Quiz 2: Date: May 5, 2022 from 11.00-12.30

Question 1 (40 marks) Score.....

Consider the Muliperiod model of consumption and portfolio choice. Let an individual in this economy has the utility function as follow:

$$\max_{C_s,\omega_s,\forall_t} E_t\left[\sum_{s=t}^{T-1}\delta^s\left(\frac{C_s^{(1-\gamma)}}{1-\gamma}\right)+\delta^T\left(\frac{W_T^{(1-\gamma)}}{1-\gamma}\right)\right]$$

Assume that there is no wage income $(y_t = 0\ \forall t)$ and a constant risk-free rate return asset , $R_{ft} = R_f$. Also asssume that n=1 and the return of a single risky asset, R_{rt},is independently and identically distributed over time. Denote the proportion of wealth invested in the risky asset at date t as ω_t.

Please read and answer the following questions carefully and completely.

Score.....

Question 1.1 (10 marks) Derive the first-order condition for the optimal consumption level and portfolio weight at date T-1, C^*_{T-1} and ω^*_{T-1}., and give an explicit expression for C^*_{T-1}

$$J(W_t, I_t, t) = \max_{C_s, \{w_{is}\}, \forall s,i} E_t\left[\sum_{s=t}^{T-1} \sigma^s \left(\frac{C_s^{(1-r)}}{1-r}\right) + \sigma^T \left(\frac{W_T^{(1-r)}}{1-r}\right)\right]$$

at T_1 ; $J(W_1, T) = E_1[B(W_1, 1)] = B(W_1, 1)$

$$J(W_{T-1}, T-1) = \max_{C_{T-1}, \{w_{i,T-1}\}} E_{T-1}[U(C_{T-1}, T-1) + B(W_T, T)]$$

$$= \max_{C_{T-1}, \{w_{i,T-1}\}} U(C_{T-1}, T-1) + E_{T-1}[B(W_1, T)]$$

$$J(W_{T-1}, T-1) = \max_{C_{T-1}, \{w_{i,T-1}\}} U(C_{T-1}, T-1) + E_{T-1}[B(S_{T-1} R_{T-1}, T)$$

$$S_{T-1} = W_{T-1} + y_{T-1} - C_{T-1}$$

$$R_{T-1} = R_{f,T-1} + \sum_{i=1}^{n} w_{i,T-1}(R_{i,T-1} - R_{f,T-1})$$

FOC $\frac{\partial J_w}{\partial C_{T-1}} = \sigma^{T-1}(1-r)\frac{C_{T-1}}{1-r} - E_{T-1}\left[\sigma^T(1-r)\frac{W_T^{-r}}{1-r} \cdot R_{T-1}\right] = 0$

$$\frac{\partial J}{\partial w_{i,T-1}} = E_{T-1}\left(\sigma^T(1-r)\frac{W_T^{-r}}{1-r}(R_{i,T-1} - R_{f,T-1})\right]$$

Score.....

Question 1.2 (10 marks) Solve for the form of J(W_{T-1},T-1).

Score.....

Question 1.3 (10 marks) Derive the first-order condition for the optimal consumption level and portfolio weight at date T-2, C^*_{T-2} and ω^*_{T-2}., and give an explicit expression for C^*_{T-2}

Score.....

Question 1.4 (10 marks) Solve for the form of J(W_{T-2},T-2).Based on the pattern for T-1 and T-2, provide expressions for the optimal consumption and portfolio weight at any date T-t, t=1,2,3,....