Chapter 1:
Nature of economics analysis and Element of mathematical economics model

1.1 Structure of economic analysis

- How do we study economics?
  - Model-based analysis
- What is model?
  - Model is an *abstract* form of the problem that we are interested in.
  - The problem that we (economists) are interested in aims at getting us some insights, helping us understand something about how the world/economy works.
  - Typically, all the results in a certain model hinges on the *assumptions* used in the analysis.

- Purpose of economics model: Developed to answer three broad classes of analytical questions;
  - *Static equilibrium analysis*
  - *Comparative static analysis*
  - *Dynamic analysis*

Example 1.A: *The Price determination model*

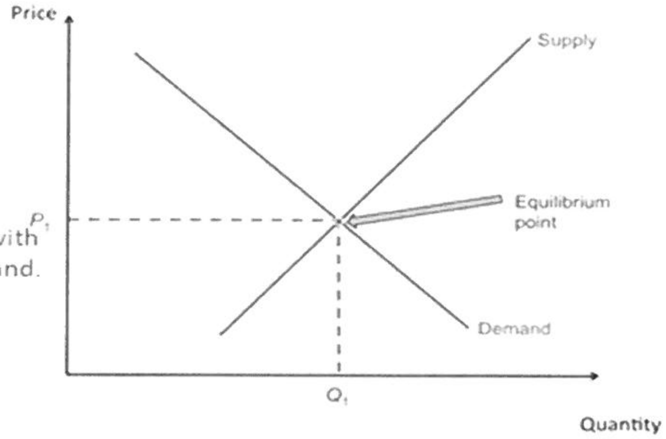
Question: At what price and quantity of output, market will trade?
- We know that there must be buyers and sellers in the market.
- To understand their trading decision, economists model the interaction of the two trading parties in the market using the basic **demand and supply model**.
- Conventionally, behavioral assumptions are that
  - Demand: Given all other factors, quantity demanded is negatively related to price
    - Downward sloping curve
  - Supply: Given all other factors, quantity supplied is positively related to price.
    - Upward sloping curve

Structure of economic analysis: Static equilibrium analysis

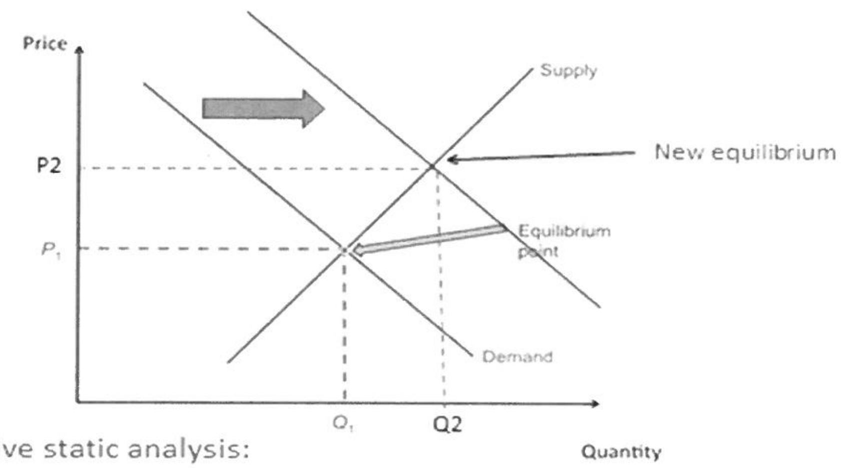
Static analysis:

- Draw demand/supply onto p-q diagram

- Equilibrium occurs when price clears the market. That is, demand matches with supply, aka no excess demand.



Structure of economic analysis: Comparative static analysis



Comparative static analysis:
For example, what if income increases?

Equilibrium Analysis → predict equ^{m} outcome

Comparative Static Analysis → Compare two ~~equilli~~ equilibria

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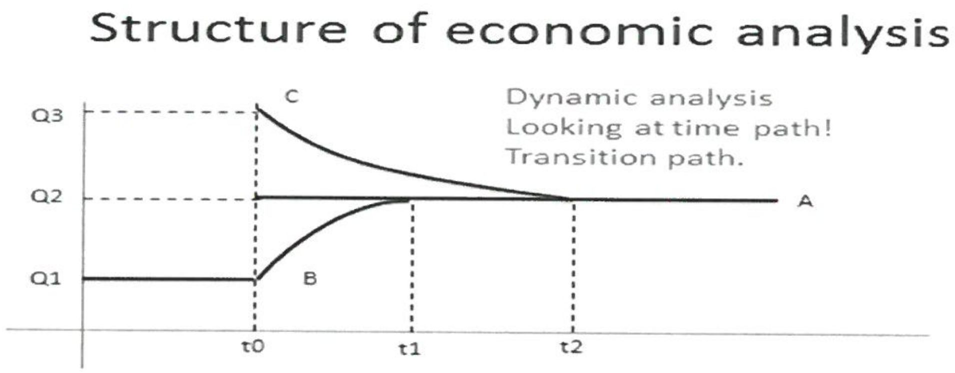
Dynamic Analysis (EE432)

Dynamic analysis



Where we are headed in this semester is to formalize all these analytical questions by using *mathematical tools.*

1.2 Elements of mathematical economic model

- Typically, any *mathematical models* involve with the following ingredients.
    - Variables
        - Quantity or amount of some objects whose value can be changed.
        - For example, Y is a variable that represents "your income in a day". X is another variable that represents how many hours you have worked in a day. (The value of both Y and X can be changed.) $y = w \cdot X$ linear/non-linear
    - Equation
        - Depending the context questions
        - An equation describes the relationship of variables, at the least two.
        - How Y and X can be linked together is summarized by an equation.
        - For example, $Y = 9X$ if hourly wage (w) is \$9.

- Nature of mathematical economic models

In general, mathematical economic models comprise of set of equations (more than one equation), each of which describes the interconnection among sets of variables. Mathematically speaking, the model has a feature of what so called *system of simultaneous equations.*

$$\begin{aligned} a_{11}x_1 + \; & a_{12}x_2 + \cdots + \; a_{1n}x_n = b_1 \\ a_{21}x_1 + \; & a_{22}x_2 + \cdots + \; a_{2n}x_n = b_2 \\ \vdots \qquad & \quad \vdots \qquad\qquad\quad \vdots \qquad \vdots \\ a_{m1}x_1 + \; & a_{m2}x_2 + \cdots + a_{mn}x_n = b_m . \end{aligned}$$

The example above is to represent a model in the ***linear system***. Model can take the ***non-linear form***.

Sometimes, mathematical economic model takes the form of set of optimality conditions derived from optimizing agent problem. Optimality condition could be a single equation or system of equations. But in any cases, the condition describes the relationship between the variables that we aim to understand their behavior and the determinant variables.

To understand common structure and basic terminologies used in mathematical economics, consider *Example 1.B* which is the reformulation of the basic market equilibrium model using mathematical approach.

Example 1.B: Consider market equilibrium model as an example.

a, b, c, d, e, f → Coefficients Parameters → kept fixed during the course of analysis

→ behavioural Equation.

Equation 1: $Q_d = a - bP - cY$; → Demand Curve

Equation 2: $Q_s = d + eP - fT$; → Supply Curve.

Equation 3: $Q_d = Q_s$ → Equilibrium Condition (Equation)

→ NO ED; NOES

Notations defined as follows:

- Q_d = quantity demanded, Q_s = quantity supplied, P = price,
- Y = income, T= temperature ➔ variables

- (a, b, c, d, e, f) = *coefficients* or *parameters* that are all positive

Treatment of the type of equations:

- Behavioral equations
- Equilibrium conditions

Treatment of the variables:

- Single equation approach:
  - Independent variable (Explanatory) (Left : Dependent Right : Independent)
  - Dependent variable
- System-based approach
  - Endogenous variable: Qd , Qs , P
  - Exogenous variable: T, y → you don't care how they are determined; take as given outside of the model.

Solve for their values.

Treated as given.

1.3.1) Static Equilibrium analysis

From example 1.B, the most common goal of studying the model is to study the equilibrium analysis, i.e. to solve for the solution of the simultaneous equations. As the exogenous variables are treated as given, solution to the system of equation would represent the *endogenous equilibrium solution*. Technically speaking, endogenous equilibrium solution is called the *reduced-form equation*.

Reduced-form equation is the mathematical function that describes the behavior of the endogenous equilibrium solution. *Typically, the function would depend on (i) exogenous variables and (ii) parameters.*

- Type of variables treated/distinguished in the model
  - Exogenous variables = treated as "*given*" in the model. Their values are given as known thing, and *will not be solved* within the model.

- o <u>Endo</u>genous variable = set of variables that will be solved for.

Example 1.C: National income model

income

parameters in the equations

A very basic/simple version of Keynesian cross model

- Consumption function: $C = C(Y, a, b)$ → general form.
- Investment equation: $I = I_0$ → Investment is Exo
- Government equation: $G = G_0$ → Exo
- Equilibrium condition: $Y = C + I + G$ Total output = Total spending.

Variable Lists:

Y = income, C = consumption

I = investment and G = government expenditure

- ***Static equilibrium analysis***: characterizing the equilibrium solution of the model by solving for a solution of the system of equations. By the definition, the reduced-form solution takes the following *generic* representation.

f^n/Equation ⇒ Evolution/Variation of Endogenous Variables.

$$Y^* = f(I_0, G_0, a, b)$$

$$C^* = g(I_0, G_0, a, b)$$

- If we assume the function form, the reduced-from solution would have an analytical (specific) solution which is specific to the assuming conditions on the behavioral equation.
  - o For example, we might assume that $C(Y, a, b) = a + bY$

↳ autonomous Consumption

↳ marginal propensity to consume (MPC)

Keywords: *Reduced-form equation, generic solution, specific solution*

1.3.2) Comparative Static analysis:

Having solved for the equilibrium solution, what economists usually ask is what would happen to the equilibrium if something, previously assumed to be fixed, has changed.

Example 1.C (cont.): National income model

$y = C + I + G$; $y = C + I_0 + G_0$

$I = I_0$

$G = G_0$

$C = C(y, a, b) = a + b \cdot y$

$y = a + by + I_0 + G_0$ → *not* reduced-form Equation

$$y^* = \frac{a + I_0 + G_0}{1 - b}$$ → reduced-form Equation output / Income.

Equilibrium Value of your Eq$^{\text{m}}$ income

(i) Exo (I_0, G_0)

(ii) Parameter (a, b)

$C = a + b \cdot y$

$$C^* = a + b \cdot y^* = a + \frac{b}{1-b}(a + I_0 + G_0)$$

reduced-form Equation for Consumption

- Delta Approach (Δ)
- Derivative Approach. (d)

(a', b', I_0', G_0')

$\hookrightarrow y^{*,'}$

$$y^{*,'} = \frac{a' + I_0' + G_0'}{1 - b'}$$

(a', b', I_0', G_0'') $\quad G_0'' > G_0'$

$$y^{*,''} = \frac{a' + I_0' + G_0''}{1 - b'}$$

Size of change in Endogenous Equ^n Sol^n

$y^{*,''} - y^{*,'}$ → multiplier : $\frac{1}{1-b} > 1$

$0 < b < 1$

$$= \frac{1}{1-b'} \underbrace{(G_0'' - G_0')}$$

b': marginal propensity to consume.

size the increase/decrease of government Expenditure (Exogenous)

- From the example 1.B, it is straightforward to solve for all the endogenous equilibrium solutions, Y^*, C^*.

- Numerically, if a = 1, I_0 = \$1, G_0 = \$1 and b = 0.5, this yields us,

Question: What if G is now changed to \$2, how big is the change in Y* and C*?

Answer:

New Y* =______________ ➔

A $1 increase in G causes an increase in Y* by ________

In the later part, we will try to figure out the level of multiplier when we don't assign any numerical values of variables.

- The idea is simple. *We just apply the derivative method to the equilibrium solution function*
- That is we calculate the value of $\frac{\partial Y^*}{\partial I_0}, \frac{\partial Y^*}{\partial G_0}, \frac{\partial Y^*}{\partial a}, \frac{\partial Y^*}{\partial b}$.
