The Solow growth model

- The basis of all modern theories of growth.

- Long-term economic growth depends on one single factor --- **technological progress**.
  - Rising **total factor productivity (z)**.
  - Sustained improvement in living standards (real per capita income or output per worker).

Population growth

- Assume population grows exogenously at a constant rate.
- N = population in the current period.
- N' = population in the future period.
- n > -1; rate of population growth.

Current people

Total number of ppl in the future period

$$N' = (1 + n)N$$

rate of new People

Consumers

Consume "C"

"work at firm" → N supply labor force

- Consumers = population = workers.
- Consumers supply labor in production.
- Consumers receive real output (Y) as (wage and dividend) income.

  → where do they get the output from?? Firm

  - Spend on consumption goods (C) and save a constant fraction (s) of Y as savings (S).

$$Y = C + S; \quad \underline{S = sY}$$

fixed fraction of your income

① "given" → saving rate

② Consumption Behaviour →

$$C = (1 - s)Y$$

Indentical Representative firm.

The representative firm

own a Production technology. → production function

- The firm produces output using current capital stock (K) and current labor input (N).
- Assuming **constant returns to scale**. *

→ ① Easy to work with / derive results
② Empirically CRTS matches data

Existing machines tools available at the time firm makes production decision

Level of technology.

$$\boxed{Y = zF(K, N)}$$

→ production function

$$\frac{Y}{N} = zF\left(\frac{K}{N}, 1\right)$$

income per ppl →

"Level of output" ← K, N.

$\frac{\text{Level of output}}{\text{worker}}$; $\frac{K}{N}$

capital-to-worker ratio

Per worker production function

"Level in the long-term"

$$Let\ y = \frac{Y}{N} = output\ per\ worker$$

→ Income per capita

analyze what is the dynamic of k

$$k = \frac{K}{N} = capital\ per\ worker$$

$$f(k) = zF\left(\frac{K}{N}, 1\right)$$

$$y = zf(k)$$

Example

$$Y = z \cdot K^{\alpha} N^{1-\alpha}$$

$$y = \frac{Y}{N} = z\,\frac{K^{\alpha} \cdot N^{1-\alpha}}{N}$$

$$= z \cdot K^{\alpha} \cdot N^{-\alpha}$$

$$= z \cdot \boxed{\left(\underbrace{\frac{K}{N}}_{k}\right)^{\alpha}}$$

$\leftarrow f(k)$

F: Aggregate

f: per-capita (per ppl.)

Property of "f" ⇒ (1) more "k" more "y" → per-worker production f^n is increasing f^n

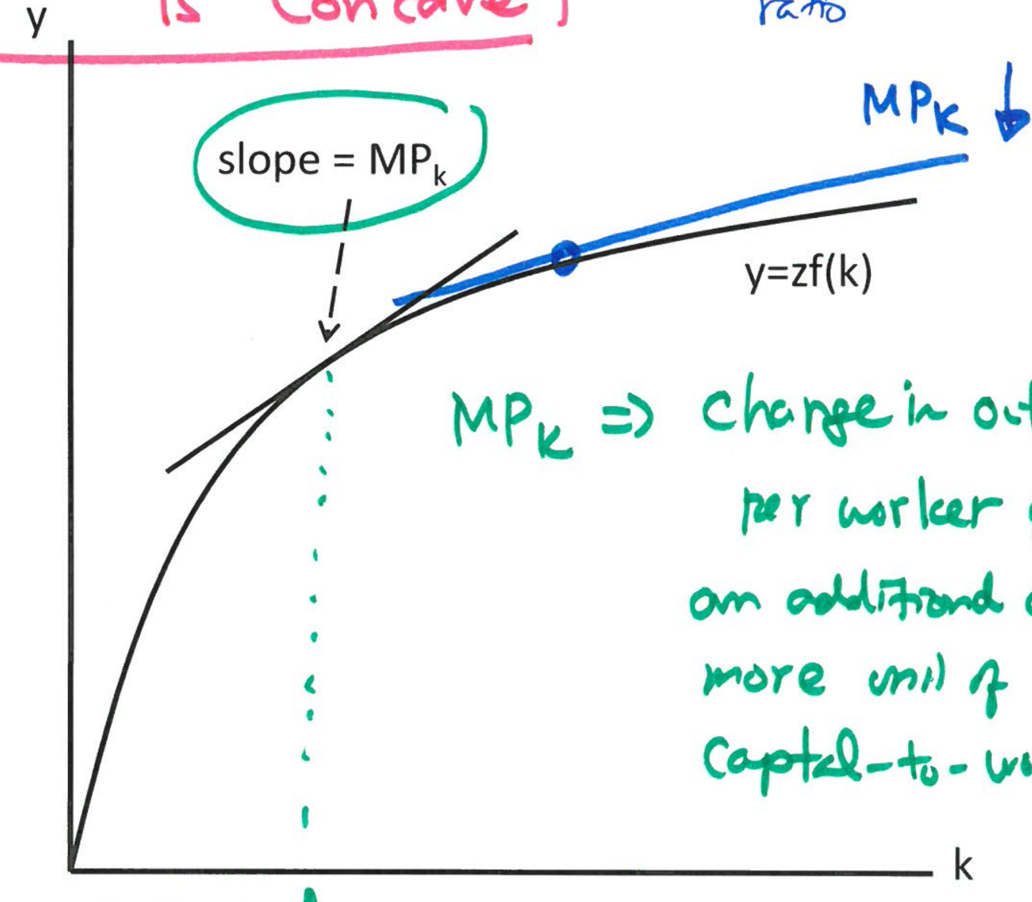
(2) MPK marginal product of "k" → decline as "k" ↑ ; follows the law of diminishing marginal product of capital-to-labor ratio

Marginal product of k

Per worker Production f^n is Concave

(y) – long-term

- Output per worker (y) increases at a decreasing rate as capital per worker (k) rises.
- Slope is the marginal product of k.



MP$_k$ ⇒ change in output per worker given an additional one more unit of capital-to-worker

"k" dynamic

speed of the increase in "y" gets slower as little "k" keeps increasing!

Growth of capital stock
Capital accumulation

- Assumer capital wears out over time at the rate of d (or depreciation).
  - where 0 > d > 1.
- I = investment = addition to capital stock.
- K’ = capital stock in the future period.

$$K' = \underbrace{(1-d)K}_{\text{net capital stock after taking into account the depreciation}} + I$$

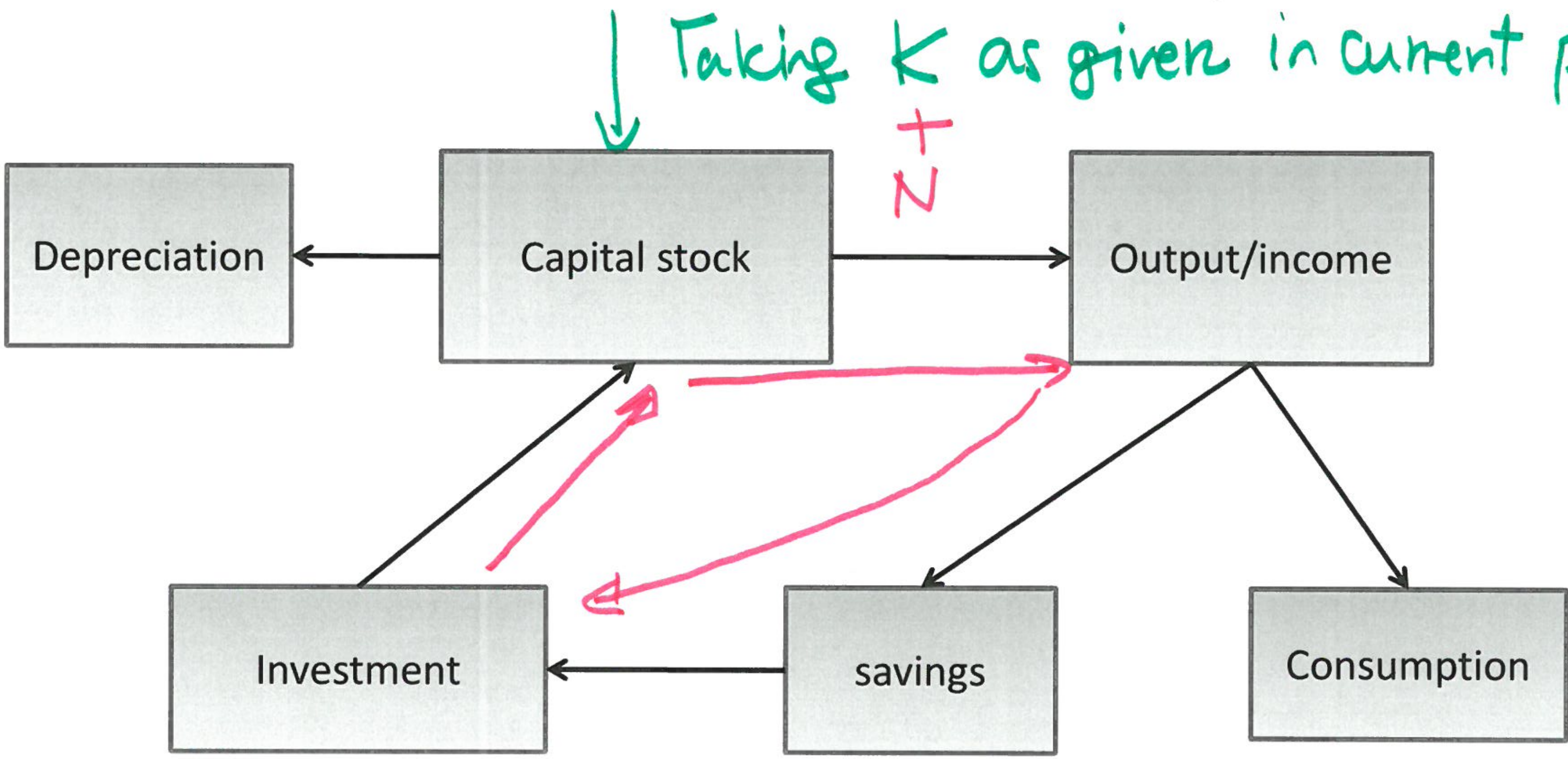
K' → tomorrow.

K → current period ~~Captl~~ Capital stock

I → new capital added to

The working of growth process: capital accumulation drives output

Taking K + N as given in current period



how long can we rely on this as the driving ~~fore~~ force to output/growth ?? } ⇒ reach any limit?

Solution of model: Equilibrium output

- At equilibrium, **savings equals investment** so that output consists of consumption and investment.

$$S = I \rightarrow \text{Eqm}^{\underline{m}}.$$

$$\boxed{S = Y - C}$$

$$Y = C + I$$

Consumption

investment

$$C = (1-s) \cdot Y$$

$$K' - (1-d) \cdot K$$

$$Y = (1-s) \cdot Y + K' - (1-d) \cdot K$$

$y \rightarrow$ income per worker

Equilibrium condition

- The future capital stock is current capital stock deducted by depreciation and added by investment (= savings).

$$Y = C + I$$

$$C = (1 - s)Y$$

$$I = K' - (1 - d)K$$

- Substitute C and I in the Y equation.

Per worker formulation

$$\cancel{Y} = (1-s)Y + K' - (1-d)K$$

~~Y~~ − sY

Equ^{m} condition for the aggregate level

rearrange the terms:

⇒ saving = investment

$$K' = sY + (1-d)K$$

K↑ → higher Y → I↑ → K' → Y' (next period output ↑)

but $Y = zF(K, N)$

this growth process occurs under a dynamic

Capital accumulation function in terms of K

so $K' = szF(K, N) + (1-d)K$

divide it by N :

$\frac{K'}{N'} = k'$

$$\frac{N'}{N'} \times \boxed{\frac{K'}{N} = sz\underbrace{\frac{F(K,N)}{N}}_{f(k)} + (1-d)\underbrace{\frac{K}{N}}_{k}}$$

$$\underbrace{\frac{K'}{N'}}_{k'} \cdot \frac{N'}{N} \Rightarrow (1+n)$$

*

$\frac{Y}{N} \Rightarrow \frac{K}{N}$

Dynamic of "k"

Future capital per worker function

$$\frac{K'}{N}\frac{N'}{N'} = szF(\frac{K}{N}, 1) + (1-d)\frac{K}{N}$$

$$where \quad k' = \frac{K'}{N'} \quad and \quad \frac{N'}{N} = (1+n)$$

$$\rightarrow k'(1+n) = szf(k) + (1-d)k$$

$$k' = \frac{szf(k)}{(1+n)} + \frac{(1-d)k}{(1+n)}$$

$\boxed{k' \text{ written in terms of } k}$

Dynamic of capital-to-labor

- Future k' as a function of current k. accumulation process