Instructions

(1) Please read the instruction carefully.
(2) Please read each question carefully and answer the questions straightforwardly. Always provide economic reasons at lease a paragraph for your analysis, or a graph when necessary, even when the question does not indicate so.
(3) Handing and submitting assignments are only available via BE Moodle.

Answering the questions and preparing answer sheets

(1) Answers are to be handwritten, in either digital or analog form, in a blank canvas or any clean paper. Make sure that your handwriting is clearly visible and readable.
(2) There is no need to rewrite the question. Just indicate the question number clearly for each of the answer, such as 1.a).
(3) Default decimal point is 4.
(4) Choose precise wordings, especially when you want to interpret the meaning of a test, confidence interval, or coefficients.
(5) When done, for the digital case, collage all the pages into a single PDF file. For those who write on sheets of paper, take photo of all pages then convert all of them into a single PDF file as well.
(6) Name your PDF file as StudentID_YourNickname, such as 640123456_Bo.

Submitting your answers

(1) Make sure your file does not exceed 10MB. This is the maximum file size for BE Moodle upload.
(2) Login to BE Moodle, head into the course, then the assignment topic.
(3) Choose your file to submit. Done. There will be timestamp for your upload date and time, so please make sure to not submit later than that.

Question 1. (12 points) Economic model of Crime.

1.a) Based on the regression results provided, write out the estimated coefficients in the form of regression equation (1.1). Interpret the estimated coefficients associated with $avgsen$. Based on Model (1.1), test whether the average sentence served from prior convictions has an impact on the number of arrests in the current year (1986). Show your work. (Use α = 0.05)

1.b) What is the overall significance of the regression from Model (1.1) and Model (1.2)? What test do you use? (Use α = 0.01)

1.c) If we are interested in testing whether "ethnic background and legal income" has an impact on the number of arrests in the current year (1986), what kind of null/alternative hypothesis would we be testing? Perform the test and discuss your finding. (Use α = 0.05)

Estimate the model (1.1) reports in the Table 1.1

$$narr86_i = \beta_1 + \beta_2 pcnv_i + \beta_3 avgsen_i + \beta_4 tottime_i + \beta_5 ptime86_i + \beta_6 qemp86_i + u_i \qquad \textbf{(1.1)}$$

Table 1.1

Source	SS	df	MS			
				Number of obs	=	2,725
				F(5, 2719)	=	24.29
Model	85.9532425	5	17.1906485	Prob > F	=	0.0000
Residual	1924.39391	2,719	.707757967	R-squared	=	0.0428
				Adj R-squared	=	0.0410
Total	2010.34716	2,724	.738012906	Root MSE	=	.84128

narr86	Coefficient	Std. err.	t	P>|t|	[95% conf. interval]
pcnv	-.1512246	.040855	Omitted for the purpose of this exam		
avgsen	-.0070487	.0124122			
tottime	.0120953	.0095768			
ptime86	-.0392585	.0089166			
qemp86	-.1030909	.0103972			
_cons	.7060607	.0331524			

Estimate the model (1.2) reports in the Table 1.2

$$narr86_i = \beta_1 + \beta_2 pcnv_i + \beta_3 avgsen_i + \beta_4 tottime_i + \beta_5 ptime86_i + \beta_6 qemp86_i + \beta_4 inc86_i + \beta_5 black_i + \beta_6 hispan_i + u_i \quad \textbf{(1.2)}$$

where

$narr86_i$	= the number of arrests in the current year (1986)
$pcnv_i$	= the proportion of prior arrests that led to a conviction
$avgsen_i$	= the average sentence served from prior convictions (in months)
$tottime_i$	= months spent in prison since age 18 prior to 1986
$ptime86_i$	= months spent in prison in 1986
$qemp86_i$	= the number of quarters that the man was legally employed in 1986
$inc86_i$	= legal income, 1986, (hundred dollars)
$black_i$	= 1 if black ethnic background
$hispan_i$	= 1 if Hispanic ethnic background

Table 1.2

Source	SS	df	MS
Model	145.390104	8	18.173763
Residual	1864.95705	2,716	.686655763
Total	2010.34716	2,724	.738012906

Number of obs	=	2,725
F(8, 2716)	=	26.47
Prob > F	=	0.0000
R-squared	=	0.0723
Adj R-squared	=	0.0696
Root MSE	=	.82865

narr86	Coefficient	Std. err.	t	P>|t|	[95% conf. interval]
pcnv	-.1332344	.0403502			
avgsen	-.0113177	.0122401			
tottime	.0120224	.0094352			
ptime86	-.0408417	.008812			
qemp86	-.0505398	.0144397			
inc86	-.0014887	.0003406			
black	.3265035	.0454156			
hispan	.1939144	.0397113			
_cons	.5686855	.0360461			

Omitted for the purpose of this exam

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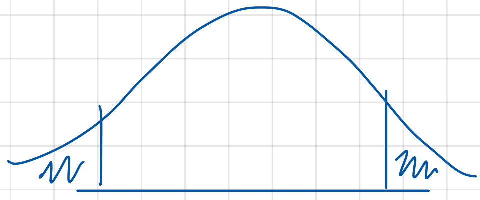
1a) $narr86_i = \beta_1 + \beta_2 pcnv_i + \beta_3 avgsen_i + \beta_4 tottime_i + \beta_5 ptime86_i + \beta_6 qemp86_i + u_i$

$narr86_i = 0.7061 - 0.1512\, pcnv_i - 0.0070\, avgsen_i + 0.0121\, tottime_i - 0.0393\, ptime86_i - 0.1031\, qemp86 + u_i$

$H_0: \beta_3 = 0$ $H_a: \beta_3 \neq 0$

$tcal = \frac{\hat{\beta}_3 - \beta_3}{se\hat{\beta}_3} = \frac{-0.0070 - 0}{0.0124} = -0.5645$ #



degree of freedom = $n-k = 2725 - 6 = 2719$ $\alpha = 0.05$

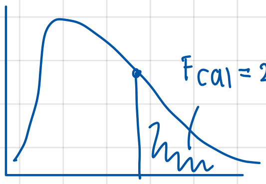
critical value = $\pm$ 1.960 #

calculated stat is not in the critical value which means we can reject the null hypothesis. we can say for sure that average sentence parameter is not zero 95 percent.-

1b) model 1.1 :

$H_0: \beta_2 = \beta_3 = \beta_4 = \beta_5 = \beta_6 = \beta_7 = \beta_8 = \beta_9$ H_a: otherwise

$Fcal = \frac{R^2/(k-1)}{1-R^2/(n-k)} = \frac{0.0428/(6-1)}{1-(.0428)/(2725-6)} = 24.3153$ #



$F_{(5,\ 2719)} = 3.02$ when $\alpha = 0.01$

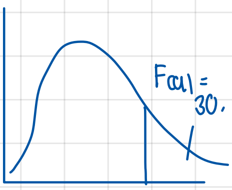
we can reject H_0 and make sure that all independent variables in this model provide significant explanation in variation of the dependent variable.

model 1.2

$H_0: \beta_2 = \beta_3 = \beta_4 = \beta_5 = \beta_6 = 0$ H_a: otherwise

$Fcal = \frac{R^2/(k-1)}{1-R^2/(n-k)} = \frac{0.0723/(8-1)}{(1-0.0723)/(2725-8)} = 30.2497$ #

$F_{(8,2716)} = 2.51$ # $\alpha = 0.01$



we can reject null hypothesis, we can make sure that the independent variables overall in this model provide significant explanation in variation of the dependent variable.

1 c) marginal conrribution:

H_0 = ethnic background and legal income has no marginal contribution to the model

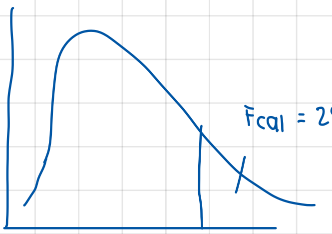
H_a = Otherwise

$$F_{cal} = \frac{R^2_{new} - R^2_{old} / (\text{no. of new regressors} \rightarrow 3)}{1 - R^2_{new} / (n - k_{new})}$$

$$F_{cal} = \frac{(0.0723 - 0.0428)/3}{(1 - 0.0723)/(2725-8)} = 28.7994 \ \#$$

$$F_{critical} = F_{(3, 2717)} = 2.60 \ \#$$

$\alpha = 0.05$



we can reject our null hypothesis.
we can make sure that variables added for ethnic background and legal income have marginal contribution to the model.

Question 2. (12 points) Dummy variables and interaction terms.

Using the Thailand labor force survey (LFS) in quarter 2 of 2019 and 2020, employees log of wage is modeled as follows. (Number of observations is 97,878 in total)

$$\ln wage_i = \beta_1 + \beta_2 civil_i + \beta_3 year_i + \beta_4 civil_i \cdot year_i + u_i$$

where $\ln wage_i$ = natural logarithmic scale of monthly wage

$civil_i$ = 1; civil servant and state employee

= 0; otherwise

$year_i$ = 1; year 2020

= 0; otherwise (2019)

This model is also known as Difference-in-Differences (DiD) and its intention is to capture the effect of COVID-19 since March of 2020 on different types of employment. During the pandemic, we assume that civil servant and state employee's wage is not reduced (control group) while others', namely employees in private firms or freelance, etc., is suspected to be reduced (treatment group). The estimation result is shown below with standard errors in parentheses. Answer the following questions.

$$\ln \widehat{wage}_i = \underset{(0.0035)}{9.1748} + \underset{(0.0072)}{0.587}\, civil_i - \underset{(0.005)}{0.0336}\, year_i + \underset{(0.0102)}{0.0444}\, civil_i \cdot year_i + u_i$$

2.a) Test all the parameters individually if each of them is significantly different from zero of not.

2.b) How much on average does a civil servant and state employee earns more or less than the others disregarding the year?

2.c) How much on average does the pandemic affect wage overall?

2.d) Are the control group and the treatment group better-off or worse-off during the pandemic. Discuss each group separately, show your work and explain with economic reasons according to the intention of this model.

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2a) t-test :

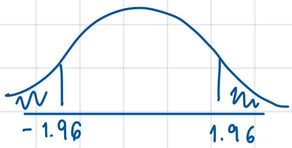
degree of freedom : 97,878 − 4 = 97874 $\alpha = 0.05$

critical value : ± 1.96

$H_0 : \beta_k = 0$ $H_a : \beta_k \neq 0$ where $k = 1, 2, 3, 4$

$t_{cal}(\beta_1) = \frac{\hat{\beta}_1 - \beta_1}{se(\hat{\beta}_1)} = \frac{9.1748 - 0}{0.0035} = 2621.3714$ #

$t_{cal}(\beta_2) = \frac{\hat{\beta}_2 - \beta_2}{se(\hat{\beta}_2)} = \frac{0.587 - 0}{0.0072} = 81.5278$ #



$t_{cal}(\beta_3) = \frac{\hat{\beta}_3 - \beta_3}{se(\hat{\beta}_3)} = \frac{-0.0336 - 0}{0.005} = -6.72$ #

$t_{cal}(\beta_4) = \frac{\hat{\beta}_4 - \beta_4}{se(\hat{\beta}_4)} = \frac{0.0444}{0.0102} = 4.3529$ #

∴ All tcal value exceeds the critical value. we can reject the null hypotheses and conclude that all the parameters are significantly different from zero #

2b) $\widehat{\ln wage_i} = 9.1748 + 0.587\ civil_i - 0.0336\ year_i + 0.0444\ civil_i \cdot year_i + u_i$

$wage_i = e^{\beta_1 + \beta_2 civil_i - \beta_3 year_i + \beta_4 civil_i \cdot year_i + u_i}$

$E(\widehat{wage_i} \mid civil_i = 1) = e^{\beta_1 + \beta_2 civil_i - \beta_3 year_i + \beta_4 civil_i \cdot year_i + u_i}$

$= e^{9.1748 + 0.587(1)}$

$E(\widehat{wage_i} \mid civil_i = 0) = e^{\beta_1 + \beta_2 civil_i - \beta_3 year_i + \beta_4 civil_i \cdot year_i + u_i}$

$= e^{9.1748}$

since we are disregarding the year we can consider β_2 to tell the difference between civil servant and other group.

$(e^{\beta_2} - 1) \times 100 = 79.86\%$ and β_2 is a positive coefficient

∴ thus, on average civil servants earn more than other groups by 79.86 % #

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2c) the pandemic effect is in year 2020 so we use β_3 to represent the effect.

$\widehat{\ln wage}_i = 9.1748 + 0.587\ civil_i - 0.0336\ year_i + 0.0444\ civil_i \cdot year_i + u_i$

pandemic happen in year 2020; so $year_i = 1, 0$

$wage_i = e^{\beta_1 + \beta_2 civil_i - \beta_3 year_i + \beta_4 civil_i \cdot year_i + u_i}$

when $year_i = 1$ $\quad E(wage_i \mid year_i = 1) = e^{\beta_1 + \beta_2 civil_i - \beta_3 year_i + \beta_4 civil_i \cdot year_i + u_i}$

$= e^{9.1748 - 0.0336(1)}$

$year_i = 0$ $\quad E(wage_i \mid year_i = 0) = e^{\beta_1 + \beta_2 civil_i - \beta_3 year_i + \beta_4 civil_i \cdot year_i + u_i}$

$= e^{9.1748}$

$(e^{\beta_3} - 1)100 = 3.30\ \%$ negative coefficient. In 2020, overall wage drops by 3.30%. for all groups.

2d) $\widehat{\ln wage}_i = 9.1748 + 0.587\ civil_i - 0.0336\ year_i + 0.0444\ civil_i \cdot year_i + u_i$

Effect of pandemic on control group & treatment group.
considering only in 2020 so $year_i = 1$

control group: $civil_i = 1$

$E(\widehat{wage}_i \mid civil_i = 1) = e^{\beta_1 + \beta_2 civil_i - \beta_3 year_i + \beta_4 civil_i \cdot year_i + u_i}$

$= e^{9.1748 + 0.587(1) - 0.0336(1) + 0.0444(1)(1)}$

Treatment group: $civil_i = 0$

$E(\widehat{wage}_i \mid civil_i = 0) = e^{\beta_1 + \beta_2 civil_i - \beta_3 year_i + \beta_4 civil_i \cdot year_i + u_i}$

$= e^{9.1748 - 0.0336(1)}$

For control group, even when there is a coefficient drop in year 2020 by 0.0336, it bounces back from the increase in β_4 coefficient (0.0444)

so to find the overall wage

$(e^{\beta_3 + \beta_4} - 1)100 = 1.09\%$, thus for control group, they are better off as their overall wage increases by 1.09%

While for the treatment group, there is no bounce back.

$(e^{\beta^3} - 1)100 = -3.3\%$. thus, the treatment group is worse off due to a decrease in overall wage by 3.3%

The result makes economic sense because civil servants wage didn't drop during the pandemic since there is no lay-off while other employment does get effect from pandemic.

Question 3. (8 points) Multicollinearity.

As cheese ages, several chemical processes take place that determine the taste of the final product. The data given pertain to concentrations of various chemicals in a sample of 30 mature cheddar cheeses and subjective measure of taste for each sample.

Estimate the model (3.1) reports in the Table 3.1

$$Taste = \beta_0 + \beta_1 acetic + \beta_2 h2s + \beta_3 lactic + u \qquad \textbf{(3.1)}$$

Where $Taste$ = Measures of taste for each sample

$acetic$ = The natural logarithm of concentration of acetic

$h2s$ = The natural logarithm of concentration of hydrogen sulfide

$lactic$ = Lactic

Table 3.1

Source	SS	df	MS
Model	5020.64468	3	1673.54823
Residual	2642.24237	26	101.624706
Total	7662.88705	29	264.237485

Number of obs	=	30
F(3, 26)	=	16.47
Prob > F	=	0.0000
R-squared	=	0.6552
Adj R-squared	=	0.6154
Root MSE	=	10.081

taste	Coefficient	Std. err.	t	P>|t|	[95% conf. interval]
acetic	1.538645	3.000501	Omitted for the purpose of this exam		
h2s	3.915242	1.153106			
lactic	18.80235	8.342614			
_cons	-34.13491	15.67628			

	acetic	h2s	lactic
acetic	1.0000		
h2s	0.2700	1.0000	
lactic	0.3607	0.6448	1.0000

Variable	VIF	1/VIF
lactic	1.83	0.546648
h2s	1.72	0.582609
acetic	1.15	0.867477
Mean VIF	1.57	

3.a) Is there evidence of multicollinearity in the data? How do you know? Explain your answers in detail and state the critical value for hypothesis testing to receive full points.

3.b) What is the property of BLUE? If there is the multicollinearity problem, is the OLS estimators still retain the property of BLUE? If not, which properties are violated?

Question 4. (8 points) Heteroscedasticity.

The data on U.S. inflation rates (%) and unemployment rates (%), 1948-2006

Estimate the model (4.1) reports in the Table 4.1

$$Inf_t = \beta_1 + \beta_2 unem_t + u_t \quad \textbf{(4.1)}$$

where Inf_t = inflation rates (%)

$unem_t$ = unemployment rates (%)

Table 4.1

Source	SS	df	MS			
				Number of obs	=	59
				F(1, 57)	=	3.85
Model	32.3284496	1	32.3284496	Prob > F	=	0.0545
Residual	478.096987	57	8.38766644	R-squared	=	0.0633
				Adj R-squared	=	0.0469
Total	510.425437	58	8.80043856	Root MSE	=	2.8961

inf	Coefficient	Std. err.	t	P>|t|	[95% conf. interval]
unem	.5054734	.2574699	Omitted for the purpose of this exam		
_cons	1.010847	1.491583			

White's general test statistic: 1.0266 Chi-sq (2)

Breusch-Pagan / Cook-Weisberg test for heteroskedasticity

chi2(1) = 1.12

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3 a)

$H_0 : \beta_k = 0 \qquad k = 1, 2, 3, 4$

$H_a : \beta_k \neq 0$

$t_{cal}(\beta_1) = \frac{\hat{\beta}_1 - \beta_1}{se(\hat{\beta}_1)} = \frac{-34.1349 - 0}{15.6763} = -2.1775$ # ; reject null

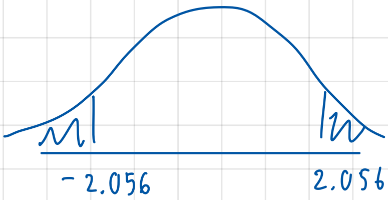
$t_{cal}(\beta_2) = \frac{\hat{\beta}_2 - \beta_2}{se(\hat{\beta}_2)} = \frac{1.5386 - 0}{3.0005} = 0.5128$ # ; accept null

$t_{cal}(\beta_3) = \frac{\hat{\beta}_3 - \beta_3}{se(\hat{\beta}_3)} = \frac{3.9152 - 0}{1.1531} = 3.3954$ # ; reject null

$t_{cal}(\beta_4) = \frac{\hat{\beta}_4 - \beta_4}{se(\hat{\beta}_4)} = \frac{18.8024 - 0}{8.3426} = 2.2538$ # ; reject null

Degree of freedom = 30 − 4 = 26 , $\alpha = 0.05$

Critical value = ± 2.056



we can reject null hypothesis for $\beta_1, \beta_3, \beta_4$ but not β_2.

Almost all the parameters are significantly different from zero. While R^2 is 0.6552 thus, we conclude that the conflicting test is not found in the model. So there is no multicollinearity.

3 b) BLUE stands for Best Linear Unbiased Estimator and it refers to the estimators having the lowest variance possible.
So it does not get affected by multicollinearity

Answer the following questions.

4.a) Interpret the intercept and slope coefficients.

4.b) According to the test statistics given after Table 4.1 below, is there any sufficient evidence to conclude that there is heteroscedasticity problem? Show your work on the hypothesis testing. (Use $\alpha = 0.05$)

4.c) Given your test results in a), do the OLS estimators still retain the property of BLUE? If not, which properties are violated?

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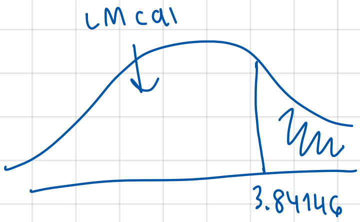
4 a) There are 2 coefficients β_1 and β_2.

β_1 is the intercept of the model. β_1 is equal to 1.01 meaning that when unemployment rate is 0, the inflation rate is 1.01.

β_2 is the slope of unemployment rate at 0.5. This means that when unemployment rate increases by 1 %, inflation increase by 0.5 %.

4b) white's general test statistic = 1.0266



so LM_{cal} for χ^2_{k-1} is 1.0266.

Find critical value χ^2_{k-1} at $\alpha = 0.05$

$$\chi^2_1 = 3.84146$$

$\therefore$ $LM_{cal} < \chi^2_1$ so we can't reject null hypothesis of homoscedasticity.

4c) since the null hypothesis in the question 2b cannot be reject, the OLS estimators still retain in the BLUE properties.