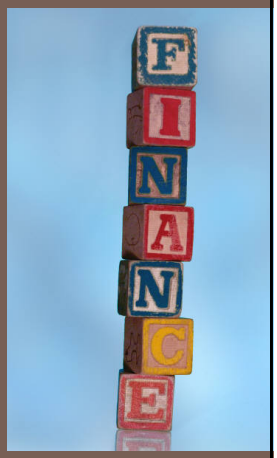
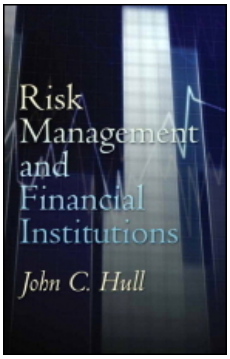
FN 201
BUSINESS FINANCE



Lecture 12
Risk and Return 1

Reference

- Excerpt from John C. Hull, Risk Management and Financial Institutions, Pearson International Edition, 2007



- Concerned with the way risks are managed by banks and other FIs, but many of the ideas and approaches are equally applicable to other types of firms

Agenda

- Quantify risk
- Portfolio Risk and Return
- Efficient Frontier
- Diversification
- Risk Preference

Key Concepts

- Risk and (expected) Return trade-off
  - Expected return defined as weighted average of possible return
    - Weights being probability of the return occurring
  - Risk defined as standard deviation of return over one year
    - Risk = sqrt($E(R^2) - [E(R)]^2$)
    - Now we can "quantify" the risk

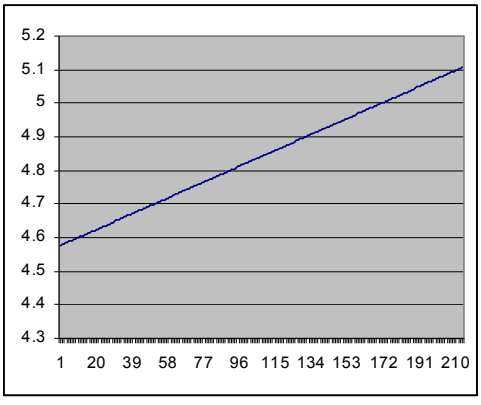
Ex1. Cal E(r) and Risk

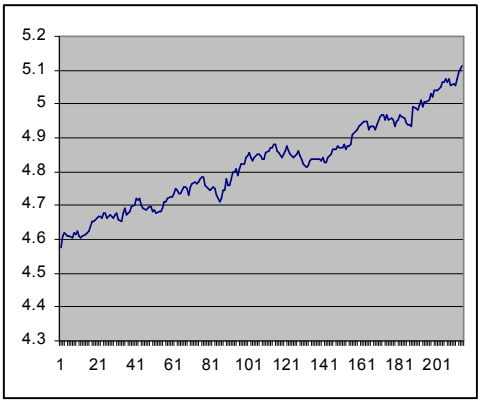
- Return in 1 year from investing 100k in equities

Probability	Return
0.05	+50%
0.25	+30%
0.40	+10%
0.25	-10%
0.05	-30%
Total = 1.00	

- $E(r) = 0.05*0.50 + 0.25*0.30 + 0.40*0.10 + 0.25*(-0.10) + 0.05*(-0.30) = \underline{0.10 \text{ or } 10\%}$
- risk or $SD(r) = \text{sqrt}(\ E(R^2) - [E(R)]^2\ )$
- $E(R^2) = 0.05*0.50^2 + 0.25*0.30^2 + 0.40*0.10^2 + 0.25*(-0.10)^2 + 0.05*(-0.30)^2 = 0.046$
- $SD(r) = \text{sqrt}\ (\ 0.046 - 0.10^2) = \underline{0.1897 \text{ or } 18.97\%}$

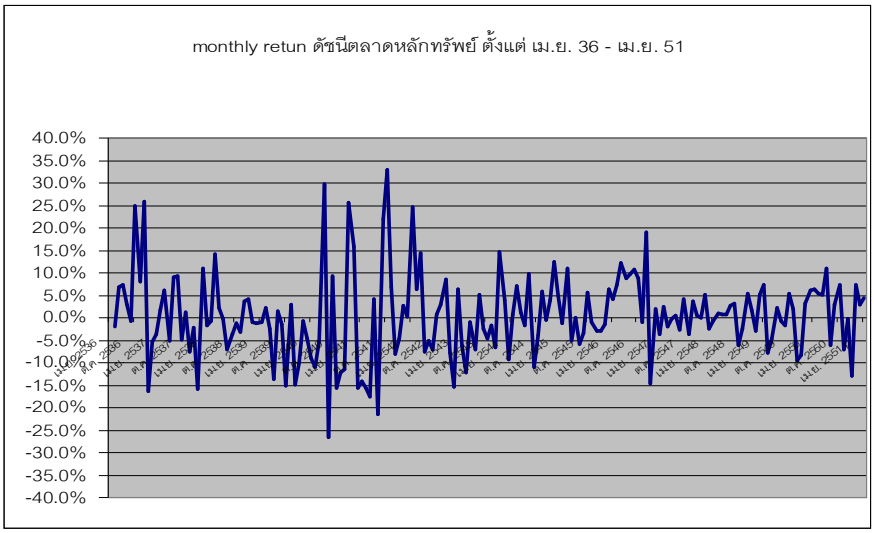
- If risk is SD. of return, how is it look liked?





- Risk is not "danger of losses" but rather "dispersion of unexpected outcome"
- How bad or good can it become?

This is Risk

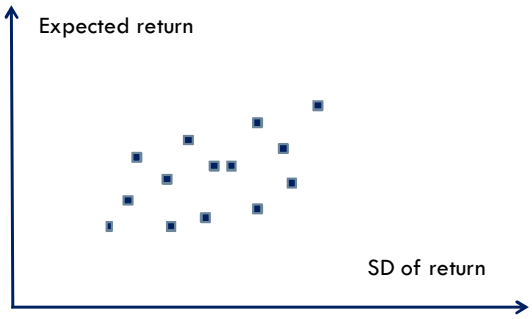


Pioneer work in the area of risk & return tradeoff

- Markowitz (1952)
  - Quantifying risk
  - Portfolio risk
  - Efficient frontier
- Sharp(1964)
  - CAPM model – express relationship between expected return and risk in terms of (one) systematic risk
- Ross (1976)
  - Arbitrage pricing theory – an extension of CAPM- there are several sources of systematic risk
- All these works have had a profound effect on the way we think about risk today
- Key ideas will be recapped here

Investment Opportunities

- Given knowing how to "quantify" risk, we can characterize every investment opportunity by expected return and SD of return



- Next question is what will happen (in terms of risk and return) if we combine investment A & B to form a portfolio?

Portfolio Risk and Return

- Expected returns $E[r_p] = w_A \cdot E[r_A] + w_B \cdot E[r_B]$
- Variance of return

$$Var(r_p) = w_A^2 Var(r_A) + w_B^2 Var(r_B) + 2 w_A w_B Cov(r_A, r_B)$$

- SD of return = sqrt(Var(r_p))
- Note:

$$\rho_{A,B} = \frac{\text{cov}_{A,B}}{\sigma_A \sigma_B}$$

Portfolio Risk

- The variance of a two stock portfolio is the sum of these four boxes from variance-covariance matrix

	Stock 1	Stock 2
Stock 1	$x_1^2\sigma_1^2$	$x_1 x_2 \sigma_{12} = x_1 x_2 \rho_{12} \sigma_1 \sigma_2$
Stock 2	$x_1 x_2 \sigma_{12} = x_1 x_2 \rho_{12} \sigma_1 \sigma_2$	$x_2^2\sigma_2^2$

Ex2.5: How about risk of 3 stocks portfolio

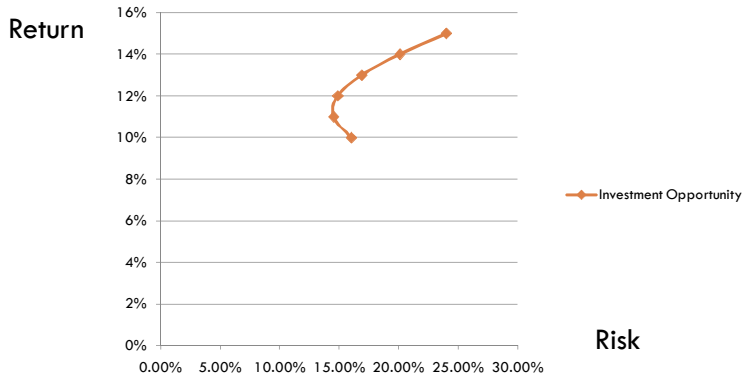
Ex2. Cal Portfolio risk and return

- Two assets have following characteristics
  - $E(r_A) = 10\%$, $SD(r_A) = 16\%$
  - $E(rB) = 15\%$, $SD(rB) = 24\%$
  - Correlation b/w the returns is 0.2

Weight A	Weight B	$E(r_p)$	$SD(r_p)$
0.0	1.0	15%	24.00%
0.2	0.8	14%	20.09%
0.4	0.6	13%	16.89%
0.6	0.4	12%	14.87%
0.8	0.2	11%	14.54%
1.0	0.0	10%	16.00%

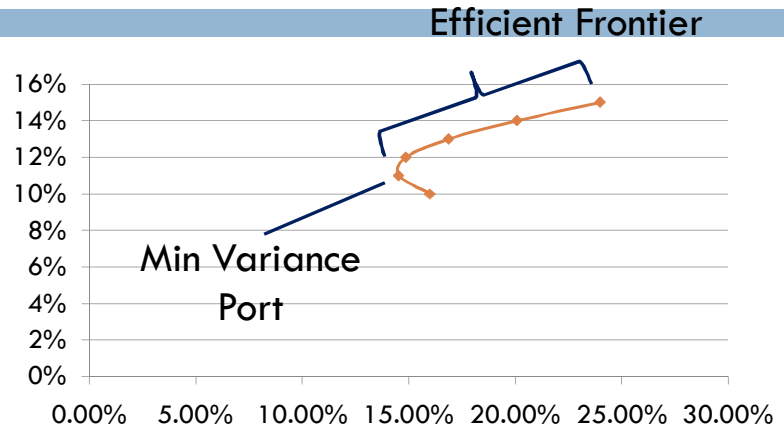
Ex2. Cal Portfolio risk and return (2)

- Make a plot



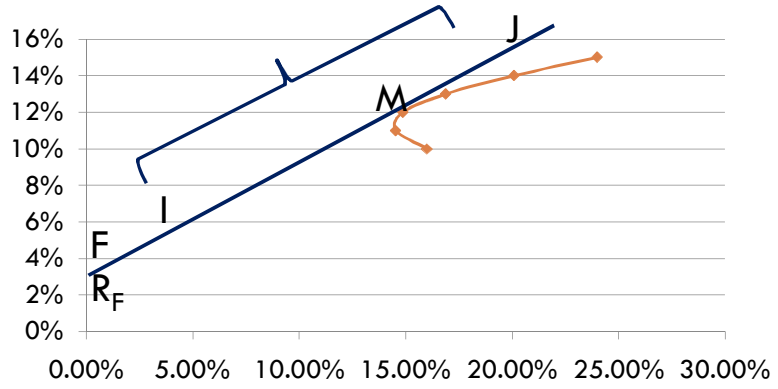
- Most investors are risk-averse
  - They want to increased expected return while reducing SD of return
  - Try to move as far as they can in a "north-west" direction

Efficient Frontier



- This represent how far we can move in a north-west direction
- Investor must invest on this efficient frontier
- There is no investment that dominates a point on the frontier in the sense that it has both a "higher expected return and lower risk
- It illustrate the best possible risk and return trade-off (in the world with only 2 assets)

Let's add one risk free asset



- F is risk-free rate
- M is pint of tangency
- I achieved by investing certain percentage (x) of fund in portfolio M and the rest in risk-free asset
- FM become our "new efficient frontier"

Implication 1

- $E(r_I) = (1-x)R_F + xE(r_M)$
- $SD(r_I) = x\ SD(r_M)$

- All points along line FM can be obtained by choosing a combination between asset F and portfolio M
- This line dominate all the point on previous frontier because of better risk and return trade-off
- If we can further assumption that we can also borrow and invest at risk free rate, we can create investments that are on the line from F to M buy beyond M

- E.g. point J can be created from borrowing at risk free to invest, i.e., invest in M more than initial fund (X>1)
  - $E(r_I) = xE(r_M) - (x-1)R_F$
  - $SD(r_I) = x\ SD(r_M)$

Implication 2

- With frontier become straight-line, there is a linear trade-off between expected return and risk. And all investors should chose the same portfolio of risky asset. This is the portfolio represented by M.

- Investors should reflect appetite for risk by combining M with borrowing or lending at risk free rate

- What is the portfolio M look liked?
- M must be portfolio of all risky investment (otherwise some assets will have zero demand and hence zero price)
- Amount of M must be proportional to amount of investment available
- The investment M is usually referred to as "Market Portfolio"

Implication 3

- With frontier become straight-line, investing along this line also maximize risk-return trade-off measured in terms of sharp ratio
- Sharp Ratio defined as $[E(r_P) - R_F] / SD(r_P)$

Ex3. Impact of Correlation

- What will happen if correlation between two assets is 1, 0 or -1?
- Suppose you invest 60% of your portfolio in Wal-Mart and 40% in IBM.
  - The expected return on Wal-Mart and IBM is 10% and 15%
  - The standard deviation of their annualized daily returns are 19.8% and 29.7%, respectively
  - Assume a correlation coefficient of 1.0 and calculate the portfolio risk
  - What will happen if correlation reduce to 0 and eventually reach -1?

rho	1.00
SD (Wal.)	19.80%
SD (IBM)	29.70%
w1	0.60
w2	0.4

Comp.1	0.014113
Comp.2	0.014113
Comp.3	0.028227
Port.Val	0.056454
SD	23.76%
Return	23.76%

rho	0.75
SD (Wal.)	19.80%
SD (IBM)	29.70%
w1	0.60
w2	0.4

Comp.1	0.014113
Comp.2	0.014113
Comp.3	0.02117
Port.Val	0.049397
SD	22.23%
Return	23.76%

rho	0.50
SD (Wal.)	19.80%
SD (IBM)	29.70%
w1	0.60
w2	0.4

Comp.1	0.014113
Comp.2	0.014113
Comp.3	0.014113
Port.Val	0.04234
SD	20.58%
Return	23.76%

rho	0.25
SD (Wal.)	19.80%
SD (IBM)	29.70%
w1	0.60
w2	0.4

Comp.1	0.014113
Comp.2	0.014113
Comp.3	0.007057
Port.Val	0.035284
SD	18.78%
Return	23.76%

rho	-1.00
SD (Wal.)	19.80%
SD (IBM)	29.70%
w1	0.60
w2	0.4

Comp.1	0.014113
Comp.2	0.014113
Comp.3	-0.02823
Port.Val	0
SD	0.00%
Return	23.76%

rho	-0.75
SD (Wal.)	19.80%
SD (IBM)	29.70%
w1	0.60
w2	0.4

Comp.1	0.014113
Comp.2	0.014113
Comp.3	-0.02117
Port.Val	0.007057
SD	8.40%
Return	23.76%

rho	-0.50
SD (Wal.)	19.80%
SD (IBM)	29.70%
w1	0.60
w2	0.4

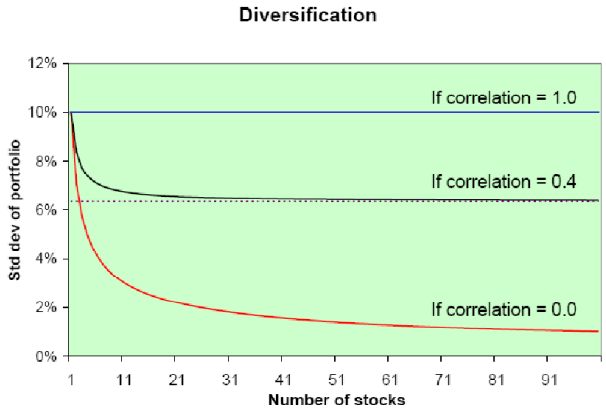
Comp.1	0.014113
Comp.2	0.014113
Comp.3	-0.01411
Port.Val	0.014113
SD	11.88%
Return	23.76%

rho	-0.25
SD (Wal.)	19.80%
SD (IBM)	29.70%
w1	0.60
w2	0.4

Comp.1	0.014113
Comp.2	0.014113
Comp.3	-0.00706
Port.Val	0.02117
SD	14.55%
Return	23.76%

Diversification-notes

- Diversification always reduces portfolio risk as long as correlation is not 1
- In well diversified portfolio, covariance among stocks are more important than individual variances.



Ex. Try it in Excel

	Stock prices		Monthly Return	
Date	PNC	GS	PNC	GS
3/2/2000	34.16	80.22		
4/2/2000	33.46	79.57		
7/2/2000	33.13	80.57	4.665%	4.066%
8/2/2000	33.83	84.03	10.297%	6.097%
9/2/2000	33.08	81.51	9.109%	2.535%
10/2/2000	32.05	81.39	1.636%	2.905%
11/2/2000	31.63	79.98	-8.395%	1.005%
14/2/2000	31.25	81.27	-11.618%	1.288%
15/2/2000	32.38	81.27	-6.313%	-0.160%
16/2/2000	31.53	77.93	-7.601%	-5.152%
17/2/2000	31.35	77.16	-5.220%	-4.548%
18/2/2000	30.03	75.93	-9.249%	-5.857%
22/2/2000	29.66	76.64	-9.482%	-0.222%
24/2/2000	28.53	84.21	-14.797%	6.627%
25/2/2000	28.25	83.86	-18.321%	1.127%
28/2/2000	29.33	82.68	-17.816%	-1.130%
29/2/2000	29.05	86.91	-17.020%	6.133%
1/3/2000	29.09	89.03	-21.423%	3.358%
2/3/2000	29.19	90.44	-21.606%	5.337%
3/3/2000	29.23	96.31	-18.158%	11.765%
6/3/2000	28.62	97.42	-17.695%	19.426%
7/3/2000	27.54	98.54	-19.471%	21.383%

	PNC	GS
E[r]	0.675%	0.956%
SD[r]	6.669%	9.085%
var[r]	0.004447	0.008254
SD[r]	6.669%	9.085%

Covar Mat	PNC	GS
PNC	0.004447	0.001895
GS	0.001895	0.008254

Covar Mat	PNC	GS
PNC	0.004447	0.001895
GS	0.001895	0.008254

$$E[r_p] = w_A \cdot E[r_A] + w_B \cdot E[r_B]$$

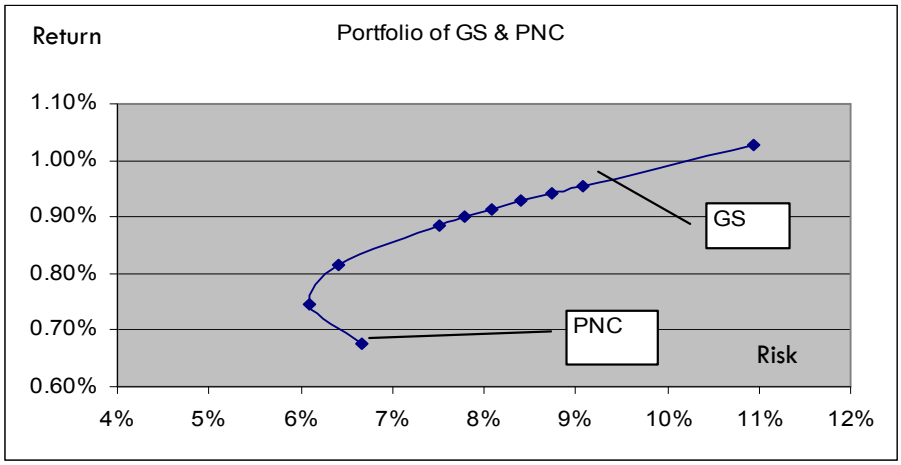
$$Var(r_p) = w_A^2 Var(r_A) + w_B^2 Var(r_B) + 2 w_A w_B Cov(r_A, r_B)$$

W(PNC)	W(GS)	E[rp]	SD[rp]
-0.25	1.25	1.0%	11%
0	1	1.0%	9%
0.05	0.95	0.9%	9%
0.1	0.9	0.9%	8%
0.15	0.85	0.9%	8%
0.2	0.8	0.9%	8%
0.25	0.75	0.9%	8%
0.5	0.5	0.8%	6%
0.75	0.25	0.7%	6%
1	0	0.7%	7%

1-Steps Calculation	0.2855	=MMULT(MMULT(w,sigma),TRANSPOSE(w))

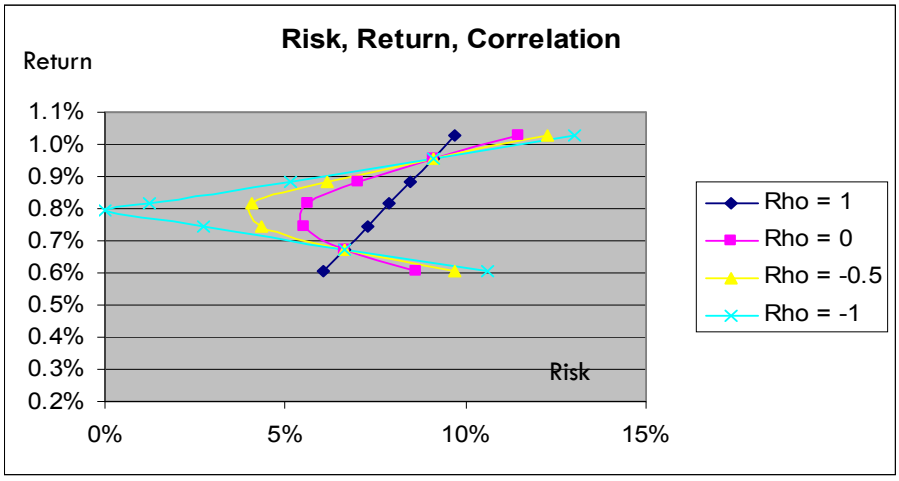
Portfolio Risk and Correlation

- Using actual monthly data on GS and PNC



Portfolio Risk and Correlation

- When correlation changes, this is how investment opportunities looks liked



Only Covariance Matters

- Let's look at 3 stocks case
- $var(Rp) = 1/N^2 * var_1 + 1/N^2 * var_2 + 1/N^2 * var_3 + 2/N^2 * cov(1,2) + 2/N^2 * cov(1,3) + 2/N^2 * cov(2,3)$
- $var(Rp) = 1/N^2 (\text{sum } var_1\ var_2\ var_3) + 2/N^2 * (\text{sum } cov(1,2)\ cov(1,3)\ cov(2,3))$
- $var(Rp) = 1/N$ (Avg.var) + $2/N$ (Avg.cov)
- Generally $var(Rp) = 1/N$ (Avg.var) + $N-1/N$ (Avg.cov)
  - As N increases, the first term tend to go to zero, only second term left
  - Hence, in the well-diversified portfolio, only covariance matters in the sense that it determines portfolio risk (variance)
  - Key idea in modern portfolio theory

Diversification Benefit

- What is it exactly?
- In plain word,
  - It is unlikely that things will turn out badly at the same time
  - Or price of all stocks in your investment portfolio unlikely to drop at the same time, some firms will perform well while others may not
- In technical term,
  - idiosyncratic risk between firms get cancelled out

This is diversification benefit

The traditional theory on diversification "Don't put your eggs in one basket"
(Photo courtesy of www.moneywise.co.uk)

The Importance of Diversification

From year to year, there's no telling which asset classes will be the best performers—a strong argument for portfolio diversification. The chart below ranks the best to worst performing asset classes from top to bottom for the years 1999 to 2008.

1999	2000	2001	2002	2003	2004	2005	2006	2007	2008
Small-Cap Growth 43.10%	Commodities 31.84%	Small-Cap Value 14.02%	Commodities 25.91%	Small-Cap Growth 48.54%	Real Estate 33.17%	Commodities 21.36%	Real Estate 36.14%	Commodities 16.23%	Long-Term Bonds 24.03%
Large-Cap Growth 33.16%	Real Estate 31.04%	Real Estate 12.36%	Unhedged Foreign Bonds 22.99%	Small-Cap Value 46.03%	Small-Cap Value 22.25%	International Stocks 14.01%	International Stocks 26.88%	Large-Cap Growth 11.82%	Unhedged Foreign Bonds 12.00%
International Stocks 27.31%	Small-Cap Value 22.80%	Interm-Term Bonds 8.44%	Long-Term Bonds 16.79%	International Stocks 39.17%	International Stocks 20.70%	Real Estate 13.99%	Small-Cap Value 23.48%	International Stocks 11.62%	Short-Term Bonds 6.61%
Commodities 24.35%	Long-Term Bonds 20.27%	Short-Term Bonds 8.30%	Interm-Term Bonds 10.26%	Large-Cap Value 30.03%	Large-Cap Value 16.49%	Large-Cap Value 7.05%	Large-Cap Value 22.21%	Unhedged Foreign Bonds 10.81%	Interm-Term Bonds 5.24%
Large-Cap Value 7.34%	Interm-Term Bonds 11.63%	High Yield Bonds 4.48%	Short-Term Bonds 5.76%	Large-Cap Growth 29.75%	Small-Cap Growth 14.31%	Long-Term Bonds 6.50%	Small-Cap Growth 13.35%	Long-Term Bonds 9.81%	Cash 1.80%
Cash 4.74%	Short-Term Bonds 8.00%	Long-Term Bonds 4.21%	Real Estate 3.60%	High Yield Bonds 28.15%	Unhedged Foreign Bonds 12.04%	Large-Cap Growth 5.27%	High Yield Bonds 11.77%	Short-Term Bonds 7.32%	High Yield Bonds -26.39%
Short-Term Bonds 3.06%	Large-Cap Value 7.02%	Cash 4.09%	Cash 1.70%	Real Estate 27.75%	High Yield Bonds 10.87%	Small-Cap Value 4.70%	Large-Cap Growth 9.09%	Small-Cap Growth 7.04%	Small-Cap Value -28.92%
High Yield Bonds 2.51%	Cash 5.95%	Unhedged Foreign Bonds -3.58%	High Yield Bonds -1.89%	Commodities 23.93%	Commodities 9.15%	Small-Cap Growth 4.15%	Unhedged Foreign Bonds 5.94%	Interm-Term Bonds 6.97%	Commodities -35.65%
Interm-Term Bonds -0.82%	Unhedged Foreign Bonds -2.48%	Large-Cap Value -5.59%	Small-Cap Value -11.42%	Unhedged Foreign Bonds 18.63%	Long-Term Bonds 7.70%	Cash 3.00%	Cash 4.76%	Cash 4.74%	Large-Cap Value -36.85
Small-Cap Value -1.49%	High Yield Bonds -5.12%	Small-Cap Growth -9.23%	Large-Cap Value -15.52%	Interm-Term Bonds 4.10%	Large-Cap Growth 6.30%	High Yield Bonds 2.74%	Interm-Term Bonds 4.33%	High Yield Bonds 2.19%	Large-Cap Growth -38.44%
Real Estate -2.57%	International Stocks -13.95%	Commodities -19.51%	International Stocks -15.64%	Long-Term Bonds 2.48%	Interm-Term Bonds 4.34%	Interm-Term Bonds 2.43%	Short-Term Bonds 3.96%	Large-Cap Value -0.17%	Small-Cap Growth -38.54%
Unhedged Foreign Bonds -6.19%	Large-Cap Growth -22.43%	Large-Cap Growth -20.42%	Large-Cap Growth -27.89%	Short-Term Bonds 1.90%	Cash 1.24%	Short-Term Bonds 1.67%	Commodities 2.07%	Small-Cap Value -9.77%	Real Estate -39.20%
Long-Term Bonds -8.74%	Small-Cap Growth -22.44%	International Stocks -21.21%	Small-Cap Growth -30.27%	Cash 1.07%	Short-Term Bonds 0.91%	Unhedged Foreign Bonds -9.24%	Long-Term Bonds 1.85%	Real Estate -17.88%	International Stocks -43.38%

Cash represented by the Citigroup 3-month T-Bill Index, an index of three-month Treasury bills. Commodities represented by the Dow Jones-AIG Commodity Total Return Index, which is composed of futures contracts on 19 physical commodities. Unhedged Foreign Bonds represented by the JPMorgan Non-U.S. Global Government Bond (Unhedged) Index, which is an unmanaged market index representative of the total return performance in U.S. dollars on an unhedged basis of major non-U.S. bond markets. High Yield Bonds represented by the Merrill Lynch US High Yield Master II Index, which tracks the performance of below investment grade (BBB), but not in default, US dollar-denominated corporate bonds publicly issued in the domestic market. Intermediate-Term Bonds represented by the Barclays Capital Aggregate Index, which is composed of securities from the Barclays Capital Government/Credit Bond Index, Mortgage-Backed Securities Index, and Asset-Backed Securities Index. It is representative of the domestic, investment-grade, fixed-rate, taxable bond market. Long-Term Bonds represented by the Barclays Capital Long Treasury Index, an index of US Treasury obligations with maturities greater than 10 years. Short-Term Bonds represented by the Merrill Lynch 1–3 Year Treasury Index, an index of US Treasury obligations with maturities from 1 to 2.99 years. International Stocks represented by the MSCI EAFE Index. The Morgan Stanley Capital International (MSCI) Europe, Australasia, Far East Index (EAFE) is an index of over 900 companies, and is a generally accepted benchmark for major overseas markets. Large-Cap Growth Stocks represented by the Russell 1000 Growth Index, which measures the performance of those Russell 1000 companies with higher price-to-book ratios and higher forecasted growth values. Large-Cap Value Stocks represented by the Russell 1000 Value Index, which measures the performance of those Russell 1000 companies with lower price-to-book ratios and lower forecasted growth values. Small-Cap

Allianz Global Investors

This is NOT diversification benefit

The idea of portfolio theory can be applied to corporation as well

- Imagine this situation
  - You are CRO of a major corporation. CEO wants your views on a major new acquisition
  - You have been flooded with report showing that new acquisition has a positive NPV and will enhance shareholder value. But what sort of analysis and ideas can you give to the CEO?
  - Maybe you want to see
    - How the new acquisition fits into company's portfolio?
    - What is the correlation of the performance of the new acquisition with the rest of company's business?
    - When the rest of business turn bad, will the new acquisition also provide a poor returns – or will it have the effects of dampening the ups and downs in the rest of business?

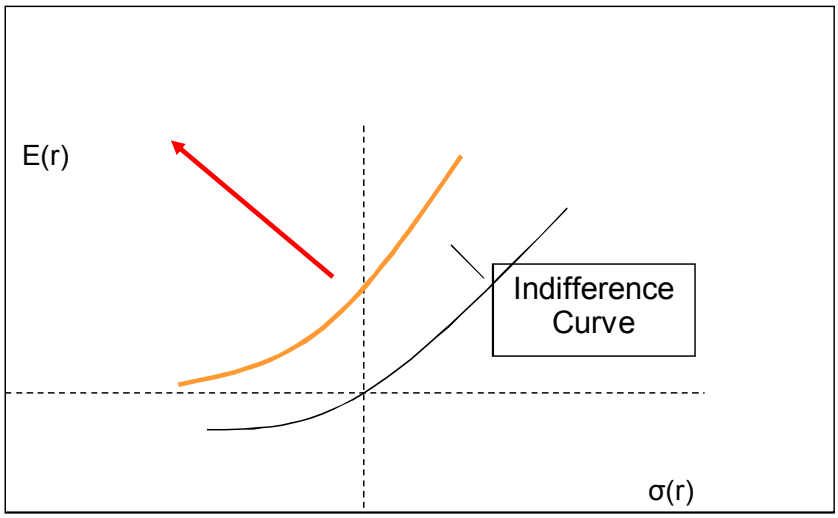
Risk Preference

Risk Preference

- At the end, individual chooses investment portfolio according to one's risk preference and objective
- 3 types of risk preferences
  - Risk averse
  - Risk neutral
  - Risk lover
- Suppose you can choose between 50 THB now with absolute certainty or equal chance of 100 or 0 THB, which one will you choose?

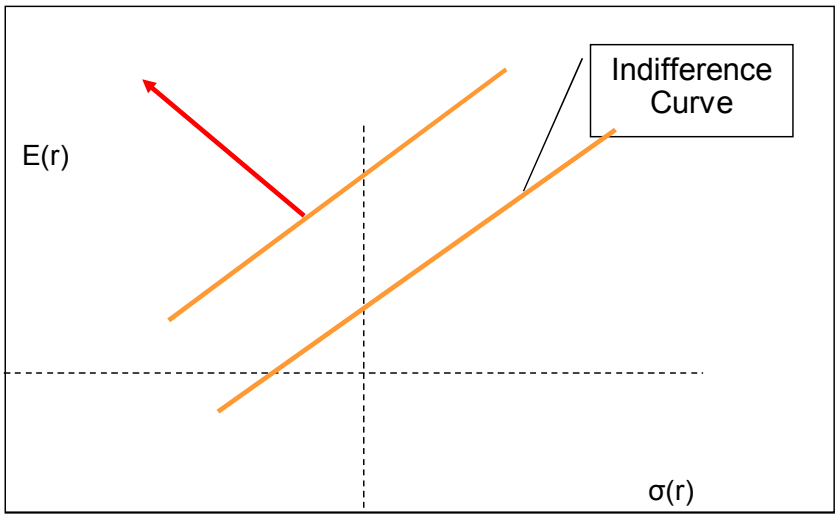
Risk Averse

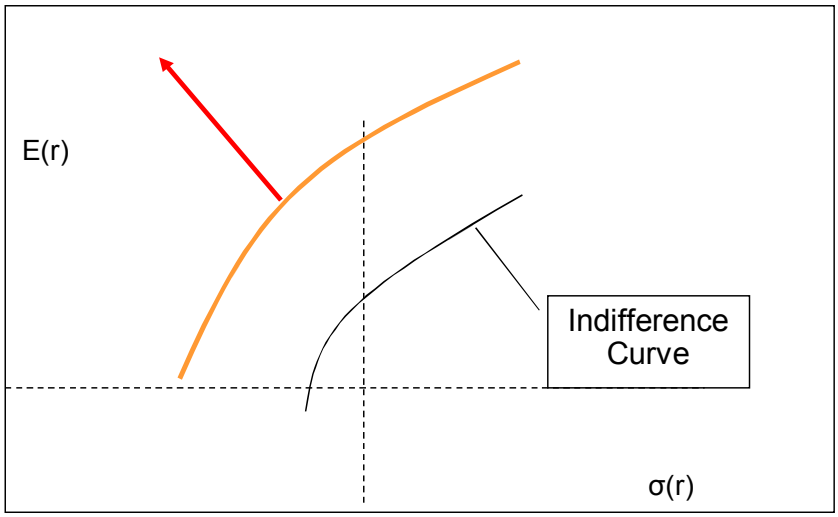
- Risk Averse
  - Prefer 50THB now than a 50% chance of 100 and 50% of 0
  - Penalize the expected rate of return of a risky portfolio by a certain %
  - The greater the risk, the larger the penalty
- To be on the same utility indifference curve, higher risk must be compensated by higher return
- Indifference curve is upward-sloping
- The more risk averse, the steeper an indifference curve



Risk Neutral & Risk Lover

- Risk neutral
  - Judge risky prospects purely by expected return
  - Indifferent between 50THB now and 50% chance of 100
  - The level of risk is irrelevant to risk-neutral investors
  - Indifference curve is horizontal
- Risk lover
  - Adjusts the expected return upward to reflect utility from the "fun" from taking risks
  - Prefer 50% chance of 100 than 50THB now
  - Indifference curve is downward sloping
  - Less risky, must be compensated by higher expected return, since losing the "fun part"


Indifference Curve
E(r)
σ(r)


E(r)
Indifference Curve
σ(r)

Q&A