Student Name Woradech Mawatchakij ID 6104640641

EE 325 Section 1 (Aj.Wanwiphang) Homework Assignment 1

Due date: 31 January 2020 before 11pm

*** Please submit this assignment on Moodle. For those who work on paper, please scan or submit the pictures of your work.***

1. Find the answers following questions (please also show your calculation)

a. $\sum_{i=1}^{5}(a+bx_i)$

$\sum_{i=1}^{5} a + \sum_{i=1}^{5} bx = 5a+b\sum_{i=1}^{5} x_i = 5a+b(x_1+x_2+x_3+x_4+x_5)$

b. $\sum_{y=0}^{5} f(x+y)$

$= f(x+0)+f(x+1)+f(x+2)+f(x+3)+f(x+4)+f(x+5)$

c. $\sum_{i=1}^{10} i^2$

$= 1^2+2^2+3^2+4^2+5^2+6^2+7^2+8^2+9^2+10^2$

$= 385$

d. $\sum_{x=1}^{2}\sum_{y=2}^{3}(2x+y)$

Σ

2. Given X is discrete random variable. The probability distribution function (PDF) of this variable is shown in the table

X	-2	-1	0	1	2	3	4
$f(x)$	0.5b	b	2.25b	2b	1.5b	0.5b	0.25b

** when b is constant number

a. Find the value of b

$\Sigma f(x) = 1$; $8b = 1$

$b = \frac{1}{8} = 0.125$

b. Find the answer for $P(X \le 2)$

$= 0.5b + b + 2.25b + 2b + 1.5b$

$= 7.25(0.125) = 0.90625$

c. Find the answer for $P(-2 \le X \le 3)$

$= 7.75(0.125)$

$= 0.96875$

d. Find the answer for $P(X \ge 1)$

$= 4.25(0.125)$

$= 0.53125$

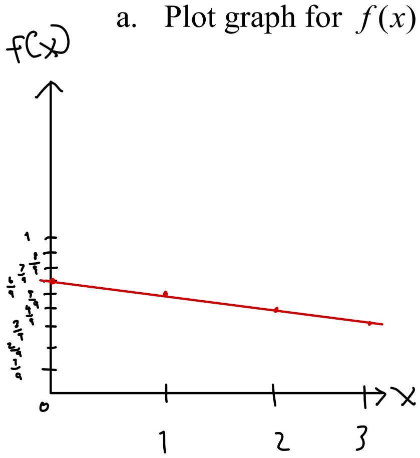
Student Name Woradech Mawatchakij ID 6104640641

3. Given X is continuous random variable. The probability distribution function (PDF) of this variable is

$$f(x) = -\frac{1}{9}x + \frac{6}{9}, 0 \le x \le 3$$

X	0	1	2	3
$f(x)$	$\frac{6}{9}$	$\frac{5}{9}$	$\frac{4}{9}$	$\frac{3}{9}$

a. Plot graph for $f(x)$



b. Find the answer for $P(1 \le X \le 3)$

$$P(1 \le x \le 3) = \int_1^3 \left(-\frac{1}{9}x + \frac{6}{9}\right) dx$$
$$= \left[-\frac{1}{18}x^2 + \frac{6}{9}x\right]_1^3$$
$$= \left(-\frac{9}{18} + 2\right) - \left(-\frac{1}{18} + \frac{6}{9}\right)$$
$$= \frac{27}{18} - \frac{11}{18} = \frac{16}{18} = \frac{8}{9}$$

c. Find the answer for $P(X \ge 2)$

$$P(x \ge 2) = \int_2^3 \left(-\frac{1}{9}x + \frac{6}{9}\right) dx + \int_3^\infty f(x)dx$$
$$= \left[-\frac{1}{18}x^2 + \frac{6}{9}x\right]_2^3 + 0$$
$$= \left(-\frac{9}{18} + 2\right) - \left(-\frac{4}{18} + \frac{12}{9}\right)$$
$$= \frac{27}{18} - \frac{20}{18}$$
$$= \frac{7}{18}$$

d. Find the expected value of X

$$E(X) = \int_0^3 x f(x) dx = \int_0^3 x\left(-\frac{1}{9}x + \frac{6}{9}\right) dx$$
$$= \int_0^3 \left(-\frac{1}{9}x^2 + \frac{6}{9}x\right) dx$$
$$= \left[-\frac{1}{27}x^3 + \frac{6}{18}x^2\right]_0^3$$
$$= (-1 + 3) - 0$$
$$= 2$$

Student Name Woradech Mawatchakij ID 6104640641

4. Let random variable X be the outcome of throwing one dice and random variable Y be the outcome of tossing one coin. Coin has two sided that has valued 1 and 0.

   a. Construct the joint probability distribution function (PDF) table of X and Y

Dice (X)

Coin (Y)		1	2	3	4	5	6
	0	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$
	1	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

   b. Find the marginal probability distribution function (PDF) of X

$P(X=1) = \frac{1}{6}$ $\quad P(X=2) = \frac{1}{6}$ $\quad P(X=3) = \frac{1}{6}$

$P(X=4) = \frac{1}{6}$ $\quad P(X=5) = \frac{1}{6}$ $\quad P(X=6) = \frac{1}{6}$

   c. Find the marginal probability distribution function (PDF) of Y

$P(Y=0) = \frac{1}{2}$ $\quad P(Y=1) = \frac{1}{2}$

   d. Find the conditional probability distribution function (PDF) of X given Y is equal to 1

$$P(X|Y=1) = \frac{P(X, Y=1)}{P(Y=1)} = \frac{P(X, Y=1)}{\frac{1}{2}} = 2(P(X, Y=1))$$

   e. Find the expected value of X given Y is equal to 1

$$E(X|Y=1) = \sum x_i \, P(X=x_i \mid Y=1)$$
$$= \sum x_i \, (2P(X, Y=1))$$
$$= 2\left[1\left(\tfrac{1}{12}\right) + 2\left(\tfrac{1}{12}\right) + 3\left(\tfrac{1}{12}\right) + 4\left(\tfrac{1}{12}\right) + 5\left(\tfrac{1}{12}\right) + 6\left(\tfrac{1}{12}\right)\right]$$
$$= 2\left(\tfrac{21}{12}\right) = \tfrac{21}{6} = 3.5$$

   f. Find the variance of X given Y is equal to 1

$$Var(X|Y=1) = E[X^2|Y=1] - [E(X|Y=1)]^2$$
$$= \sum x_i^2 \, P(X_i|Y=1) - (3.5)^2$$
$$= (1^2)(2)\left(\tfrac{1}{12}\right) + 2^2(2)\left(\tfrac{1}{12}\right) + \ldots + 6^2(2)\left(\tfrac{1}{12}\right) - (3.5)^2$$
$$= \tfrac{1}{6}(91) - (3.5)^2 = 2.9667$$

Student Name Woradech Mawatchaki ID 6704640641

5. If X_1, X_2, X_3 is a random sample from a population with mean u and variance σ^2. X_1, X_2, X_3 are not independent

$$Cov(X_1, X_2) = Cov(X_1, X_3) = Cov(X_2, X_3) = \frac{1}{4}\sigma^2$$

$\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{3}(X_1 + X_2 + X_3)$

Find $E(\bar{X})$ and $\text{var}(\bar{X})$

$$E(\bar{X}) = E\left(\frac{1}{3}(X_1 + X_2 + X_3)\right)$$
$$= \frac{1}{3}\left[E(X_1) + E(X_2) + E(X_3)\right]$$
$$= \frac{1}{\cancel{3}}(\cancel{3}\mu_X)$$
$$\therefore E(\bar{X}) = \mu_X$$

$$Var(\bar{X}) = Var\left[\frac{1}{3}(X_1 + X_2 + X_3)\right]$$
$$= \left(\frac{1}{3}\right)^2 Var(X_1 + X_2 + X_3)$$
$$= \frac{1}{9}\left[Var(X_1) + Var(X_2) + Var(X_3) + 2Cov(X_1, X_2) + 2Cov(X_1, X_3) + 2Cov(X_2, X_3)\right]$$
$$= \frac{1}{9}\left[3\sigma^2 + \cancel{2}\left(\frac{3\sigma^2}{\cancel{4}\,2}\right)\right]$$
$$= \frac{1}{\cancel{9}}\left(\frac{\cancel{9}}{2}\sigma^2\right)$$
$$\therefore Var(\bar{X}) = \frac{\sigma^2}{2}$$

6. Given X_1, X_2, X_3, X_4 are independent identically distributed random variables from population with mean u and variance σ^2. $\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{4}(X_1 + X_2 + X_3 + X_4)$

a. Find $E(\bar{X})$ and $\text{var}(\bar{X})$ in term of μ and σ

$$E(\bar{X}) = E\left[\frac{1}{4}(X_1 + X_2 + X_3 + X_4)\right]$$
$$= \frac{1}{4}\left[E(X_1) + E(X_2) + E(X_3) + E(X_4)\right]$$
$$= \frac{1}{\cancel{4}}(\cancel{4}\mu_x)$$
$$\therefore E(\bar{X}) = \mu_X$$

$$Var(\bar{X}) = Var\left[\frac{1}{4}(X_1 + X_2 + X_3 + X_4)\right]$$
$$= \left(\frac{1}{4}\right)^2 Var(X_1 + X_2 + X_3 + X_4)$$

Student Name Woradeeh Mawatchakij ID 610464064

$$= \frac{1}{16}\left[Var(X_1)+Var(X_2)+Var(X_3)+Var(X_4)\right]$$

$$= \frac{1}{\cancel{16}\,4}(\cancel{4}\sigma^2)$$

$$\therefore Var(\bar{X}) = \frac{\sigma^2}{4}$$

b. Given $\tilde{X} = \frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4$ is another estimator of μ. Show that $\tilde{X}$ is an unbiased estimator of u

$$E(\tilde{X}) = E\left[\frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4\right]$$

$$= E\left(\frac{1}{8}X_1\right) + E\left(\frac{1}{4}X_2\right) + E\left(\frac{1}{8}X_3\right) + E\left(\frac{1}{2}X_4\right)$$

$$= \frac{1}{8}E(X_1) + \frac{1}{4}E(X_2) + \frac{1}{8}E(X_3) + \frac{1}{2}E(X_4)$$

$$= \frac{1}{8}\mu + \frac{1}{4}\mu + \frac{1}{8}\mu + \frac{1}{2}\mu$$

$\therefore E(\tilde{X}) = \mu$ which indicates that $\tilde{X}$ is an unbiased estimator of μ

c. Between $\bar{X}$ and $\tilde{X}$, which one is the better estimator for μ? Why?

$$Var(\tilde{X}) = Var\left(\frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4\right)$$

$$= \left(\frac{1}{8}\right)^2 Var(X_1) + \left(\frac{1}{4}\right)^2 Var(X_2) + \left(\frac{1}{8}\right)^2 Var(X_3) + \left(\frac{1}{2}\right)^2 Var(X_4)$$

$$= \frac{1}{64}\sigma^2 + \frac{1}{16}\sigma^2 + \frac{1}{64}\sigma^2 + \frac{1}{4}\sigma^2$$

$$Var(\tilde{X}) = \frac{11}{32}\sigma^2$$

Since $Var(\bar{x}) < Var(\tilde{x})$, $\bar{x}$ is better estimator for μ