Chapter 4 Part 1: Solow Growth Model

EE312

Macroeconomics, Stephen Wiliamson, Chapter 4,5

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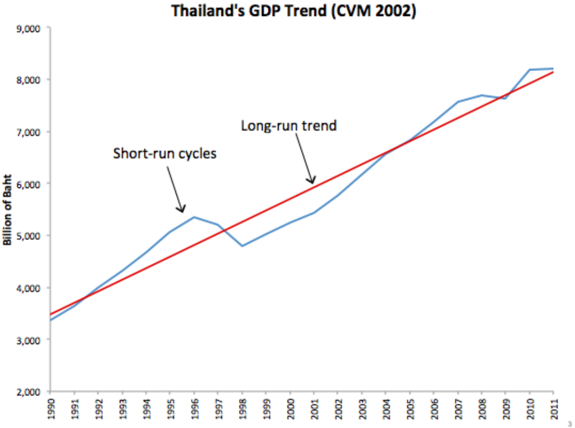
Outline

1. Importance of Growth

- The standards of living in the long term depend on economic growth.
  - Short-run fluctuations tend to cancel out in the long run.
- What determines economic growth?
  - Models of economic growth.
  - The Solow growth model.
  - Endogenous growth models.


Thailand's GDP Trend (CVM 2002)
Billion of Baht
9,000
8,000
7,000
6,000
5,000
4,000
3,000
2,000
Long-run trend
Short-run cycles
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
2008
2009
2010
2011

Economic growth facts

- Before the Industrial Revolution in about 1800, standards of living differed little over time and across countries.
- Since the Industrial Revolution in about 1800, per capita income growth has been sustained in the richest countries (e.g., about 2% in US since 1869).
- Across countries, we observe that
    - The higher the rate of investment, the higher output per worker.
    - The higher population growth, the lower output per worker. High population growth corresponds with low living standards.

- Are standard of living converging across countries of the world?
  - International differences in living standards widen between developed and developing countries (except East Asia).
  - Growth convergence occurs among developed countries.
  - No growth convergence among the poorest countries.

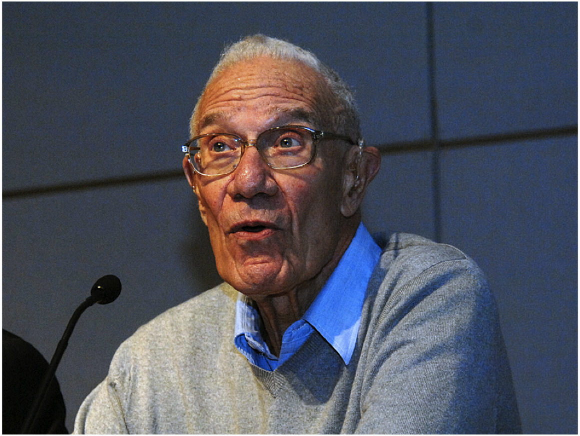
GDP per Capita Current $

Country	1996	2012(World Bank est.)
Thailand	3,055	5,480
Singapore	25,796	51,709
China	703	6,091
Malaysia	4,744	10,432

Source: world bank

- Economies are divided according to 2012 GNI per capita, calculated using the World Bank Atlas method.
  - The groups are: low income, $1,035 or less;
  - lower middle income, $1,036 - $4,085;
  - upper middle income, $4,086 - $12,615;
  - and high income,$12,616 or more.

- “ONCE you start thinking about growth, it’s hard to think about anything else.” ,
  [said Robert Lucas, a Nobel prize-winning economist, “On the Mechanics of Economic Development”, Journal of Monetary Economics, 1988]
- “Judging by their rhetoric, the world’s policymakers are indeed thinking about little else. The statement released after the most recent meeting of G20 leaders in Toronto in June mentioned the word “growth” 29 times in nine pages. Mr Obama says his economic policy is all about “laying the foundations for long-term growth”. Britain’s prime minister, David Cameron, used his first speech in office to lay out a “strategy for economic growth”. Japan’s government unveiled a ten-year “new growth strategy” in June.”
  The Economist, 2010



Robert M. Solow (b.1924), Nobel Prize 1987

- The basis of all modern theories of growth.
- Long-term economic growth depends on one single factor — **technological progress**.
  - Rising total factor productivity (z).
  - Sustained improvement in living standards (real per capita income or output per worker).

2.1 Population growth

- Assume population grows exogenously at a constant rate.
- N= population (or workers) in the current period.
- N' = population in the future period.
- n= rate of population growth in the current period

$$N' = (1 + n)N$$

$n > -1$

2.2 Consumers:

- Consumers = population = workers.
- Consumers supply labor in production.
- Consumers receive real output (Y) as (wage and dividend) income.
- Spend on consumption goods (C) and save a constant fraction (s) of Y as saving (S).

$$
\begin{aligned}
Y &= C + S \\
S &= sY \\
C &= (1 - s)Y
\end{aligned}
$$

- The Neoclassical Production Function
- The firm produces output using the current capital stock (K) and the current labor input (N).
- Assuming
  1. Constant returns to scale.
     $F(\lambda K, \lambda N) = \lambda F(K, N)$; for all $\lambda > 0$
  2. Positive and diminishing returns to private inputs:
     For all $K > 0$ and $L > 0$, F exhibits positive and diminishing marginal products with respective to each input.
  3. Inada conditions:
     The marginal product of capital (or labor) approaches infinity as capital (or labor) goes to 0 and approaches 0 as capital (or labor) goes to infinity.

- **Production function :** $Y = zF(K, N)$
- **Per worker production function:**
  Let y=$\frac{Y}{N}$ = output per worker and $\frac{K}{N} = k$ = capital per worker

$$\frac{Y}{N} = \frac{1}{N} zF(K, N)$$

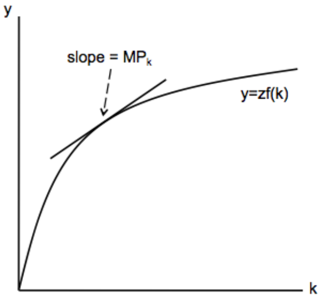
$$\text{CTS} : F(\lambda K, \lambda N) = \lambda F(K, N)$$

$$\frac{Y}{N} = zF(\frac{K}{N}, 1)$$

$$\text{Let } f(k) F(\frac{K}{N}, 1) = F(k, 1)$$

$$y = zf(k)$$

- Marginal Product of Capital



Production function, fixed labor

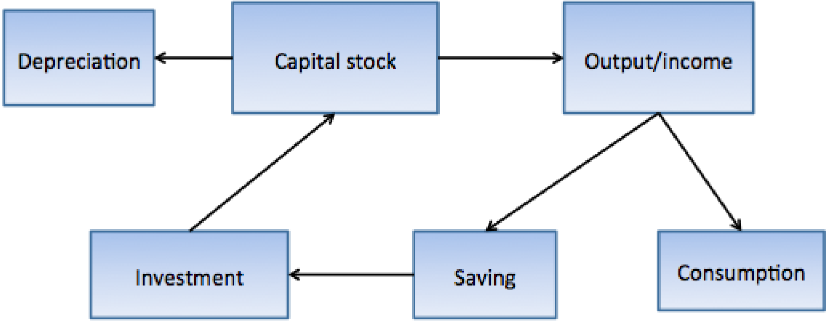
- Output per worker (y) increases at a decreasing rate as capital per worker (k) rises.
- Slope is the marginal product of capital (MP_K).

2.4 Growth of capital stock

- Assume capital wears out over time at the rate of d (or depreciation), where 0 > d > 1.
- I = (gross) investment = addition to capital stock.
- K' = capital stock in the future period.

$$K' = (1 - d)K + I$$

2.5 The working of growth



2.6 Equilibrium output

- At equilibrium, saving equals investment so that output consists of consumption and investment.

$$\begin{aligned} S &= I \\ S &= Y - C \\ Y &= C + S \\ Y &= C + I \end{aligned}$$

- Equilibrium condition: The future capital stock is the current capital stock deducted by depreciation and added by investment (= saving).

$$
\begin{aligned}
Y &= C + I \\
C &= (1 - s)Y \\
I &= K' - (1 - d)K
\end{aligned}
$$

- Substitute C and I in the Y equation

- Per worker formulation

$$Y = (1-s)Y + K' - (1-d)K$$

rearraging the term

$$K' = Y - (1-s)Y + (1-d)K$$
$$= sY + (1-d)K$$

From $Y = zF(K,N)$,

$$K' = szF(K,N) + (1-d)K$$

divided by N

$$\frac{K'}{N} = sz\frac{F(K,N)}{N} + (1-d)\frac{K}{N}$$

$$\frac{K'}{N} = szf(k) + (1-d)k$$

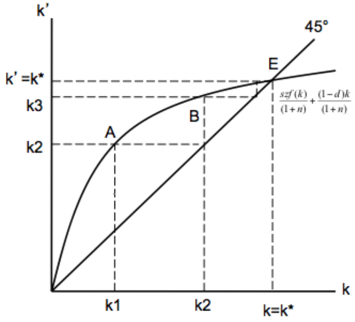
- Future capital per worker function ($k' = \frac{K'}{N'}$)

$$\frac{K'}{N} \times \frac{N'}{N'} = sz\frac{F(K,N)}{N} + (1-d)\frac{K}{N}$$

$$\text{Let } k' = \frac{K'}{N'} \text{and } \frac{N'}{N} = (1+n)$$

$$k'(1+n) = szf(k) + (1-d)k$$

$$k' = \frac{szf(k)}{(1+n)} + \frac{(1-d)k}{(1+n)}$$



- At A, k2 > k1; k is growing.
- At B, k3 > k2; k is growing.
- k = k*; steady-state capital per worker.

- Diminishing returns on k
- At E, $k = k' = k^*$ so that k^* is steady.
- To the left of k^*, $k' > k$ so that k is increasing.
- To the right of k^*, $k' < k$ so that k is decreasing.
- As k is increasing, MPk is falling so that **y is increasing at a decreasing rate.**
- Finally, investment (or new capital) is just sufficient to keep up with population growth and depreciation, so that k (and y) is stagnant.

Steady-state aggregates

- With k* at the steady state, y*, c* and szf(k*) are all at the steady-state.
  - No further improvement in output per worker (y).
- Given population growth (n), total factor productivity (z) and the saving rate (s), the steady-state growth rate is ‘n’ for aggregate quantities:
  - Capital stock (K) and output (Y);
  - Consumption (C), saving (S) and investment (I).

2.8 Analysis of the steady-state

$$k' = \frac{szf(k)}{(1+n)} + \frac{(1-d)k}{(1+n)}$$

$$k* = \frac{szf(k*)}{(1+n)} + \frac{(1-d)k*}{(1+n)}$$

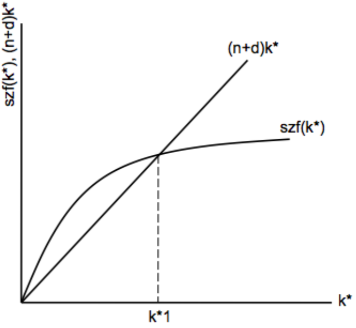
Multiply both sides by $(1+n)$

$$(1+n)k* = szf(k*) + (1-d)k*$$

$$(n+d)k* = szf(k*)$$

Steady State Investment = Steady State Saving

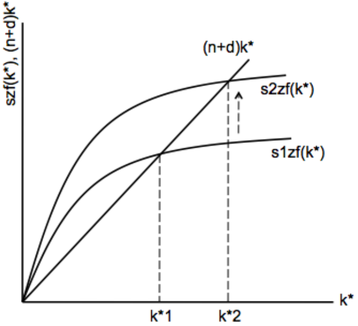
- szf(k*) = saving per worker;
- (n+d)k* = investment per worker needed to keep up with population growth and depreciation.
- At k*, the capital stock is still growing, but just sufficient to equip each worker with the same k and depreciation (so k* is steady).
    - **'Capital widening'**: growing K just to keep the steady k and y.



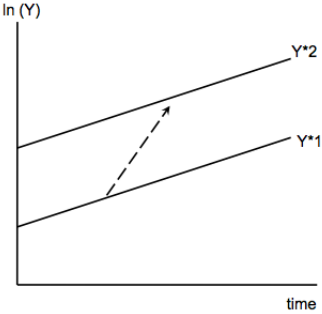
- szf(k*) is concave due to zf(k*).
- (n+d)k* has the slope = (n+d).

2.10 Effect of an increase in s

- s may increase due to changes in consumers' propensity or government policy.
- Assume a permanent increase in s:
  - szf(k*) rotates upwards.
  - Higher steady-state k* and y* (on a different 'growth path').
  - Higher growth of K and Y is transitional.
  - Convergence to the same steady-state growth rate of 'n'.



- Higher saving rate results in a higher k* and y*.
- **A rise in s raises k*.**



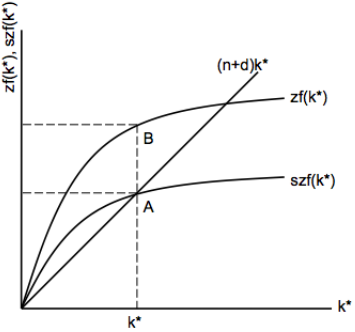
- **Temporary gain in growth rate**
- K and Y move to new ‘growth paths’.
- Higher growth rates of K and Y are transitional, converging to n.

2.11 Steady-state consumption per worker

$$
\begin{aligned}
y* &= zf(k*) \\
\frac{S}{N} &= szf(k*) \\
c* &= zf(k*) - szf(k*) \\
&= (1-s)zf(k*)
\end{aligned}
$$

At $k*$, $szf(k*) = (n+d)k*$

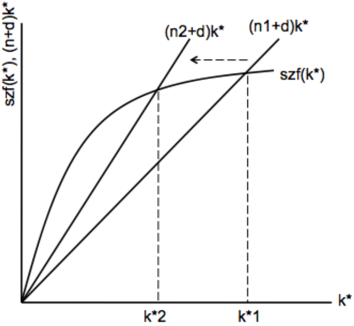
$$
c* = zf(k*) - (n+d)k*
$$



- $c* = y * -szf(k*) = zf(k*) - (n+d)k*$.
- AB = c*

2.12 Effect of an increase in n

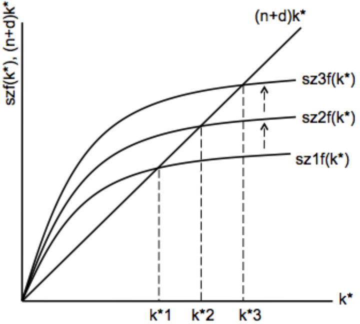
- The increase in population growth (n1 to n2) rotates (n+d)k* upwards.
- Decreased steady-state capital (k*) and output per worker (y*).
  - More workers (N*) produce larger output (Y*).
  - But falling productivity of labor results in lower output per worker (y*).
- The steady-state growth rate is higher at n2 for the capital stock (K) and total output (Y).



- Higher population growth (n) results in lower k* and y*.
- **A higher n with lower k***

2.13 Effect of an increase in z

- A rising s or falling n raises steady-state output per worker (living standards).
  - But the improvement will cease at some point (s cannot exceed 1; n cannot fall indefinitely).
- An increase in total factor productivity (z) raises steady-state capital (k*) and output per worker (y*).
  - Sustained increases in z cause sustained increases in output per worker (y).



- Sustained increases in z cause sustained improvements in y*.

Sources of sustained growth

- Growth from increases in productive inputs:
  - Physical capital accumulation, F(K, N). Human capital accumulation, F(K, H).
- Growth from total factor productivity (z):
  - Technical progress, inventions, better management and organization.
  - Weather, improved government regulations, falling input prices.

- Growth since the Industrial Revolution has come mainly from rising z.
- Will this continue indefinitely into the future?
- Growth accounting: identify sources of growth. Increases in productive inputs (K, N) or in total factor productivity (z).
- Calculation based on the production function and the Solow residual.

Thailand's production function

$$Y = zK^{\alpha}N^{(1-\alpha)},$$

where $0 < \alpha < 1$. Note that $\alpha + (1 - \alpha) = 1$

- Assume constant returns to scale (CRS).
- α = share of the capital input in GDP.
- 1 -α = share of the labor input in GDP.

$$Y = zK^{0.6}N^{0.4}.$$

The Solow residual for Thailand

$$
\begin{aligned}
Y &= zK^{0.6}N^{0.4} \\
z &= \frac{Y}{K^{0.6}N^{0.4}}
\end{aligned}
$$

- Estimated z = Solow residual.
- It measures the level of total factor productivity for Thailand.

Thailand's Solow Residual
Productivity growth
Productivity slump
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
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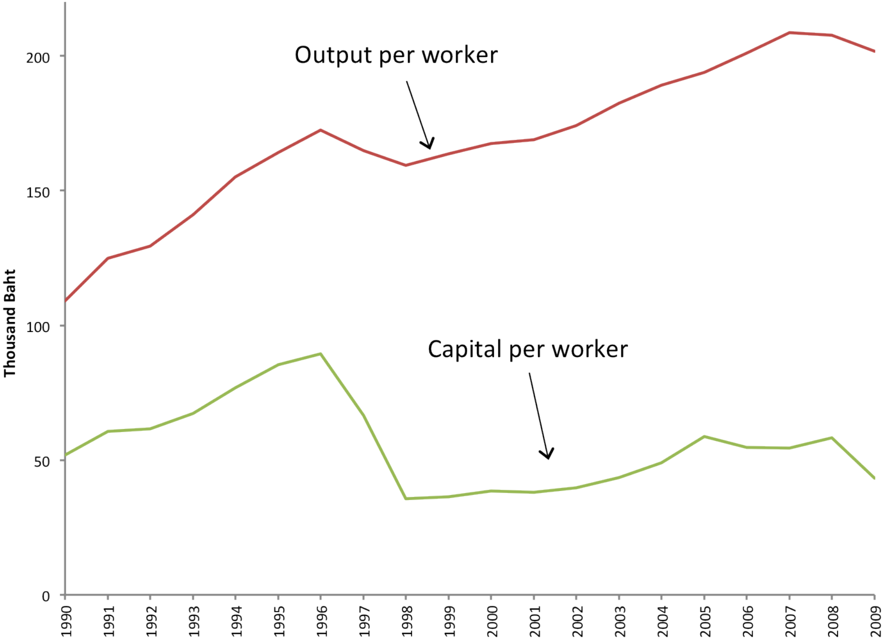
2005
2006
2007
2008
2009


Output per worker
Capital per worker
Thousand Baht
0
50
100
150
200
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
2008
2009

Thailand's growth accounting

Year	Output	Capital	Labor	Solow residual
1990-1997	6.2%	3.9%	0.3%	3.7%
1998-2009	4.2%	3.8%	2.0%	1.1%

East Asian Growth Accounting(Average Annual Growth Rates)

	Output	Capital	Labor	Total Factor Productivity
Hong Kong (1966–1991)	7.3%	7.7%	2.6%	2.3%
Singapore (1966–1990)	8.7%	10.8%	4.5%	0.2%
South Korea (1966–1990)	10.3%	12.9%	5.4%	1.7%
Taiwan (1966–1990)	9.4%	11.8%	4.6%	2.6%
United States (1966–1990)	3.0%	3.2%	2.0%	0.6%

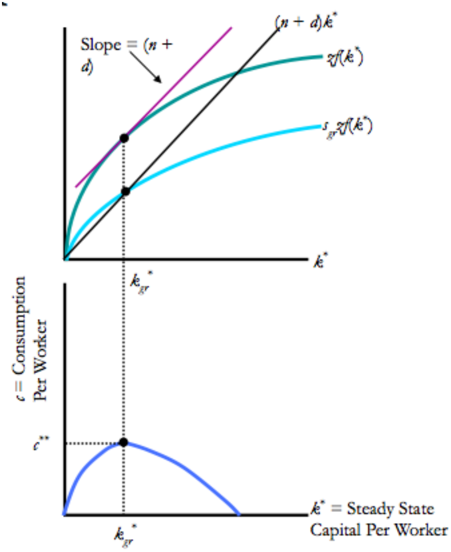
- Consumption per worker in the steady state is $c^* = y^* - szf(k*) = zf(k*) - (n+d)k^*$.
- Golden rule quantity of capital per worker, k^{**}, gives the maximum consumption per worker, c^{**}.
- Maximized c^{**}

$$c^* = zf(k^*) - (n+d)k^*$$

Set $\frac{dc^*}{ck^*}$=0

$$\frac{dzf(k^*)}{dk^*} = (n+d)$$

$$MP_k = n+d$$



Slope = (n + d)
(n + d)k*
zf(k*)
s_gr zf(k*)
k*
k_gr*
c = Consumption Per Worker
c**
k* = Steady State Capital Per Worker
k_gr*

In this economy, there are two markets in the current period.

1. consumption goods are traded for current labor
2. consumption goods are traded for captial

Consumers save by accumulating capital.

At equilibrium, $S = I$ so that $Y = C + I$.

Equilibrium condition: The future capital stock is the current capital stock deducted by depreciation and added by investment (= saving)

$$\begin{aligned} K &= k^*N \\ Y &= y^*N = zf(k^*)N \\ I &= sY = szf(k*)N \\ C &= (1-s)Y = (1-s)zf(k^*)N \end{aligned}$$

Given population growth (n), total factor productivity (z) and the saving rate (s), the steady-state growth rate is 'n' for aggregate quantities. "while the growth rate of per worker variable = 0 at steady state"

Solow model tells us that growth in key macroeconomic aggregates is determined by exogenous labor force growth when the saving rate, the labor force growth rate, and total factor productivity are constant.

Solow's model states that investment in capital cannot drive long run growth in GDP per worker.

Policy lesson: don't advise poor countries to invest without due regard for technology and incentives. Capital deepening (an increase in capital per worker) cannot lead to a sustained economic growth in the long run.

the original steady state	in transition	the new steady state

the original steady state	in transition	the new steady state

the original steady state	in transition	the new steady state

the original steady state	in transition	the new steady state