Debraj Ray (textbook) Ch. 14.3.6 For lecture 6/2

need to borrow loan size = L

2 types of borrower

$R' > R$

① safe type : get the project return = R

② risky type : → get higher return R' with prob. p ⇒ P(success in project investment)

↘ get 0 with prob. $(1-p)$

Lender wants to max profit of lending & Borrower wants to max profit of investment

i = interest rate of loan (borrowing)

① safe type : net return : $R-(1+i)L \geq 0$ (zero profit)

$R-L-iL=0$

$R-L=iL$

⇒ highest i that lender can charge $= \dfrac{R-L}{L} = \dfrac{R}{L} - 1$

② Risky type: net return: $R' - (1+i)L \geq 0$

$\Rightarrow$ highest $i = \dfrac{R'}{L} - 1$

base on prob. of success of risky type

If compare interest rates b/w safe (i_1) and risky type (i_2)

$$i_1 = \frac{R}{L} - 1 < i_2 = \frac{R'}{L} - 1 \quad \text{because } R' > R$$

For lender

(A) if lender charges i_2 (expensive rate) $\rightarrow$ only risky type who can borrow

expected profit of lender: $\pi_2 = [p(1+i_2)L + \underbrace{0(1-p)}_{=0}] - L$

Profit fn of lender (certainty)

$(1+i)L - L = L + iL - L = \underline{iL}$

$\therefore \pi_2 = p(1+i_2)L - L$

(B) if lender charges i_1 $\rightarrow$ both safe and risky types can borrow

expected profit of lender: $\pi_1 = \underbrace{\frac{1}{2} i_1 L}_{\text{from safe type}} + \frac{1}{2}[p(1+i_1)L - L]$

2 types of borrowers

50% safe 50% risky

For lender, he has 2 options $\nearrow$ Charge i_2 (higher rate) $\Rightarrow \pi_2$

$\searrow$ Charge i_1 (lower rate) $\Rightarrow \pi_1$

since we have no information of clients (asymmetric information)

the lender will charge i_1 when $\boxed{\pi_1 > \pi_2}$

$$\frac{1}{2} i_1 L + \frac{1}{2}\left[P(1+i_1)L - L\right] > P(1+i_2)L - L$$

substitute $i_1 = \frac{R}{L} - 1$ and $i_2 = \frac{R'}{L} - 1$ [solve in form of $P \gtrless \boxed{?}$]

$$\frac{1}{2}\left(\frac{R}{L} - 1\right)L + \frac{1}{2}\left[P(\cancel{1} + \frac{R}{L} \cancel{-1})L - L\right] > P(\cancel{1} + \frac{R'}{L} \cancel{-1})L - L$$

$$\frac{1}{2}R - \cancel{\frac{1}{2}L} + \frac{1}{2}PR - \cancel{\frac{1}{2}L} > PR' - \cancel{L}$$

$$R + PR > 2PR'$$

$$R > 2PR' - PR$$

$$R > P(2R' - R)$$

$$\boxed{\frac{R}{2R' - R} > P}$$

$\pi_1 > \pi_2$ when $\frac{R}{2R'-R} > P$

or prob. of success for risky project is very low.