EE311
Oligopoly (2)

- Stackelberg Model of Quantity Competition
- Bertrand Model of Price Competition
  - Case 1: when both sell homogeneous products
  - Case 2: When both sell differentiated products
- Dominant Firm Model (or Price Leadership Model)

Stackelberg Model of Quantity Competition

Born	October 31, 1905 Moscow, Russian Empire
Died	October 12, 1946 (aged 40) Madrid, Spain
Nationality	Germany
Alma mater	University of Cologne (Ph.D. and Habilitation)
Known for	Industrial Organization
	Scientific career
Fields	Economics
Institutions	University of Berlin University of Bonn Complutense University of Madrid

Stackelberg Duopoly: Assumptions

Assumptions

1) There are 2 firms: Firm 1 and Firm 2.
2) The firms set quantity of outputs, i.e., Q_1 and Q_2 simultaneously.
3) The two firms sell homogeneous Products.

In Cournot Model, each firm sets output simultaneously.

In Stackelberg (duopoly) Model, **firm 1 sets its output, Q_1 first, and then firm 2 observes the output chosen by firm 1 (i.e., Q_1) and sets its output, Q_2.**

Hence, we have the "leader" and "follower" in this model.

Stackelberg Duopoly: The Model

In a Stackelberg duopoly model (i.e., two firms in the market),

1. The follower (Firm 2) will set Q_2 depending on what Firm 2 "thinks" how much the leader will supply its output. The follower will form its best response function and set $Q_2^* = BR_2(Q_1)$
2. The leader (Firm 1) knows what the follower will do, so it will set Q_1 to maximize its profit, *taking into account the action of the follower*, i.e., knowing that $Q_2^* = BR_2(Q_1)$.

Stackelberg Duopoly: An Example

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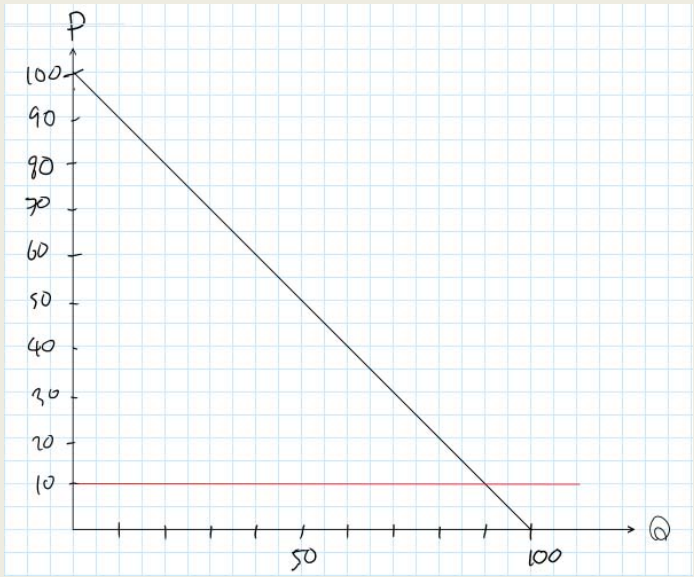
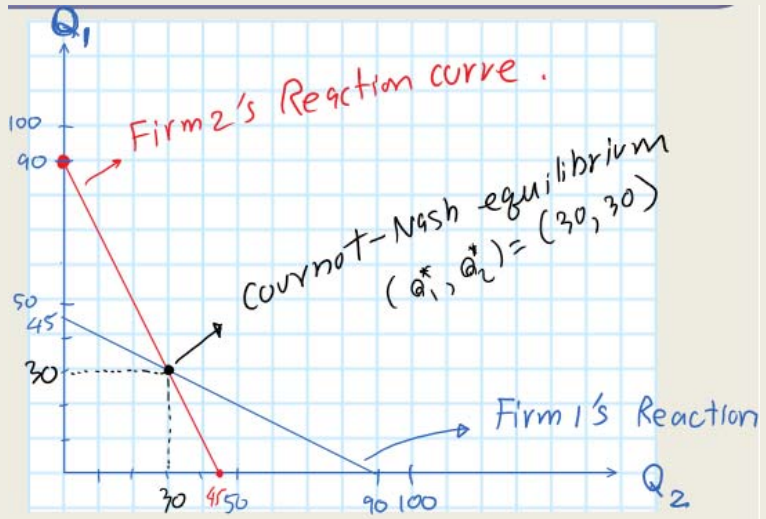


Stackelberg Duopoly: The Model



Stackelberg Duopoly: The Model

Model	Q1	Q2	P(mkt)	π1	π2
Stackelberg	45	22.5	32.5	1,012.5	506.25
Cournot	30	30	40	900	900
Cartel	22.5	22.5	55	1,012.5	1,012.5

We can see that in the Stackelberg Model, the leader has some advantage. We call this **"the first-mover advantage"**.

This is due to the leader able to control resources or the market.

Bertrand Model of Price Competition

Born	11 March 1822 Paris, France
Died	5 April 1900 (aged 78) Paris, France
Residence	France
	Scientific career
Fields	Mathematics

Bertrand Duopoly: Assumptions

Assumptions

1) **2 Firms: Firm 1 and Firm 2**
2) The two firms choose P_1 and P_2 simultaneously.

We will study TWO cases of Bertrand Oligopoly:
- Bertrand Duopoly with **Homogeneous Products**
- Bertrand Duopoly with **Differentiated Products**

Note that in Bertrand Model, firms engage in Price Competition.

In **Cournot and Stackelberg models**, firms engage in **Quantity Competition**.

Bertrand Duopoly: Homogeneous Products

Homogeneity implies that consumers will buy from the low-price seller.

Further, each firm realizes that the demand that it faces depends both on its own price and on the price set by other firms.

Any firm charging a higher price than its rivals will sell no output.

Any firm charging a lower price than its rivals will obtain the entire market demand.

Bertrand Duopoly: Homogeneous Products

Any firm charging a higher price than its rivals will sell no output.

Any firm charging a lower price than its rivals will obtain the entire market demand.

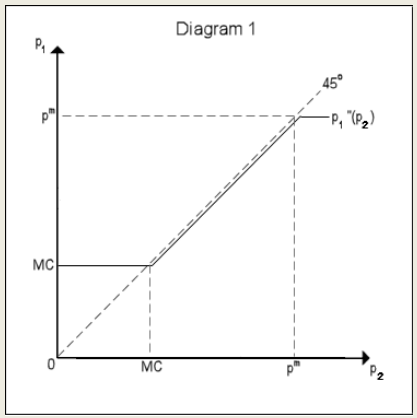
Symbolically, the quantity sold of Firm 1 is given by

$$Q_1 = D(P_1, P_2) \begin{cases} = D(P_1, P_2)/2 & \text{IF } \_\_\_\_\_ \\ = D(P_1) & \text{IF } \_\_\_\_\_ \\ = 0 & \text{IF } \_\_\_\_\_ \end{cases}$$

We use the following result to derive $P_1^* = BR_1(P_2)$.

Bertrand Duopoly: Homogeneous Products

$P_1^* = BR_1(P_2)$ refers to the profit-maximizing price (best response function) of Firm 1 which ***depends on*** the price of Firm 2.



P_m = monopoly price

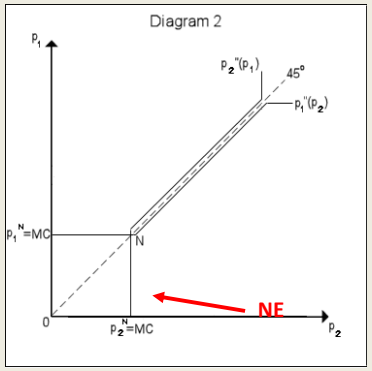
$\underline{P_1^* = BR_1(P_2)}$

$P_1 = MC$ **when** $0 \le P_2 \le MC$
$P_1 < P_2$ **when** $MC < P_2 \le P_m$
$P_1 = P_m$ **when** $P_m < P_2$

Bertrand Duopoly: Homogeneous Products

Bertrand-NE occurs when $P_1^* = BR_1(P_2^*)$ AND $P_2^* = BR_2(P_1^*)$.

This is where $P_1 = P_2 = MC$.



$\underline{P_1^* = BR_1(P_2)}$
$P_1 = MC$ **when $0 \le P_2 \le MC$**
$P_1 < P_2$ when $MC < P_2 \le P_m$
$P_1 = P_m$ when $P_m < P_2$

$\underline{P_2^* = BR_2(P_1)}$
$P_2 = MC$ **when $0 \le P_1 \le MC$**
$P_2 < P_1$ when $MC < P_1 \le P_m$
$P_2 = P_m$ when $P_m < P_1$

Bertrand Duopoly: Homogeneous Products

Bertrand Paradox

The paradox describes a situation in which two players (firms) reach Nash equilibrium where both firms charge P = MC.

The paradox is that in models such as Cournot competition, an increase in the number of firms is associated with a convergence of prices to marginal costs.

In other words, even with very few firms, the oligopoly market can reach the perfectly competitive outcome.

Bertrand Duopoly: Homogeneous Products

Bertrand Paradox (Explained)

Neither A nor B will set a higher price than the other because doing so would yield the entire market to their rival.

If they set the same price, the companies will share both the market and profits.

If either firm were to lower its price, even a little, it would gain the whole market and substantially larger profits.

Since both A and B know this, they will each try to undercut their competitor until the product is selling at zero economic profit.

Bertrand Duopoly: Differentiated Products

Assumptions

1) 2 Firms: Firm 1 and Firm 2.
2) The two firms offer differentiated products.
3) The two firms choose P_1 and P_2 simultaneously.

Product Differentiation means that lowering price below your rivals' will not result in capturing the entire market, nor will raising price mean losing the entire market.

Bertrand Duopoly: Differentiated Products

In this model, we solve for the Bertrand-Nash Equilibrium in a similar way as when we solve for the Cournot-Nash Equilibrium.

1) Find the best response function (BR) of each firm, i.e., $P_1^* = BR_1(P_2)$ for Firm 1 and $P_2^* = BR_2(P_1)$ for Firm 2.

2) Bertrand-NE is determined where the two BR's intersect, i.e. where
$P_1^* = BR_1(P_2^*)$ AND $P_2^* = BR_2(P_1^*)$

That is, Firm 1 is doing its best given what Firm 2 is doing, AND Firm 2 is doing its best given what Firm 1 is doing.

Bertrand Duopoly: Differentiated Products

Example

$Q_1 = 100 - 2P_1 + P_2$ *"Coke's demand"*
$Q_2 = 100 - 2P_2 + P_1$ *"Pepsi's demand"*
$MC_1 = MC_2 = 5$

To find $BR_1(P_2)$, we set $MR_1 = MC_1$, but first we need TR_1.

$TR_1 = P_1Q_1$ with $P_1 = (100 + P_2 - Q_1)/2$

$$TR_1 = P_1Q_1 = \left(50 + \frac{1}{2}P_2 - \frac{1}{2}Q_1\right)Q_1$$
$$MR_1 = \left(50 + \frac{1}{2}P_2 - Q_1\right)$$

Bertrand Duopoly: Differentiated Products

To find $BR_1(P_2)$, we set $MR_1 = MC_1$.

$MR_1 = \left(50 + \frac{1}{2}P_2 - Q_1\right) = MC_1 = 5$

$Q_1 = 45 + \frac{1}{2}P_2$

Recall that $Q_1 = 100 - 2P_1 + P_2$. We have

$100 - 2P_1 + P_2 = 45 + \frac{1}{2}P_2$

$55 + \frac{1}{2}P_2 = 2P_1$

$BR_1(P_2)$ ➔ $P_1 = 27.5 + \frac{1}{4}P_2$

Bertrand Duopoly: Differentiated Products

To find Bertrand-Nash Equilibrium,

$P_1 = 27.5 + P_2/4$ $P_2 = 27.5 + P_1/4$

Solving these two equations yields

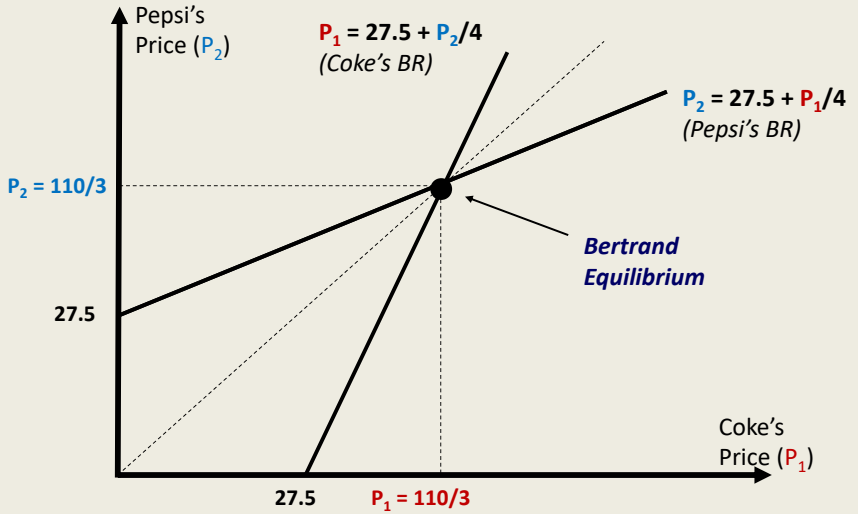
$P_1^* = P_2^* = 110/3$

Plugging these prices into demand, we have

$Q_1^* = Q_2^* = 190/3$

$\pi_1^* = \pi_2^* = 2{,}005.55$ $\Pi = 4{,}011.10$

Bertrand Duopoly: Differentiated Products



Bertrand Duopoly: Differentiated Products

LEARNING-BY-DOING EXERCISE 13.4

Computing a Bertrand Equilibrium with Horizontally Differentiated Products

Suppose Coca-Cola's and Pepsi's demand curves are given by $Q_1 = (64 + 2P_2) - 4P_1$ and $Q_2 = (50 + P_1) - 5P_2$, respectively. [These correspond to equations (13.1) and (13.2) with terms rearranged and with parentheses used to highlight terms that the firm views as fixed.] Coca-Cola's marginal cost is \$5 per unit, and Pepsi's marginal cost is \$4 per unit.

Problem

(a) What is Coca-Cola's profit-maximizing price when Pepsi's price is \$8?

(b) What is the equation of Coca-Cola's price reaction function (i.e., Coca-Cola's profit-maximizing price when Pepsi sets an arbitrary price P_2)?

(c) What are Coca-Cola's and Pepsi's profit-maximizing prices and quantities at the Bertrand equilibrium?

Dominant Firm Model (Price Leadership Model)

Dominant Firm Markets

In some industries, a single company with a very large market share (**dominant firm**) competes against small firms (**fringe firms**).

The dominant firm sets the market price and splits the market demand with fringe firms.

Fringe firms supply identical products and act as perfect competitors. Since they are small firms, **they are price-takers**.

Fringe firms takes the market price as given and decide how much to supply. **In other words, their output will be determined by the fringe firms' supply curve.**

Dominant Firm Markets

The dominant firm's problem is to find P* that maximizes its profit.

However, it is important to note that

P* will determine how much fringe firms will supply. AND,

How much fringe firms will supply affect the dominant firm's residual demand (= Market Demand – Supply of Fringe Firms).

The dominant firm faces the following "trade-off":

If the dominant firm sets high P*, fringe firms will supply a lot. As a result, the dominant firms can sell less to the market.

Dominant Firm Markets

Dominant Firm Markets

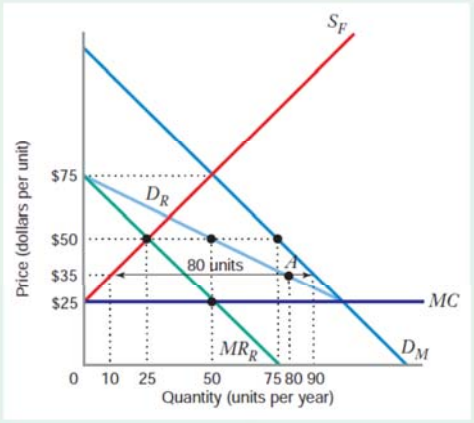


FIGURE 13.7 Dominant Firm Market
The dominant firm's residual demand curve D_R is the horizontal difference between the fringe's supply curve S_F and the market demand curve D_M. The dominant firm's profit-maximizing quantity is 50 units, and its profit-maximizing price is $50 per unit. At this price, the fringe supplies 25 units.

STACKELBERG | BERTRAND | DOMINANT FIRM

Dominant Firm Markets

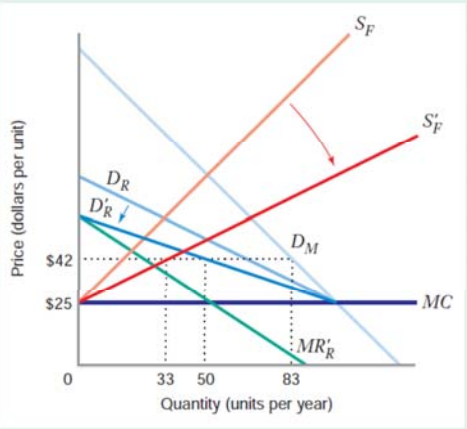


FIGURE 13.8 Dominant Firm Market When the Size of the Competitive Fringe Grows
When the size of the fringe grows, the fringe's supply curve rotates rightward to S'_F, causing the residual demand curve to rotate leftward to D'_R. The new profit-maximizing quantity for the dominant firm is 50 units, and the profit-maximizing price is $42. At this price, the fringe supplies 33 units of the total market demand of 83 units.

STACKELBERG | BERTRAND | DOMINANT FIRM

Dominant Firm Markets

LEARNING-BY-DOING EXERCISE 13.3

Computing the Equilibrium in the Dominant Firm Model

Suppose that the market demand curve in a global mining industry is given by $Q^d = 110 - 10P$, where Q^d is measured in millions of units of product mined per year and P is measured in dollars per unit. The industry is dominated by a large firm with a constant marginal cost of $5 per unit. There also exists a competitive fringe of 200 firms, each of whom has a marginal cost given by $MC = 5 + 100q$, where q is the output of a typical fringe firm.

Problem

(a) What is the equation of the supply curve for the competitive fringe?

(b) What is the equation of the dominant firm's residual demand curve?

(c) What is the profit-maximizing quantity of the dominant firm? What is the resulting market price? At this price, how much does the competitive fringe produce, and what is the fringe's market share (i.e., the fringe quantity divided by total industry quantity)? What is the dominant firm's market share?

STACKELBERG | BERTRAND | DOMINANT FIRM

Oligopoly: A Summary

Model	Product Diff.	Setting/ Timing	Cooperation	Max. Profit by setting
Cartel	NO	Simultaneous-move	Cooperative	As if it were a monopolist
Cournot	NO	Simultaneous-move	Quantity Competition	Quantity
Stackelberg (Quantity Leadership)	NO	Sequential-move	One firm is the quantity leader.	Quantity
Dominant Firm (Price Leadership)	NO	Sequential-move	One firm is the price leader.	Price
Bertrand Homogeneous	NO	Simultaneous-move	Price Competition	Price
Bertrand Differentiated	YES	Simultaneous-move	Price Competition	Price