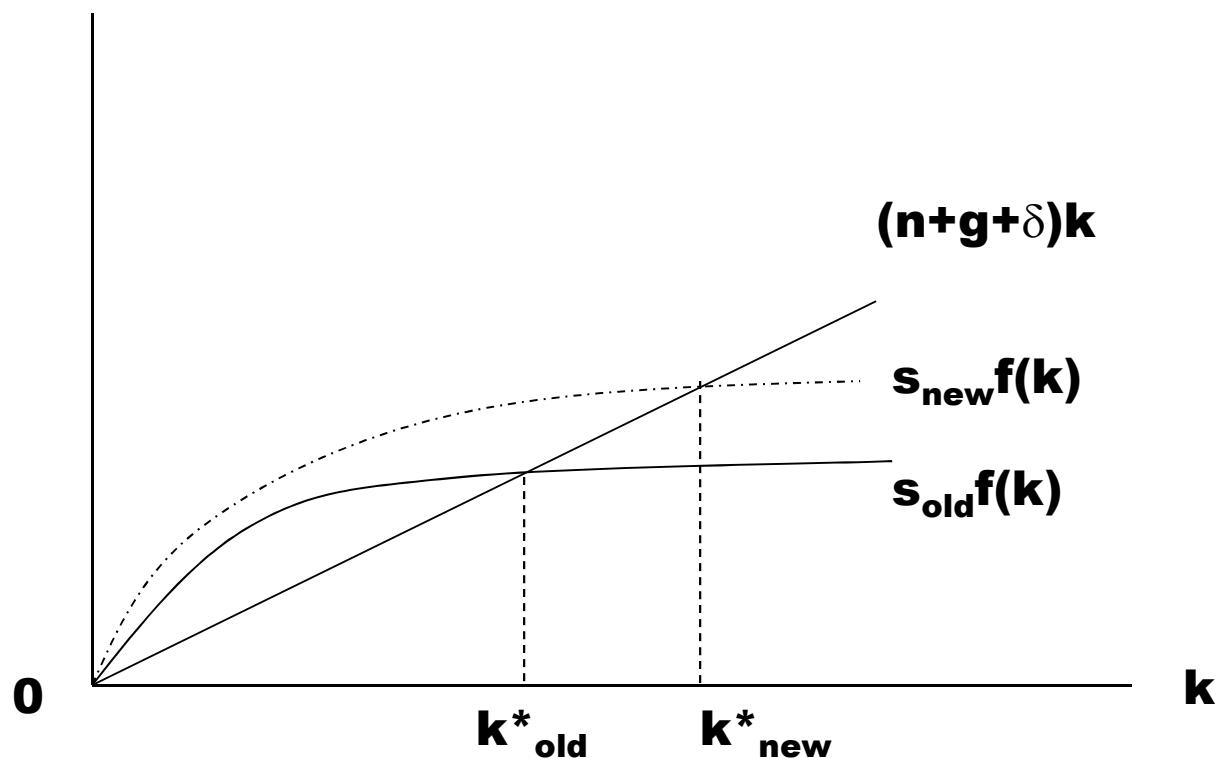


Macroeconomics

Lecture 10

The impact of a change in the saving rate on output



The impact of a change in the saving rate on output

- In sum, a change in the saving rate has a level effect but not a growth effect.
- Implication from Solow growth model, the change in the rate of technological progress (i.e., endogenous change in $A(t)$) may have growth effects.

The Solow Model and the Central Questions of Growth Theory

- Only growth in the effectiveness of labor can lead to permanent growth in output per worker.
- The impact of change in capital per worker on output per worker is modest.
- Hence, only differences in the effectiveness of labor can account for the vast differences in the wealth across time and space.

A model that resemble the Solow growth model

- **Infinite-horizon model (The Ramsey-Cass-Koopmans model):**
Competitive firms rent capital and hire labor to produce output, and a fixed number of households supply labor, hold capital, consume and save.

The Ramsey-Cass-Koopmans Model

- There is a large number of identical firms.
- Each firm has access to the production function $Y = F(K, AL)$.
- The firms hire workers and rent capital in a competitive factor markets, and sell their output in a competitive output market.
- Firms are owned by households, so any profits they earn accrue to the households.

The Ramsey-Cass-Koopmans Model

- There is a large number of households.
- The size of each household grows at rate n .
- Each member of household supplies one unit of labor at every point in time.
- The household rents capital its own to firms. It has initial capital holding of $K(0)/H$.

The behavior of firms

- Because the production function has constant returns and the economy is competitive, firms earn zero profits.
- Capital earns its marginal product. Assuming that there is no depreciation, the real rate of return to capital equals its earnings per unit time, or $r(t) = \partial F(K, AL) / \partial K = f'(k(t))$.
- The real wage per unit of effective labor is
- $w(t) = \partial F(K, AL) / \partial AL = f(k(t)) - k(t)f'(k(t))$

The behavior of households

- The representative household takes the paths of $r(t)$ and $w(t)$ as given.
- Its objective function is to maximize

$$U = \int_{t=0}^{\infty} e^{-\rho t} u(C(t)) \frac{L(t)}{H} dt$$

where $C(t)$ is the consumption of each member of the household at time t .

$u(\cdot)$ is the instantaneous utility function, and takes the form of a constant – relative – risk – aversion utility

$$u(C(t)) = \frac{C(t)^{1-\theta}}{1-\theta}, \quad \theta > 0, \quad \rho - n - (1-\theta)g > 0$$

$L(t)$ is total population in the economy.

The behavior of households

- **The household's budget constraint is**

$$\int_{t=0}^{\infty} e^{-R(t)} C(t) \frac{L(t)}{H} dt \leq \frac{K(0)}{H} + \int_{t=0}^{\infty} e^{-R(t)} A(t) w(t) \frac{L(t)}{H} dt \quad (5)$$

where $R(t) = \int_{\tau=0}^t r(\tau) d\tau.$

One unit of capital good invested at time 0 yields $e^{R(t)}$ units of good at t .

- **Let $c(t)$ be consumption per effective labor,
 $c(t)=[C(t)L(t)/H]/[A(t)L(t)/H].$**

The behavior of households

- Eq(5) can be rewritten as

$$\int_{t=0}^{\infty} e^{-R(t)} c(t) \frac{A(t)L(t)}{H} dt \leq k(0) \frac{A(0)L(0)}{H} + \int_{t=0}^{\infty} e^{-R(t)} w(t) \frac{A(t)L(t)}{H} dt \quad (6)$$

$$\text{Or} \quad \frac{K(0)}{H} + \int_{t=0}^{\infty} e^{-R(t)} [w(t) - c(t)] \frac{A(t)L(t)}{H} dt \geq 0 \quad (7)$$

$$\lim_{s \rightarrow \infty} \left[\frac{K(0)}{H} + \int_{t=0}^s e^{-R(t)} [w(t) - c(t)] \frac{A(t)L(t)}{H} dt \right] \geq 0 \quad (8)$$

Note that the household's capital holdings at time s are

$$\frac{K(s)}{H} = \frac{K(0)}{H} e^{R(s)} + e^{R(s)} \int_{t=0}^s e^{-R(t)} [w(t) - c(t)] \frac{A(t)L(t)}{H} dt \quad (9)$$

$\therefore$ Eq(9) is $e^{R(s)}$ times the expression in the brackets in Eq(8), hence

$$\lim_{s \rightarrow \infty} e^{-R(s)} \frac{K(s)}{H} \geq 0 \quad (10)$$

Since $A(t)L(t) = A(0)L(0)e^{(n+g)t}$, Eq(6) can be rewritten as

$$\int_{t=0}^{\infty} e^{-R(t)} c(t) e^{(n+g)t} dt \leq k(0) + \int_{t=0}^{\infty} e^{-R(t)} w(t) e^{(n+g)t} dt \quad (11)$$

Eq(10) and (11) together give

$$\lim_{s \rightarrow \infty} e^{-R(s)} e^{(n+g)s} k(s) \geq 0 \quad (12)$$