CHAPTER 9: THE CAPITAL ASSET PRICING MODEL

1. $$E(r_P) = r_f + \beta_P \times [E(r_M) - r_f]$$

$$.18 = .06 + \beta_P \times [.14 - .06] \rightarrow \beta_P = \frac{.12}{.08} = 1.5$$

4. The expected return is the return predicted by the CAPM for a given level of systematic risk.

$$E(r_i) = r_f + \beta_i \times [E(r_M) - r_f]$$

$$E(r_{\$1\ Discount}) = .04 + 1.5 \times (.10 - .04) = .13 = 13\%$$

$$E(r_{Everything\ \$5}) = .04 + 1.0 \times (.10 - .04) = .10 = 10\%$$

6. The expected return of a stock with a $\beta = 1.0$ must, on average, be the same as the expected return of the market which also has a $\beta = 1.0$.

9. a. Call the aggressive stock A and the defensive stock D. Beta is the sensitivity of the stock's return to the market return, i.e., the change in the stock return per unit change in the market return. Therefore, we compute each stock's beta by calculating the difference in its return across the two scenarios divided by the difference in the market return:

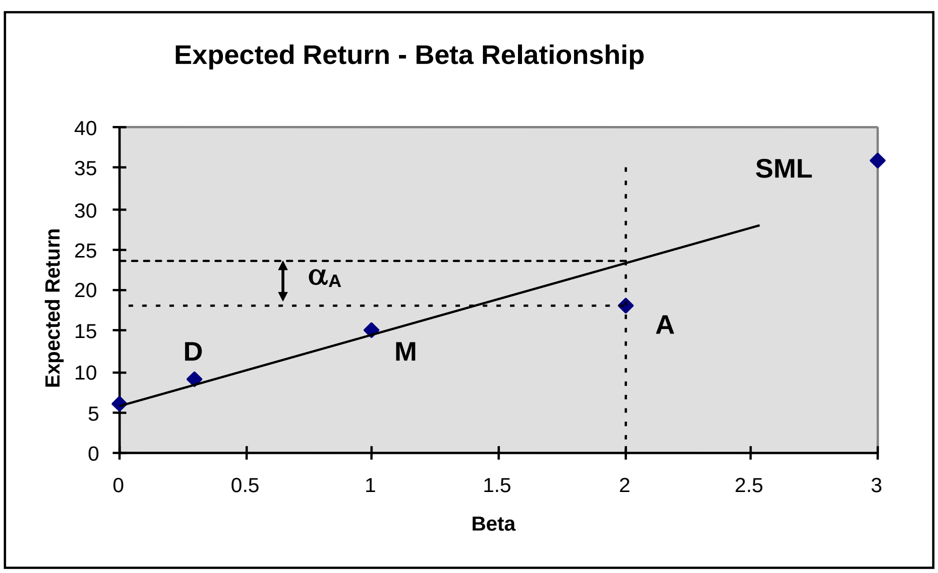
$$\beta_A = \frac{-.02 - .38}{.05 - .25} = 2.00 \qquad \beta_D = \frac{.06 - .12}{.05 - .25} = 0.30$$

b. With the two scenarios equally likely, the expected return is an average of the two possible outcomes:

$E(r_A) = 0.5 \times (-.02 + .38) = .18 = 18\%$

$E(r_D) = 0.5 \times (.06 + .12) = .09 = 9\%$

c. The SML is determined by the market expected return of $[0.5 \times (.25 + .05)] = 15\%$, with $\beta_M = 1$, and $r_f = 6\%$ (which has $\beta_f = 0$). See the following graph:



The equation for the security market line is:

$$E(r) = .06 + \beta \times (.15 - .06)$$

d. Based on its risk, the aggressive stock has a required expected return of:

$$E(r_A) = .06 + 2.0 \times (.15 - .06) = .24 = 24\%$$

The analyst's forecast of expected return is only 18%. Thus the stock's alpha is:

α_A = actually expected return – required return (given risk)

= 18% – 24% = –6%

Similarly, the required return for the defensive stock is:

$$E(r_D) = .06 + 0.3 \times (.15 - .06) = 8.7\%$$

The analyst's forecast of expected return for D is 9%, and hence, the stock has a positive alpha:

α_D = actually expected return – required return (given risk)

= .09 – .087 = +0.003 = +0.3%

The points for each stock plot on the graph as indicated above.

e. The hurdle rate is determined by the project beta (0.3), not the firm's beta. The correct discount rate is 8.7%, the fair rate of return for stock D.

10. Not possible. Portfolio A has a higher beta than Portfolio B, but the expected return for Portfolio A is lower than the expected return for Portfolio B. Thus, these two portfolios cannot exist in equilibrium.

11. Possible. If the CAPM is valid, the expected rate of return compensates only for systematic (market) risk, represented by beta, rather than for the standard deviation, which includes nonsystematic risk. Thus, Portfolio A's lower rate of return can be paired with a higher standard deviation, as long as A's beta is less than B's.

12. Not possible. The reward-to-variability ratio for Portfolio A is better than that of the market. This scenario is impossible according to the CAPM because the CAPM predicts that the market is the most efficient portfolio. Using the numbers supplied:

$$S_A = \frac{.16 - .10}{.12} = 0.5 \qquad S_M = \frac{.18 - .10}{.24} = 0.33$$

Portfolio A provides a better risk-reward tradeoff than the market portfolio.

13. Not possible. Portfolio A clearly dominates the market portfolio. Portfolio A has both a lower standard deviation and a higher expected return.

14. Not possible. The SML for this scenario is: $E(r) = 10 + \beta \times (18 - 10)$

Portfolios with beta equal to 1.5 have an expected return equal to:

$$E(r) = 10 + [1.5 \times (18 - 10)] = 22\%$$

The expected return for Portfolio A is 16%; that is, Portfolio A plots below the SML ($\alpha_A = -6\%$), and hence, is an overpriced portfolio. This is inconsistent with the CAPM.

15. Not possible. The SML is the same as in Problem 14. Here, Portfolio A's required return is: $.10 + (0.9 \times .08) = 17.2\%$

This is greater than 16%. Portfolio A is overpriced with a negative alpha:

$\alpha_A = -1.2\%$

16. Possible. The CML is the same as in Problem 12. Portfolio A plots below the CML, as any asset is expected to. This scenario is not inconsistent with the CAPM.

18. The series of $1,000 payments is a perpetuity. If beta is 0.5, the cash flow should be discounted at the rate:

$$.06 + [0.5 \times (.16 - .06)] = .11 = 11\%$$

$$PV = \$1{,}000/0.11 = \$9{,}090.91$$

If, however, beta is equal to 1, then the investment should yield 16%, and the price paid for the firm

should be: $PV = \$1{,}000/0.16 = \$6{,}250$

The difference, $2,840.91, is the amount you will overpay if you erroneously assume that beta is 0.5 rather than 1.

20. $r_1 = 19\%$; $r_2 = 16\%$; $\beta_1 = 1.5$; $\beta_2 = 1$

a. To determine which investor was a better selector of individual stocks we look at abnormal return, which is the ex-post alpha; that is, the abnormal return is the difference between the actual return and that predicted by the SML. Without information about the parameters of this equation (risk-free rate and market rate of return) we cannot determine which investor was more accurate.

b. If $r_f = 6\%$ and $r_M = 14\%$, then (using the notation alpha for the abnormal return):

$$\alpha_1 = .19 - [.06 + 1.5 \times (.14 - .06)] = .19 - .18 = 1\%$$

$$\alpha_2 = .16 - [.06 + 1 \times (.14 - .06)] = .16 - .14 = 2\%$$

Here, the second investor has the larger abnormal return and thus appears to be the superior stock selector. By making better predictions, the second investor appears to have tilted his portfolio toward underpriced stocks.

c. If $r_f = 3\%$ and $r_M = 15\%$, then:

$$\alpha_1 = .19 - [.03 + 1.5 \times (.15 - .03)] = .19 - .21 = -2\%$$

$$\alpha_2 = .16 - [.03 + 1 \times (.15 - .03)] = .16 - .15 = 1\%$$

Here, not only does the second investor appear to be the superior stock selector, but the first investor's predictions appear valueless (or worse).

21. a. Since the market portfolio, by definition, has a beta of 1, its expected rate of return is 12%.

b. β = 0 means no systematic risk. Hence, the stock's expected rate of return in market equilibrium is the risk-free rate, 5%.

c. Using the SML, the *fair* expected rate of return for a stock with β = –0.5 is:

$$E(r) = 0.05 + [(-0.5) \times (0.12 - 0.05)] = 1.5\%$$

The *actually* expected rate of return, using the expected price and dividend for next year is:

$$E(r) = \frac{\$41 + \$3}{\$40} - 1 = 0.10 = 10\%$$

Because the actually expected return exceeds the fair return, the stock is underpriced.

CFA PROBLEMS

3. a.

4. d. From CAPM, the fair expected return = 8 + 1.25 × (15 – 8) = 16.75%

Actually expected return = 17%

α = 17 – 16.75 = 0.25%

6. c.

8. d. [You need to know the risk-free rate]

12. a.

	Expected Return	Alpha
Stock X	5% + 0.8 × (14% – 5%) = 12.2%	14.0% – 12.2% = 1.8%
Stock Y	5% + 1.5 × (14% – 5%) = 18.5%	17.0% – 18.5% = –1.5%