Equity Funds
Solutions

1. To reduce the problem, I will work only with return of Michelin.
1.1 Simple return per month is R(t)=P(t)/P(t-1), the first few observations is given below.

Date	Michelin	SimpleReturn(Rt)
1/2/2007	64.61	
2/1/2007	72.36	R1 = 0.119950472
3/1/2007	76.08	R2 = 0.051409619
4/2/2007	86.34	R3 = 0.134858044

1.2. Gross return is 1+Rt, for simplicity, you can just write 1+R1, 1+R2, and so on.

Date	Michelin	SimpleReturn	Gross Return
1/2/2007	64.61		
2/1/2007	72.36	0.119950472	1.11995
3/1/2007	76.08	0.051409619	1.05141
4/2/2007	86.34	0.134858044	1.134858

1.3 Taking natural log of simple return is easy using =ln( ) function in excel.

Date	Michelin	SimpleReturn	Gross Return	Ln(Pt/Pt-1)	Ln(1+R)
1/2/2007	64.61				
2/1/2007	72.36	0.119950472	1.11995	0.1132845	0.1132845
3/1/2007	76.08	0.051409619	1.05141	0.0501318	0.0501318
4/2/2007	86.34	0.134858044	1.134858	0.1265076	0.1265076

In the last two columns, you can either take Ln(Pt/Pt-1) or ln(1+R), the result is the same.

1.4 Average arithmetic return is what the investor who rebalances his portfolio will earn each month over 11 months of evaluation (we miss Jan return bec. no december 2006 price data). Rebalancing means the investor buys at beginning of each month and sell and ending of each month. Each time starting with new money.

AM per month = $\frac{1}{11}\sum_{t=1}^{11} R_t$

= (0.119+ 0.051 + ...+ -0.03127887)/11 = 1.454% per month.

1.5 The product of 11 months of gross return is (1+R1)*(1+R2) + ...(1+R11) = 1.136

1.6 If we divide 12/07 by 01/07 price, we get 1.136

1.7 Summing up ln(1+R1) + ln(1+R2) +....+ln(1+R11) gives value 0.12755.

1.8 We can see that solution in 1.5 and 1.6 are the same, we have the gross return for the holding period of 11 months and this number is 1.136. You can think of this 1.136 as future value of your initial investment of 1.0 reinvested and compounded each month with returns, R1, R2, ..., to R11.

I can rewrite a familiar equation,

FV = PV(1+R)^T
1.136 = (1+R)^12

Solving for R, I get the geometric average monthly return of approx. 1.06%

Now to relate computation in question 1.7 which gives value 0.12755. This is exactly the same as taking ln(P(december)/P(January)) = ln(1.136070) = 0.12755.

Recall from your high school maths that if exp(x) = y, then x = ln (y). So when I write

ln(P(t)/P(t-k)) = 0.12755, it has the same representation as

exp(0.12755) = (P(t)/P(t-k)) = 1.136

If you think back about continuous compounding where we have $P_T = P_o e^{RT}$, then is exactly what we're doing.

$$\frac{P_T}{P_{T-k}} = 1.136 = e^{12R}$$

to solve for R in this case, take ln(1.136) = 12R, this gives R approx 1.06% as well.

1.9 The value 1.06% is the average monthly geometric return that gives the investor who has a BUY and HOLD (no rebalancing) strategy a portfolio value of 1.136 at end of evaluation period. This translate into a holding period return of 13.6% over approx 11 months.

The arithmetic mean (AM) 1.454% is what the investor who rebalances his portfolio every single month will earn on average. Note that in reality, this AM is likely to overstate his true performance because if you take into account commissions on buy and sell every single month!

2. CFA Examination Level III (Ch. 26 Q10)
During the annual review of Acme's pension plan, several trustees questioned Lucy Graham, a pension consultant, about various aspects of performance measurement and risk assessment. In particular, one trustee asked about the appropriateness of using each of the following benchmarks:

Market index
Median of manager's performance

a) Can you identify strengths and weaknesses of using these two benchmarks to measure the performance of a portfolio?
Ans: From our class discussion, we can attribute fund manager's performance to market, style, and active management, $P = M + S + A = M + (B-M) +A$.

If the benchmark is the market then $P = M + A$. The problem with this evaluation is that we don't know how much of A is actually the manager's true and special ability and which part is due to style (for example, the fact that the pension fund has a policy style to invest in high dividend firms like other pension funds).

If the median of fund manager's performance is also used, let's denote this by B, then we can better separate $P = M + (B-M) +A$. Think of the median pension manager's performance B is comprised of performance driven by market conditions and performance driven by all pension funds.

Another trustee asked how to distinguish between the following:

b) The Sharpe ratio, Treynor ratio, and Jensen's alpha
Risk-adjusted approach is better than above.

3. a) Risk adjusted equity returns of the 2 equity ports are computed from,

Risk adj. returns = (Realized return – Rf)/beta + Rf

Good Samaritan port: (11.8% - 7.8%)/1.20 +7.8% = 11.1%
Atkin's port: (10.7%-7.8%)/1.05 + 7.8% = 10.6%

Both ports outperform S&P500 both on absolute and risk adjusted basis. GS outperformed AK before risk adjustment but only by 0.5% after.

3b) Factors that can account for differences in reported performance
Different asset mixes
Different operating costs

Too short period to judge

4 a)
S = (10-6)/18 = 22.2% (underperformed)
Sm = (12-6)/13 = 46.2%

T = (10-6)/0.6 = 6.7% (outperformed)
Tm = (12-6%)/1.00 =6%

4b) Port X has large amount of unsystematic risk so Sharpe shows underperformance, but has high T measure because of low systematic risk.

5. A. The lower limit of a one standard deviation confidence interval is the sample mean return (0.56 percent) minus the sample standard deviation (8.86 percent): 0.56% − 8.86% = −8.30%. The upper limit is the sample mean return (0.56 percent) plus the sample standard deviation (8.86 percent): 0.56% + 8.86% = 9.42%. Summarizing, the one standard deviation confidence interval runs from −8.30 percent to 9.42 percent, written as [−8.30%, 9.42%]. If the portfolio return is normally distributed, approximately 68 percent (precisely 68.27 percent) of monthly returns will fall in this interval.

B. The lower limit of a 95 percent confidence interval is the sample mean return minus 1.96 standard deviations: 0.56% − 1.96 × 8.86% = −16.81%. The upper limit is the sample mean return plus 1.96 standard deviations: 0.56% + 1.96 × 8.86% = 17.93%. Summarizing, an exact 95 percent confidence interval runs from −16.81 percent to 17.93 percent, written as [−16.81%, 17.93%]. Only 5 percent of a large number of returns should fall outside of this interval, under the normality assumption. In that sense, we have 95 percent confidence in this interval.

C. The lower limit of a 99 percent confidence interval is the sample mean return minus 2.58 standard deviations: 0.56% − 2.58 × 8.86% = −22.30%. The upper limit is the sample mean return plus 2.58 standard deviations: 0.56% + 2.58 × 8.86% = 23.42%. Summarizing, an exact 99 percent confidence interval runs from −22.30 percent to 23.42 percent, written as [−22.30%, 23.42%]. Only 1 percent of a large number of returns should fall outside of this interval, under the normality assumption.
In that sense, we have 99 percent confidence in this interval.

6. $P(r < 0) = P(z < (0-0.02)/0.04) = P(z < -0.5) = N(-0.5) = 0.308$

7. a)

Monthly	Port A	Port B
Avg Return	0.0125	0.0107
Variance	0.0039	0.0055
Std	0.0623	0.0739

Avg RF ret 0.00291

*Note, you can annualize return by (1+R)^12-1
and variance with STD*(12)^0.5

Sharpe A = (0.0125-0.00291)/0.0623 = 0.1534
Sharpe B = (0.0107-0.00291)/0.0739 = 0.1049

Now run regressions to obtain alpha and beta,

Ri – Rf = alpha + beta (Rm – Rf)

Alpha A = 0.003327
Alpha B = 0.000413

	Portfolio A	Portfolio B
Beta	0.9322279	1.098925324

Treynor A = (0.0125-0.00291)/ 0.9322 = 0.0102
Treynor B = (0.0107-0.00291)/ 1.098 = 0.0071

	Port(A-RM)	Port(B-RM)
Avg Return	0.0029	0.0011
Variance	0.0012	0.0018
Std	0.0354	0.0424

Information ratio A = 0.0029/0.0354 = 0.0813
Information ratio B = 0.0011/0.0424 = 0.0253

b) Tracking error is difference between portfolio return of each month and the market (benchmark) return.

	Portfolio A	Portfolio B
Variance of TE	0.0012	0.0018
Std of TE	0.0354	0.0424
Annualized TE	0.1225	0.1469

c) Using shorter time series data 1993-1995 to forecast 1996 performance.
There are many ways to forecast 1996 performance, the easiest is to use history averages as a proxy for 1996 monthly performance average. For more advanced approaches, the analysts can use financial econometric techniques to compute means and standard deviations.

This table is just the average of monthly returns, variance, and STD of portfolio A and portfolio B between 1993-1995. We will assume it is best forecast of 1996 return performance.

	Portfolio A	**Portfolio B**
Average	0.01784	0.01293
Variance	0.002519257	0.0020606
Std	0.050192204	0.04539383
Lower Bound 5%	-0.064729954	-0.061742072
Lower Bound 1%	-0.099111613	-0.092836846

d) The monthly portfolio STD is 5.02% for portfolio A, and 4.54% for portfolio B. This means monthly changes in position of 5.02% * 500 mn = 26 mn (Port A) and 4.54% * 500 mn = 22.5 mn for (Port B)

Investment size $ mn	500	500	
Expected Profit ($ mn)	8.92	6.47	[Investment size x avg return of 1.78% and 1.293% for port A, B, respectively]
Confidence 5%			
Expected loss ($ mn)	-32.36	-30.87	[Investment size x lower bound return of -6.47% and -6.17% for port A, B, respectively]
Confidence 1%			
Expected loss ($ mn)	-49.56	-46.42	[Investment size x lower bound return of -9.91% and -9.28% for port A, B, respectively]

e)

	Portfolio A	**Portfolio B**
Sharpe ratio (Monthly)	**0.1534**	**0.1050**
Sharpe ratio (annualized)	0.5784	0.3919
Treynor (Monthly)	0.0103	0.0071
Jensen's alpha (Month)	0.0033	0.0004
Information ratio (Month)	0.0813	0.0253

Based on these performance ratios, portfolio A appears superior. Better conclusion can be drawn by dividing data into different sub-periods to see if performance of A is superior to B in all periods of resampling time.