

Homework #1

Consider the problem of an oil company that wants to maximize profits from an oil well.

Revenue at time t is given as

$$R_t = p_t \cdot u_t,$$

Where p_t is the price of oil at t , and where u_t is the amount of oil that is extracted and sold. The company's cost function is quadratic in the amount of oil that is extracted,

$$C_t = 0.05 u_t^2$$

The amount of oil remaining in the well follows the recursion or transition equation

$$x_{t+1} = x_t - u_t,$$

where x_t is known as state variable in dynamic programming language. The company applies the discount factor $\beta = 0.9$ for profit streams that occur in the future. We also assume that the company intends to have the oil well depleted in 4 years, which means that $x_4=0$. This is known as the boundary condition in the dynamic programming. The question to be solved is how much oil should be pumped at time t , $t = 0, 1, 2, 3$ to maximize the discount profit stream.

Therefore, the above problem can also be stated as follows,

$$\text{Maximize} \quad \sum_{t=0}^3 \beta^t (p_t u_t - 0.05 u_t^2)$$

subject to the transition equations

$$x_{t+1} = x_t - u_t, \quad t = 0, 1, 2, 3$$

and the boundary condition

$$x_4 = 0$$

and it is assumed that $\beta = 0.9$.

Then, solve the above problem for optimal policy rules, u_t for $t = 0, 1, 2, 3$, with the help of the Bellman equations.