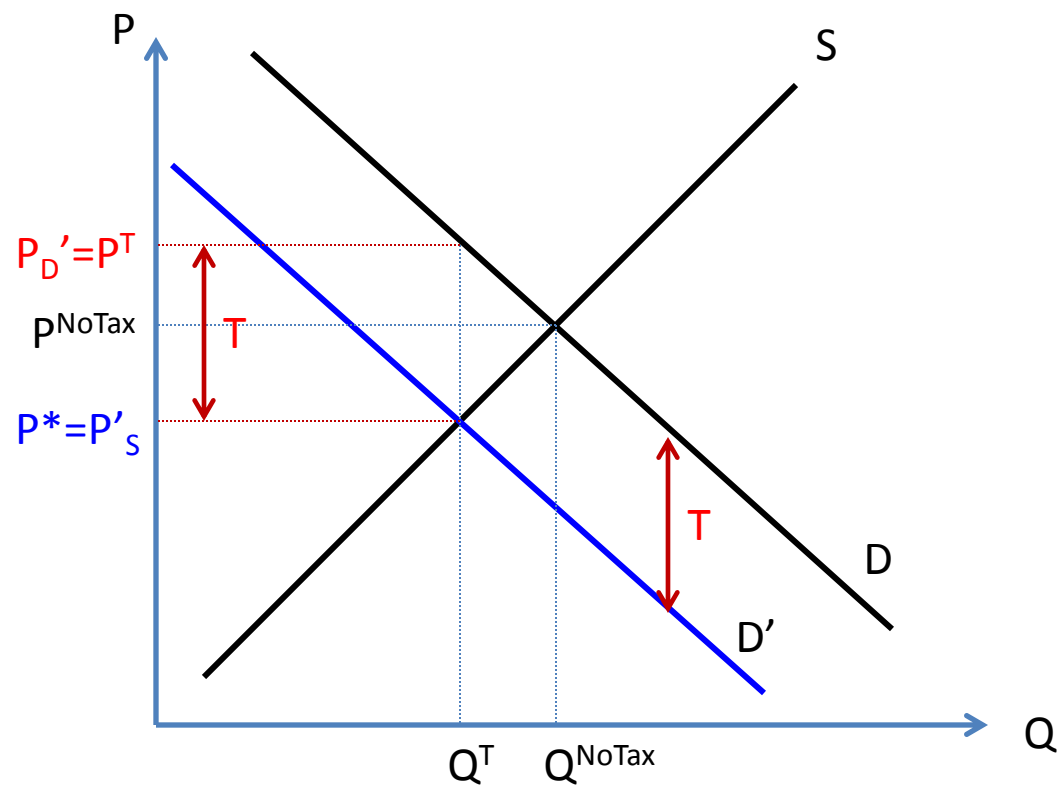
➔ $P^* = \frac{a + c - bT}{b + d} = P_S'$: Price received by producer

➔ $P^t = P + T = \frac{a + c + dT}{b + d} = P_D'$: Price (tax included) paid by consumer

Specific Tax (imposed on consumers) – Cont'd

- Consumer tax burden: $P^t - P^{NoTax} = \frac{dT}{b + d}$
- Producer tax burden: $P^{NoTax} - P'_S = -\frac{bT}{b + d}$
- Equilibrium quantity after tax: $Q^* = \frac{ad - bc - bdT}{b + d}$
- Tax revenue: $Tax = TQ^* = \frac{(ad - bc)T - bdT^2}{b + d}$

Graph: Specific Tax (imposed on consumers)



Specific Tax (imposed on producers)

- Suppose the government imposes a T baht specific tax on a good. Find the P^* , Q^* , P^t (price excluding tax received by producer), and tax revenue.

➤ Without tax: $Q_D = a - bP$ and $Q_S = -c + dP$

➤ With tax: $P_S' = P - T = P^t$ and $P_D = P$,

At the new equilibrium, $Q_D = Q_S'$.

$$a - bP = -c + d(P - T)$$

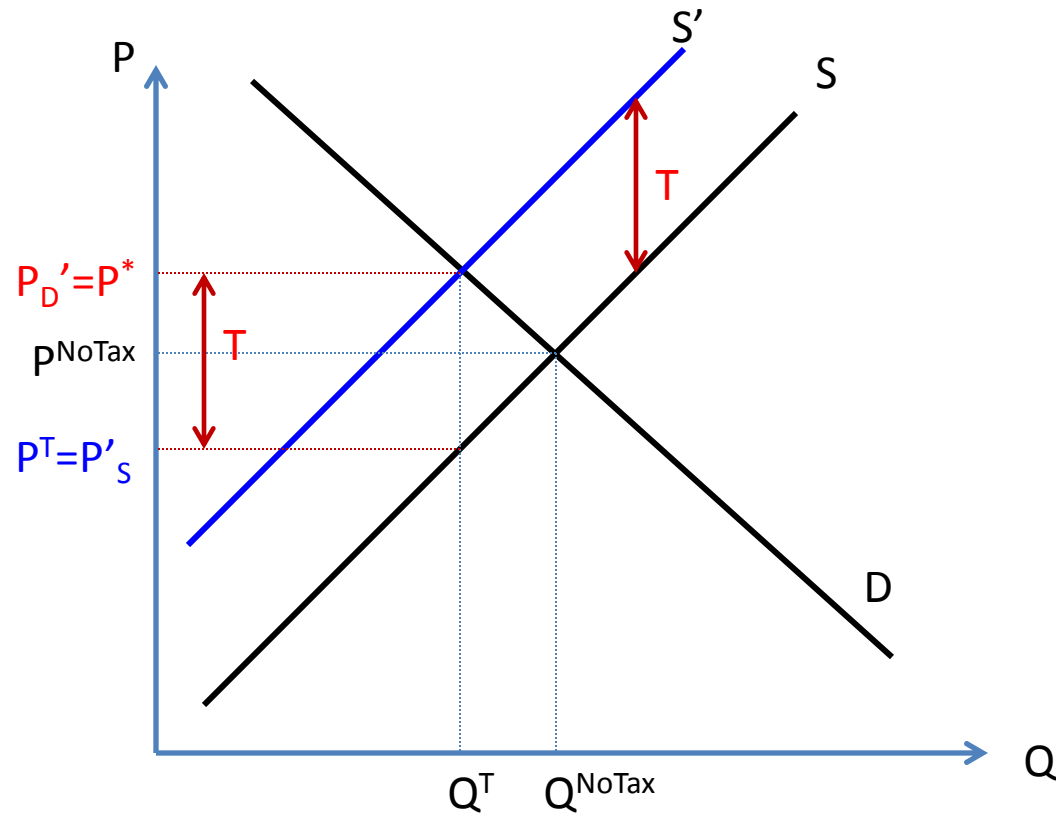
➔ $P^* = \frac{a + c + dT}{b + d} = P_D'$: Price (tax included) paid by consumer

➔ $P^T = P_D' - T = \frac{a + c - bT}{b + d} = P_S'$: Price received by producer

Specific Tax (imposed on producers) – Cont'd

- Consumer tax burden: $P_D' - P^{NoTax} = \frac{dT}{b + d}$
 - Producer tax burden: $P^{NoTax} - P_S' = -\frac{bT}{b + d}$
 - Equilibrium quantity after tax: $Q^* = \frac{ad - bc - bdT}{b + d}$
 - Tax revenue: $Tax = TQ^* = \frac{(ad - bc)T - bdT^2}{b + d}$
- The results (P_D' , P_S' , Q^* , Tax) are exactly the same as regardless of whom the government imposes the specific tax on.

Graph: Specific Tax (imposed on producers)



Ad-Valorem Tax (imposed on consumers)

- Suppose the government imposes a $t\%$ ad-valorem tax on a good. Find the P^* , Q^* , and P^t (market price).

➤ Without tax: $Q_D = a - bP$ and $Q_S = -c + dP$

➤ With tax: $P_D' = P + tP = (1+t)P = P^t$ and $P_S = P$,

At the new equilibrium, $Q_D' = Q_S$.

$$a - b[(1+t)P] = -c + dP$$

➔ $P^* = \frac{a + c}{b(1+t) + d} = P_S'$: Price received by producer

➔ $P^t = P_S' + T = \frac{(1+t)(a + c)}{b(1+t) + d} = P_D'$: Price (tax included) paid by consumer

➔ Equilibrium quantity after tax: $Q^* = \frac{ad - (1+t)bc}{b(1+t) + d}$

Ad-Valorem Tax (imposed on producers)

- Suppose the government imposes a $t\%$ ad-valorem tax on a good. Find the P^* , Q^* , and P^t (price excluding tax received by producer).

➤ Without tax: $Q_D = a - bP$ and $Q_S = -c + dP$

➤ With tax: $P_D = P$ and $P_S' = P - tP = (1-t)P$,

At the new equilibrium, $Q_D = Q_S'$.

$$a - bP = -c + d[(1-t)P]$$

➔ $P^* = \frac{(a + c)}{b + d(1-t)} = P_D'$: Price paid by consumer

➔ $P^t = P_D' - T = \frac{(1-t)(a + c)}{b + d(1-t)} = P_S'$: Price received by producer

➔ Equilibrium quantity after tax: $Q^* = \frac{ad(1-t) - bc}{b + d(1-t)}$

Elasticity Concept

- Recall the concept of price elasticity:

$$E_p = \frac{\% \Delta Q}{\% \Delta P} = \frac{\Delta Q / Q}{\Delta P / P} = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q}$$

- When Δ is very small, we use notation d instead of Δ : $E_p = \frac{dQ}{dP} \cdot \frac{P}{Q}$

- Price elasticity of demand:

$$\eta = - \frac{dQ_d}{dP} \cdot \frac{P}{Q_d}$$

- Price elasticity of supply:

$$\varepsilon = \frac{dQ_s}{dP} \cdot \frac{P}{Q_s}$$

Tax Incidence and Price Elasticity (1)

- Case 1 - Specific tax imposed on consumers.
- Consumer's price: $P^t = \frac{a + c + dT}{b + d}$. Find $dP^t/dT = ?$

Suppose $\Delta T = T_1 - T_0$. We want to know $\Delta P^t = ?$

$$\text{➤ } P^{t0} = \frac{a + c + dT_0}{b + d} \text{ and } P^{t1} = \frac{a + c + dT_1}{b + d}$$

$$\text{➤ } \Delta P^t = P^{t1} - P^{t0} = \frac{a + c + dT_1}{b + d} - \frac{a + c + dT_0}{b + d}$$

$$\text{➤ } \Delta P^t = \frac{d}{b + d} (T_1 - T_0) = \frac{d}{b + d} \Delta T$$

➔ The impact of tax change on equilibrium price when tax is imposed on consumer is:

$$\frac{\Delta P^t}{\Delta T} = \frac{d}{b + d} \quad \text{-- (1)}$$

Specific tax imposed on consumers (cont'd)

- Recall $Q_D = a - bP$ and $Q_S = -c + dP$

➤ Thus, $\Delta Q_D / \Delta P = -b$ and $\Delta Q_S / \Delta P = d$.

- From (1),
$$\frac{\Delta P^t}{\Delta T} = \frac{d}{b + d} = \frac{\Delta Q_S / \Delta P}{-\Delta Q_D / \Delta P + \Delta Q_S / \Delta P} \quad \text{-- (2)}$$

- Recall $\eta = \frac{\Delta Q_d}{\Delta P} \cdot \frac{P}{Q_d}$ and $\varepsilon = \frac{\Delta Q_s}{\Delta P} \cdot \frac{P}{Q_s}$

- Rearrange (2) and we obtain:

$$\frac{dP^t}{dT} \approx \frac{\Delta P^t}{\Delta T} = \frac{\varepsilon}{\eta + \varepsilon}$$

➤ If $\varepsilon = 0$, then $dP^t/dT = 0$.

➤ If $\eta = 0$, then $dP^t/dT = 1$.

Tax Incidence and Price Elasticity (2)

- Case 2 - Specific tax imposed on producers.

- Producer's price: $P^t = \frac{a + c - bT}{b + d}$. Find $dP^t/dT = ?$

- Suppose $\Delta T = T_1 - T_0$. We want to know $\Delta P^t = ?$

$$\text{➤ } P^{t0} = \frac{a + c - bT_0}{b + d} \text{ and } P^{t1} = \frac{a + c - bT_1}{b + d}$$

$$\text{➤ } \Delta P^t = \frac{-b}{b + d} (T_1 - T_0) = \frac{-b}{b + d} \Delta T$$

➔ The impact of tax change on equilibrium price when tax is

imposed on producer is: $\frac{\Delta P^t}{\Delta T} = \frac{-b}{b + d}$ -- (3)

Specific tax imposed on producers (cont'd)

- From (3) and following the same method, we get:

$$\frac{\Delta P^t}{\Delta T} = \frac{-b}{b+d} = \frac{\Delta Q_D / \Delta P}{-\Delta Q_D / \Delta P + \Delta Q_S / \Delta P}$$

$$\frac{\Delta P^t}{\Delta T} = \frac{\Delta Q_D / \Delta P}{-\Delta Q_D / \Delta P + \Delta Q_S / \Delta P} \cdot \frac{P/Q}{P/Q}$$

$$\Rightarrow \boxed{\frac{dP^t}{dT} \approx \frac{\Delta P^t}{\Delta T} = \frac{-\eta}{\eta + \varepsilon}}$$

- If $\varepsilon = 0$, then $dP^t/dT = -1$.
- If $\eta = 0$, then $dP^t/dT = 0$.