

## Proof t-test<sup>2</sup> = F-test

### **\*Run regression**

```
. reg y x1 x2 x3 x4 x5
```

Source	SS	df	MS	Number of obs	=	23
Model	1129.30649	5	225.861299	F(5, 17)	=	57.63
Residual	66.622243	17	3.91895547	Prob > F	=	0.0000
-----				R-squared	=	0.9443
-----				Adj R-squared	=	0.9279
Total	1195.92874	22	54.3603971	Root MSE	=	1.9796

y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
x1	.0048894	.0049619	0.99	0.338	-.0055794	.0153581
x2	-.6518875	.1743999	-3.74	0.002	-1.019839	-.2839359
x3	.2432418	.0895442	2.72	0.015	.05432	.4321636
x4	.1043175	.0706436	1.48	0.158	-.0447275	.2533625
x5	-.0711103	.0983811	-0.72	0.480	-.2786762	.1364556
_cons	38.59691	4.214487	9.16	0.000	29.70512	47.4887

### **\*Obtain estimated coefficients**

```
. mat beta=e(b)
```

```
. mat list beta
```

```
beta[1,6]
```

	x1	x2	x3	x4	x5	_cons
y1	.00488935	-.65188752	.24324181	.10431748	-.07111027	38.596911

```
. sca b4=e1(beta,1,4)
```

```
. sca b5=e1(beta,1,5)
```

### **\*Obtain Variance-Covariance matrix of estimated coefficients**

```
. mat V=e(V)
```

```
. mat list V
```

```
symmetric V[6,6]
```

	x1	x2	x3	x4	x5	_cons
x1	.00002462					
x2	.00022013	.03041532				
x3	-.00017787	-.01166837	.00801817			
x4	-.00023346	-.00540615	.00392333	.00499052		
x5	.00001652	.00554156	-.00610215	-.00473864	.00967883	
_cons	.00729795	-.55783048	.18926952	.03656747	-.18572828	17.761903

```
. sca Vb4=e1(V,4,4)
```

```
. sca Vb5=e1(V,5,5)
```

```
. sca Vb4b5=e1(V,5,4)
```

```

. sca list Vb4 Vb5 Vb4b5
      Vb4 = .00499052
      Vb5 = .00967883
      Vb4b5 = -.00473864

*Compute Standard Error b4+b5
. sca SEb4b5=sqrt(Vb4+Vb5+2*(Vb4b5))

. sca list SEb4b5
      SEb4b5 = .07205608

*Compute t-test of b4+b5
. sca ttest=(b4+b5)/SEb4b5

. sca pvalue=ttail(17,abs(ttest))*2

. sca list ttest pvalue
      ttest = .46085224
      pvalue = .65074559

*Compute t-test^2
. sca F=ttest^2

. sca list F
      F = .21238479

*Perform Restricted-F-test using STATA command
. test x4+x5=0

( 1)  x4 + x5 = 0

      F( 1, 17) = 0.21
      Prob > F = 0.6507

```

## Proof STATA Command test and Restricted-Unrestricted F-test Formula

**\*Run Unrestricted Model and Perform the test using STATA command**

```
. reg y x1 x2 x3 x4 x5
```

Source	SS	df	MS	Number of obs	=	23
-----+-----				F(5, 17)	=	57.63
Model	1129.30649	5	225.861299	Prob > F	=	0.0000
Residual	66.622243	17	3.91895547	R-squared	=	0.9443
-----+-----				Adj R-squared	=	0.9279
Total	1195.92874	22	54.3603971	Root MSE	=	1.9796

y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
-----+-----						
x1	.0048894	.0049619	0.99	0.338	-.0055794	.0153581
x2	-.6518875	.1743999	-3.74	0.002	-1.019839	-.2839359
x3	.2432418	.0895442	2.72	0.015	.05432	.4321636
x4	.1043175	.0706436	1.48	0.158	-.0447275	.2533625
x5	-.0711103	.0983811	-0.72	0.480	-.2786762	.1364556
_cons	38.59691	4.214487	9.16	0.000	29.70512	47.4887

```
. sca rss_ur=e(rss)
```

```
. test x4 x5
```

```
( 1) x4 = 0
```

```
( 2) x5 = 0
```

```

F( 2, 17) = 1.17
Prob > F = 0.3354

```

```
. sca F_x4x5=r(F)
```

```
. sca p_Fx4x5=r(p)
```

**\*Run Restricted Model**

```
. reg y x1 x2 x3
```

Source	SS	df	MS	Number of obs	=	23
-----+-----				F(3, 19)	=	93.65
Model	1120.17021	3	373.390069	Prob > F	=	0.0000
Residual	75.7585287	19	3.98729099	R-squared	=	0.9367
-----+-----				Adj R-squared	=	0.9267
Total	1195.92874	22	54.3603971	Root MSE	=	1.9968

y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
-----+-----						
x1	.0108762	.0023812	4.57	0.000	.0058922	.0158602
x2	-.5410846	.1579697	-3.43	0.003	-.8717189	-.2104503
x3	.1740546	.0625307	2.78	0.012	.0431764	.3049328
_cons	38.6472	3.6496	10.59	0.000	31.0085	46.2859

```
. sca rss_r=e(rss)

*Compute Restricted-Unrestricted F-test using Formula
. sca F_r_ur=((rss_r-rss_ur)/2)/(rss_ur/17)

. sca p_F_r_ur=Ftail(2,17,F_r_ur)

*Compare the tests
. sca list F_r_ur p_F_r_ur F_x4x5 p_Fx4x5
      F_r_ur = 1.1656532
      p_F_r_ur = .33542514
      F_x4x5 = 1.1656532
      p_Fx4x5 = .33542514
```

## Proof Chow-test and Dummy Variables Technique

### \*Chow-Structural-Break-Test

\*Estimate using all observations

. reg y x1 x2 x3 x4 x5

Source	SS	df	MS	Number of obs	=	
-----+-----				F(5, 17)	=	57.63
Model	1129.30649	5	225.861299	Prob > F	=	0.0000
Residual	66.622243	17	3.91895547	R-squared	=	0.9443
-----+-----				Adj R-squared	=	0.9279
Total	1195.92874	22	54.3603971	Root MSE	=	1.9796

y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
-----+-----						
x1	.0048894	.0049619	0.99	0.338	-.0055794	.0153581
x2	-.6518875	.1743999	-3.74	0.002	-1.019839	-.2839359
x3	.2432418	.0895442	2.72	0.015	.05432	.4321636
x4	.1043175	.0706436	1.48	0.158	-.0447275	.2533625
x5	-.0711103	.0983811	-0.72	0.480	-.2786762	.1364556
_cons	38.59691	4.214487	9.16	0.000	29.70512	47.4887
-----+-----						

. sca rss1=e(rss)

\*Estimate two sub-sample periods

. reg y x1 x2 x3 x4 x5 if obs<1972

Source	SS	df	MS	Number of obs	=	
-----+-----				F(5, 6)	=	292.39
Model	205.672619	5	41.1345239	Prob > F	=	0.0000
Residual	.844097685	6	.140682947	R-squared	=	0.9959
-----+-----				Adj R-squared	=	0.9925
Total	206.516717	11	18.774247	Root MSE	=	.37508

y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
-----+-----						
x1	.0181174	.0033991	5.33	0.002	.0098	.0264348
x2	-.2427424	.1023535	-2.37	0.055	-.4931923	.0077075
x3	.244597	.1293168	1.89	0.107	-.0718297	.5610238
x4	.0683535	.1537339	0.44	0.672	-.3078199	.4445268
x5	-.0912344	.2909469	-0.31	0.764	-.8031558	.620687
_cons	19.23903	4.279744	4.50	0.004	8.76687	29.71118
-----+-----						

. sca rss2=e(rss)

. reg y x1 x2 x3 x4 x5 if obs>=1972

Source	SS	df	MS	Number of obs	=	
-----+-----				F(5, 5)	=	75.54
Model	240.176756	5	48.0353511	Prob > F	=	0.0001
Residual	3.17961169	5	.635922339	R-squared	=	0.9869
-----+-----				Adj R-squared	=	0.9739

Total | 243.356367                    10   24.3356367    Root MSE                    =           .79745

y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
x1	.008905	.0023553	3.78	0.013	.0028504	.0149595
x2	-.0725872	.1149666	-0.63	0.556	-.3681184	.2229439
x3	.0107341	.0509495	0.21	0.841	-.1202356	.1417039
x4	.1114002	.0297072	3.75	0.013	.0350353	.1877651
x5	-.1610332	.0441186	-3.65	0.015	-.2744438	-.0476226
_cons	39.218	3.041486	12.89	0.000	31.39961	47.03639

. sca rss3=e(rss)

**\*Compute Chow-Test**

. sca Chowtest=((rss1-rss2-rss3)/6)/((rss2+rss3)/(23-12))

. sca pvalue\_Chow=Ftail(6,11,Chowtest)

**\*Dummy Variable Technique**

**\*Generate Dummy Variable**

. \*g d=(obs>=1972)  
 . \*g dx1=d\*x1  
 . \*g dx2=d\*x2  
 . \*g dx3=d\*x3  
 . \*g dx4=d\*x4  
 . \*g dx5=d\*x5

**\*Estimate Model with Intercept and Slope Dummy**

. reg y x1 x2 x3 x4 x5 d dx1 dx2 dx3 dx4 dx5

Source	SS	df	MS	Number of obs	=	23
				F(11, 11)	=	296.22
Model	1191.90503	11	108.355002	Prob > F	=	0.0000
Residual	4.02370938	11	.365791762	R-squared	=	0.9966
				Adj R-squared	=	0.9933
Total	1195.92874	22	54.3603971	Root MSE	=	.60481

y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
x1	.0181174	.0054811	3.31	0.007	.0060536	.0301812
x2	-.2427424	.1650436	-1.47	0.169	-.606001	.1205162
x3	.244597	.2085216	1.17	0.266	-.214356	.7035501
x4	.0683535	.247894	0.28	0.788	-.4772575	.6139644
x5	-.0912344	.4691481	-0.19	0.849	-1.123822	.9413535
d	19.97897	7.276354	2.75	0.019	3.963824	35.99412
dx1	-.0092125	.0057648	-1.60	0.138	-.0219008	.0034759
dx2	.1701551	.1866606	0.91	0.382	-.2406822	.5809925
dx3	-.2338629	.2120718	-1.10	0.294	-.7006298	.232904
dx4	.0430467	.2489158	0.17	0.866	-.5048132	.5909066
dx5	-.0697988	.4703398	-0.15	0.885	-1.10501	.9654121
_cons	19.23903	6.901031	2.79	0.018	4.049959	34.42809

**\*Compute Test for Intercept and All Slope Dummy Coefficients**

```
. test d dx1 dx2 dx3 dx4 dx5
```

```
( 1) d = 0  
( 2) dx1 = 0  
( 3) dx2 = 0  
( 4) dx3 = 0  
( 5) dx4 = 0  
( 6) dx5 = 0
```

```
      F( 6, 11) = 28.52  
      Prob > F = 0.0000
```

```
. sca ChowDummy=r(F)
```

```
. sca p_ChowDummy=r(p)
```

**\*Compare Chow-test & Dummy Variable Technique**

```
. sca list Chowtest pvalue_Chow ChowDummy p_ChowDummy  
      Chowtest = 28.521935  
      pvalue_Chow = 4.335e-06  
      ChowDummy = 28.521935  
      p_ChowDummy = 4.335e-06
```

**Proof One-way ANOVA & ANOVA Regression Model**

**\*Run One-way ANOVA test**

. oneway y Day, tabulate

Day	Summary of Y		Freq.
	Mean	Std. Dev.	
1	-.13904595	1.9925471	42
2	-.27028511	1.533583	45
3	-.19944609	1.9328141	46
4	.123915	2.0204847	46
5	.39229522	1.6774294	44
Total	-.01890673	1.8409723	223

Source	Analysis of Variance			F	Prob > F
	SS	df	MS		
Between groups	13.3272886	4	3.33182216	0.98	0.4178
Within groups	739.070471	218	3.39023152		
Total	752.39776	222	3.3891791		

Bartlett's test for equal variances: chi2(4) = 4.8570 Prob>chi2 = 0.302

**\*Run Model with Dummy Variables**

. reg y d2 d3 d4 d5

Source	SS	df	MS	Number of obs	=	223
Model	13.3272886	4	3.33182216	F(4, 218)	=	0.98
Residual	739.070471	218	3.39023152	Prob > F	=	0.4178
				R-squared	=	0.0177
				Adj R-squared	=	-0.0003
Total	752.39776	222	3.3891791	Root MSE	=	1.8413

y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
d2	-.1312392	.3950421	-0.33	0.740	-.9098299	.6473516
d3	-.0604001	.3929637	-0.15	0.878	-.8348945	.7140942
d4	.2629609	.3929637	0.67	0.504	-.5115334	1.037455
d5	.5313412	.3972034	1.34	0.182	-.2515092	1.314192
_cons	-.1390459	.2841123	-0.49	0.625	-.6990045	.4209126

## Dummy Variable Regression Models

### ANOVA Model

In the study of seasonal effect (Monday and Friday effects), ANOVA model is estimated using both models with and without intercept term:

Model with Intercept term: 
$$Y_t = \beta_1 + \beta_2 D_{2t} + \beta_3 D_{3t} + \beta_4 D_{4t} + \beta_5 D_{5t} + u_t$$

Model without Intercept term: 
$$Y_t = \beta_1 D_{1t} + \beta_2 D_{2t} + \beta_3 D_{3t} + \beta_4 D_{4t} + \beta_5 D_{5t} + u_t$$

where:  $Y_t$  = Return on asset at time  $t$ .

$D_{1t}$  = 1 on Monday and = 0 otherwise.

$D_{2t}$  = 1 on Tuesday and = 0 otherwise.

$D_{3t}$  = 1 on Wednesday and = 0 otherwise.

$D_{4t}$  = 1 on Thursday and = 0 otherwise.

$D_{5t}$  = 1 on Friday and = 0 otherwise.

. reg y d2 d3 d4 d5

Source	SS	df	MS	Number of obs =	995
Model	.00681682	4	.001704205	F( 4, 990) =	7.60
Residual	.221894717	990	.000224136	Prob > F =	0.0000
Total	.228711537	994	.000230092	R-squared =	0.0298
				Adj R-squared =	0.0259
				Root MSE =	.01497

y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
d2	.0044637	.0015009	2.97	0.003	.0015184 .0074089
d3	.0044351	.0015009	2.96	0.003	.0014898 .0073803
d4	.0040519	.0015009	2.70	0.007	.0011067 .0069972
d5	.0082592	.0015009	5.50	0.000	.005314 .0112045
_cons	-.0040211	.0010613	-3.79	0.000	-.0061037 -.0019385

. reg y d1 d2 d3 d4 d5, noconstant

Source	SS	df	MS	Number of obs =	995
Model	.006865358	5	.001373072	F( 5, 990) =	6.13
Residual	.221894717	990	.000224136	Prob > F =	0.0000
Total	.228760075	995	.00022991	R-squared =	0.0300
				Adj R-squared =	0.0251
				Root MSE =	.01497

y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
d1	-.0040211	.0010613	-3.79	0.000	-.0061037 -.0019385
d2	.0004426	.0010613	0.42	0.677	-.0016401 .0025252
d3	.000414	.0010613	0.39	0.697	-.0016687 .0024966
d4	.0000308	.0010613	0.03	0.977	-.0020518 .0021134
d5	.0042381	.0010613	3.99	0.000	.0021555 .0063207

Interpretations of the estimated results of the two models are as follows:

Model	With intercept term	Without intercept
Monday	$\hat{Y}_t = \hat{\beta}_1 = -0.004021$	$\hat{Y}_t = \hat{\beta}_1 = -0.004021$
Tuesday	$\hat{Y}_t = \hat{\beta}_1 + \hat{\beta}_2 = -0.004021 + 0.004464 = 0.000443$	$\hat{Y}_t = \hat{\beta}_2 = 0.000443$
Wednesday	$\hat{Y}_t = \hat{\beta}_1 + \hat{\beta}_3 = -0.004021 + 0.004435 = 0.000414$	$\hat{Y}_t = \hat{\beta}_3 = 0.000414$
Thursday	$\hat{Y}_t = \hat{\beta}_1 + \hat{\beta}_4 = -0.004021 + 0.004052 = 0.000031$	$\hat{Y}_t = \hat{\beta}_4 = 0.000031$
Friday	$\hat{Y}_t = \hat{\beta}_1 + \hat{\beta}_5 = -0.004021 + 0.008259 = 0.004238$	$\hat{Y}_t = \hat{\beta}_5 = 0.004238$

For the purpose of testing only Monday and Friday effect, the model can be stated as:

Model to test Monday and Friday effect:  $Y_t = \beta_0 + \beta_1 D_{1t} + \beta_5 D_{5t} + u_t$

. reg y d1 d5

Source	SS	df	MS			
Model	.006795783	2	.003397892	Number of obs =	995	
Residual	.221915754	992	.000223705	F( 2, 992) =	15.19	
Total	.228711537	994	.000230092	Prob > F =	0.0000	
				R-squared =	0.0297	
				Adj R-squared =	0.0278	
				Root MSE =	.01496	

  

y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
d1	-.0043169	.0012243	-3.53	0.000	-.0067194	-.0019144
d5	.0039423	.0012243	3.22	0.001	.0015399	.0063448
_cons	.0002958	.0006121	0.48	0.629	-.0009055	.001497

Interpretation of the estimated result of the model is as follow:

Model	
Tuesday Wednesday Thursday	$\hat{Y}_t = \hat{\beta}_0 = 0.000296$
Monday	$\hat{Y}_t = \hat{\beta}_0 + \hat{\beta}_1 = 0.000296 - 0.004317 = -0.004021$
Friday	$\hat{Y}_t = \hat{\beta}_0 + \hat{\beta}_5 = 0.000296 - 0.003942 = 0.004238$

## ANCOVA Model

If the model also includes quantitative independent variable ( $X_t$ ), the model can be stated as:

Model with intercept dummy: 
$$Y_t = \beta_0 + \beta_1 X_{1t} + \gamma_0 D_{1t} + \lambda_0 D_{5t} + u_t$$

This model can be interpreted as:

Model for Tuesday, Wednesday and Thursday: 
$$Y_t = \beta_0 + \beta_1 X_{1t} + u_t$$

Model for Monday: 
$$Y_t = (\beta_0 + \gamma_0) + \beta_1 X_{1t} + u_t$$

Model for Friday: 
$$Y_t = (\beta_0 + \lambda_0) + \beta_1 X_{1t} + u_t$$

```
. reg y x1 d1 d5
```

Source	SS	df	MS			
Model	.007808227	3	.002602742	Number of obs =	995	
Residual	.220903311	991	.000222909	F( 3, 991) =	11.68	
Total	.228711537	994	.000230092	Prob > F =	0.0000	
				R-squared =	0.0341	
				Adj R-squared =	0.0312	
				Root MSE =	.01493	

  

y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
x1	1.90e-06	8.91e-07	2.13	0.033	1.50e-07	3.65e-06
d1	-.0043131	.0012221	-3.53	0.000	-.0067113	-.0019149
d5	.0039385	.0012221	3.22	0.001	.0015403	.0063368
_cons	-.0708331	.0333809	-2.12	0.034	-.1363383	-.0053278

Model with intercept and slope dummy variables:

$$Y_t = \beta_0 + \gamma_0 D_{1t} + \lambda_0 D_{5t} + \beta_1 X_{1t} + \gamma_1 D_{1t} X_{1t} + \lambda_1 D_{5t} X_{1t} + u_t$$

This model can be interpreted as:

Model for Tuesday, Wednesday and Thursday: 
$$Y_t = \beta_0 + \beta_1 X_{1t} + u_t$$

Model for Monday: 
$$Y_t = (\beta_0 + \gamma_0) + (\beta_1 + \gamma_1) X_{1t} + u_t$$

Model for Friday: 
$$Y_t = (\beta_0 + \lambda_0) + (\beta_1 + \lambda_1) X_{1t} + u_t$$

```
. g x1d1 = x1*d1
```

```
. g x1d5 = x1*d5
```

```
. reg y d1 d5 x1 x1d1 x1d5
```

Source	SS	df	MS			
Model	.008022894	5	.001604579	Number of obs =	995	
Residual	.220688644	989	.000223143	F( 5, 989) =	7.19	
Total	.228711537	994	.000230092	Prob > F =	0.0000	
				R-squared =	0.0351	
				Adj R-squared =	0.0302	
				Root MSE =	.01494	

  

y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
d1	-.0541205	.086225	-0.63	0.530	-.2233254	.1150844

d5	.0576593	.0862319	0.67	0.504	-.1115592	.2268777
x1	1.92e-06	1.15e-06	1.67	0.096	-3.39e-07	4.18e-06
x1d1	1.33e-06	2.30e-06	0.58	0.564	-3.19e-06	5.85e-06
x1d5	-1.43e-06	2.30e-06	-0.62	0.533	-5.95e-06	3.08e-06
_cons	-.0716146	.0431142	-1.66	0.097	-.1562204	.0129911

## Interaction Effect

Model with interaction effect dummy:

$$Y_t = \beta_0 + \gamma_0 D_{1t} + \lambda_0 D_{JANt} + \delta_0 D_{1t} D_{JANt} + \beta_1 X_{1t} + u_t$$

where:  $D_{1t} = 1$  for Monday and  $= 0$  otherwise.  
 $D_{JANt} = 1$  for January and  $= 0$  otherwise.

The meaning of  $D_{1t} D_{JANt}$

$D_{1t} D_{JANt} = 1$  for Monday in January and  $= 0$  otherwise.

```
. g d1djan = d1*djan
. reg y d1 djan d1djan x1
```

Source	SS	df	MS	Number of obs =	995
Model	.006773371	4	.001693343	F( 4, 990) =	7.55
Residual	.221938166	990	.00022418	Prob > F =	0.0000
Total	.228711537	994	.000230092	R-squared =	0.0296
				Adj R-squared =	0.0257
				Root MSE =	.01497

y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
d1	-.0059289	.0012667	-4.68	0.000	-.0084147 - .0034432
djan	.0019612	.0017065	1.15	0.251	-.0013876 .0053099
d1djan	.004773	.0036262	1.32	0.188	-.0023429 .0118889
x1	2.08e-06	8.98e-07	2.32	0.021	3.21e-07 3.85e-06
_cons	-.0769402	.033645	-2.29	0.022	-.1429639 -.0109166