Assignment 3: Due date: March 31, 2022 before 2.00 pm

On page 134-138, Chapter 4: Consumption-savings and state pricing, work on the following questions:

$$2, 3, 4, 6$$

2.) Since, $p_s = \pi_s m_s$,

$$m_s = \frac{\delta u'(c_s^*)}{u'(c_0^*)} = \frac{\delta c_0^*}{c_s^*}$$

$$p_s = \delta \pi_s \frac{c_0^*}{c_s^*}$$

3) Since, $m_{01} = \frac{\delta u'(c_1)}{u'(c_0)} = \delta (c_1/c_0)^{\gamma-1} R_s$

Total diff:

$$0 = \pi_s \delta(\gamma-1)(c_s/c_0)^{\gamma-2} R_s \, d(c_s/c_0) + \pi_s \delta (c_s/c_0)^{\gamma-1} dR_s$$

$$\frac{R_s}{c_s/c_0} \frac{d(c_s/c_0)}{dR_s} = \frac{1}{1-\gamma}$$

4.) a. Let $p = \begin{bmatrix} 1/1.05 \\ 6 \end{bmatrix}$ and $X = \begin{bmatrix} 1 & 10 \\ 1 & 5 \end{bmatrix}$

$$[p_1 \quad p_2] = p' X^{-1} = \begin{bmatrix} \frac{1}{1.05} & 6 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 0.2 & -0.2 \end{bmatrix} = \begin{bmatrix} 0.2476 & 0.7048 \end{bmatrix}$$

The risk-neutral prob. are $p_1 R_f = 0.26$ and $p_2 R_f = 0.74$

b. $m_1 = p_1/\pi_1 = 0.495$, $m_2 = p_2/\pi_2 = 1.410$

6.) a. $0 = E[m(R_i - R_f)]$

$= E[(a + bR_m)(R_i - R_f)]$

$= aE[R_i] - aR_f + bE[R_m R_i] - bR_f E[R_m]$

$= a(E(R_i) - R_f) + b(E(R_m)E(R_i) + COV[R_m, R_i] - R_f E(R_m))$

$= E(R_i) - R_f)(a + bE(R_m)) + b\,COV(R_m, R_i)$

So, $E[R_i] - R_f = \dfrac{-b\,COV(R_m, R_i)}{a + bE(R_m)}$

$= -\dfrac{COV(R_m, R_i)}{\sigma_m^2} \dfrac{b\sigma_m^2}{a + bE(R_m)}$

$= -\beta_i \dfrac{b\sigma_m^2}{a + bE(R_m)}$

$\gamma = -\dfrac{b\sigma_m^2}{a + bE(R_m)}$

b) $\dfrac{1}{R_f} = E[a + bR_m]$; Risk-free

$1 = aE(R_m) + b(\sigma_m^2 + E[R_m]^2)$; market portfolio

$b = -\dfrac{E(R_m) - R_f}{R_f \sigma_m^2}$