Supplement Questions: Production and Cost, Derivation of Labor and Capital Demand

1. A firm production function is $Q = K^3L$.
   a. Determine the firm's marginal product of labor function. (**Ans.**: $MP_L = K^3$)
   b. Determine if the marginal product of labor function exhibits diminishing marginal returns. (**Ans.**: the marginal product of labor function does not exhibit diminishing marginal returns, but it exhibits increasing marginal return)
   c. Determine the firm's marginal product of capital function. (**Ans.**: $MP_K = 3K^2L$)
   d. Determine if the marginal product of capital function exhibits diminishing marginal returns. (**Ans.**: the marginal product of capital function does not exhibit diminishing marginal returns, but it exhibits increasing marginal return)
   e. Determine the function for the marginal rate of technical substitution. (**Ans.**: $MRTS = \frac{1}{3}\left(\frac{K}{L}\right)$)

2. The production for the personal computers of DISK, Inc., is given by $Q = 10K^{0.5}L^{0.5}$, where *Q* is the number of computers produced per day, *K* is hours of machine time, and *L* is hours of labor input. DISK's competitor, FLOPPY, Inc., is using the production function; $Q = 10K^{0.6}L^{0.4}$.
   a. If both companies use the same amounts of capital and labor, which will generate more output? (**Ans.**: they both generate the same output with the same inputs.)
   b. Assume that capital is limited to 9 machine hours, but labor is unlimited in supply. In which company is the marginal product of labor greater? (**Ans.**: DISK, Inc.)

3. Suppose that a firm faces the production function; $Q = 50\sqrt{LK}$ and the total cost function; TC = $\omega L + rK$. What are the demand curves for labor and capital?

   (**Ans.**: Demand Curves for Labor is $K = \frac{Q}{50}\sqrt{\frac{\omega}{r}}$ and Demand Curves for Labor is $L = \frac{Q}{50}\sqrt{\frac{r}{\omega}}$)

4. Given Cobb-Douglas production function $Q = AL^{\alpha}K^{\beta}$, show that:
   a. When $\alpha + \beta = 1$, it exhibits **constant return to scale**.
   b. When $\alpha + \beta < 1$, it exhibits **decreasing return to scale**.
   c. When $\alpha + \beta > 1$, it exhibits **increasing return to scale**.

5. A firm's production is of the form: $Q = 50\sqrt{LK}$. if the price of labor (ω) is \$5 per unit and the price of capital (r) is \$20 per unit. What is the cost minimizing input combination if firm wants to produce 1,000 units? (**Ans.:** K = 10 and L = 40)

Note:

- It is important to understand **the distinction between the concepts of returns to scale and marginal returns.** Returns to scale pertains to the impact of an increase in all input quantities simultaneously, while marginal returns (i.e., marginal product) pertains to the impact of an increase in the quantity of a single input, such as labor, holding the quantities of all other inputs fixed.
- Golden Rule for Least-Cost Combination**:** $\mathbf{MRTS} = \frac{MP_l}{MP_k} = \frac{\omega}{r}$