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1. From table 1 we get:

$$\sum_{i=1}^{n} x_i = 63+72+78+81+87+75+75+90 \boxed{=621}$$

$$\sum_{i=1}^{n} y_i = 2.8+3.4+3.0+3.5+3.6+3.0+2.7+3.7 = \boxed{25.7}$$

$$\bar{x}_i = \frac{63+72+78+81+87+75+75+90}{8} = \frac{621}{8} = \boxed{77.625}$$

$$\bar{y}_i = \frac{2.8+3.4+3.0+3.5+3.6+3.0+2.7+3.7}{8} = \boxed{3.2125}$$

$$\sum_{i=1}^{n} x_i^2 = (63)^2+(72)^2+(78)^2+(81)^2+(87)^2+(75)^2+(75)^2+(90)^2 = \boxed{48717}$$

$$\sum_{i=1}^{n} x_i y_i = 63(2.8)+72(3.4)+78(3.0)+81(3.5)$$
$$+87(3.6)+75(3.0)+75(2.7)+90(3.7) = \boxed{2012.4}$$

By $$\hat{\beta}_1 = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} y_i}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2} = \frac{8(2012.4) - 621(25.7)}{8(48717) - (621)^2}$$

$$= \frac{16099.2 - 15959.7}{389736 - 385641} = \frac{139.5}{4095} = \boxed{0.0341}$$

From $\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{x}_i = 3.2125 - 0.0341(77.625)$

$= \boxed{0.5655}$

1.2

$\beta_0 = 0.5655, \ \beta_1 = 0.0341$

$$\boxed{\hat{y}_i = \beta_0 + \hat{\beta}_1 x_i}$$

$i = 1 \quad \hat{y}_1 = 0.5655 + 0.0341(68) = 2.7138$

$i = 2 \quad \hat{y}_2 = 0.5655 + 0.0341(72) = 3.0207$

$i = 3 \quad \hat{y}_3 = 0.5655 + 0.0341(78) = 3.2253$

$i = 4 \quad \hat{y}_4 = 0.5655 + 0.0341(81) = 3.3276$

$i = 5 \quad \hat{y}_5 = 0.5655 + 0.0341(87) = 3.5322$

$i = 6 \quad \hat{y}_6 = 0.5655 + 0.0341(75) = 3.123$

$i = 7 \quad \hat{y}_7 = 0.5655 + 0.0341(75) = 3.123$

$i = 8 \quad \hat{y}_8 = 0.5655 + 0.0341(90) = 3.6345$

$\hat{u}_i = y_i - \hat{y}_i$

$i=1, \hat{\mu}_1 = 2.8 - 2.7138 = 0.0862$

$i=2, \hat{\mu}_2 = 3.4 - 3.0207 = 0.3793$

$i=3, \hat{\mu}_3 = 3.0 - 3.2253 = -0.2253$

$i=4, \hat{\mu}_4 = 3.5 - 3.3276 = 0.1724$

$i=5, \hat{\mu}_5 = 3.6 - 3.5322 = 0.0678$

$i=6, \hat{\mu}_6 = 3.0 - 3.123 = -0.123$

$i=7, \hat{\mu}_7 = 2.7 - 3.123 = -0.423$

$i=8, \hat{\mu}_8 = 3.7 - 3.6345 = 0.0655$

$$\sum_{i=1}^{n} \hat{\mu}_i = 0.0862 + 0.3793 - 0.2253 + 0.1724 + 0.0678 + (-0.123) + (-0.423) + 0.0655 \approx 0$$

$$\sum_{i=1}^{n} \hat{\mu}_i = \boxed{0.4347275}$$

1.3

$$Var(\hat{\mu}_i) = \frac{\sum_{i=1}^{n} \hat{\mu}_i^2}{n-2} = \frac{0.4347275}{8-2}$$

$$= 0.07245458333$$

- $$Var(\hat{\beta}_1) = \frac{\sum_{i=1}^{n} X_i^2}{n \sum_{i=1}^{n} x_i^2} \sigma^2$$

$$= \frac{48717}{8(511.875)} (0.0724545 8333)$$

$$= \boxed{0.86197068}$$

- $$Var(\hat{\beta}_2) = \frac{\sigma^2}{\sum_{i=1}^{n} x_i^2} = \frac{0.07245833}{511.875} = \boxed{0.00014154741}$$

2

i	X_i	Y_i	$x_i = X_i - Y_i$	$y_i = Y_i - \bar{Y}$	$x_i y_i$	x_i^2	X_i^2
1	10	0	-10	-9.1	91	100	100
2	12	2	-8	-7.1	56.8	64	144
3	14	5	-6	-4.1	24.6	36	196
4	16	6	-4	-3.1	12.4	16	256
5	18	7	-2	-2.1	4.2	4	324
6	22	10	2	0.9	1.8	4	484
7	24	10	4	0.9	3.6	16	576
8	26	15	6	5.9	35.4	36	676
9	28	16	8	6.9	55.2	64	784
10	30	20	10	10.9	109.	100	900
Total	200	91			394	440	4440

$$\sum_{i=1}^{n} x_i = 200 \quad , \quad \sum_{i=1}^{n} y_i = 91$$

$$\mu = \frac{200}{10} = 20$$

$\boxed{2.1}$ Find $\hat{\beta}_1 , \hat{\beta}_2$

$$\hat{\beta}_1 = \bar{y} - \bar{\beta}_2 \bar{x}$$

$$\hat{\beta}_2 = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2}$$

$$\therefore \hat{\beta}_2 = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2} = \frac{394}{440} = \boxed{0.895454}$$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X} = 9.1 - \left(\frac{394}{440}\right) 20 = 9.1 - 17.90909$$

$$= \boxed{-8.80909}$$

$\boxed{2.2}$ Find $\hat{y}_i$ & $\hat{\mu}_i$, Show $\sum_{i=1}^{n} \mu_i \approx 0$

x_i	Y_i	$\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 x_i$	$\hat{\mu}_i = Y_i - (\hat{\beta}_1 + \hat{\beta}_2 x_i)$	$\hat{\mu}_i^2$
10	0	0.14545	-0.14545	0.0211557
12	2	1.93636	0.06364	0.00405
14	5	3.72727	1.27273	1.619842
16	6	5.51818	0.48182	0.23215
18	7	7.30909	-0.30909	0.09554
22	10	10.890909	-0.890909	0.79372
24	10	12.681818	-2.681818	7.19214
26	15	14.472727	0.527273	0.27802
28	16	16.263636	-0.263636	0.069504
30	20	18.054545	1.945455	3.784795
Total 200	91		$\sum_{i=1}^{n} \hat{\mu}_i = 0.000015$	$\sum_{i=1}^{n} \hat{\mu}_i^2 = 14.0909$

$$\Rightarrow \sum_{i=1}^{n} \hat{\mu}_i \approx 0$$

$$\sum_{i=1}^{n} x_i = 200$$

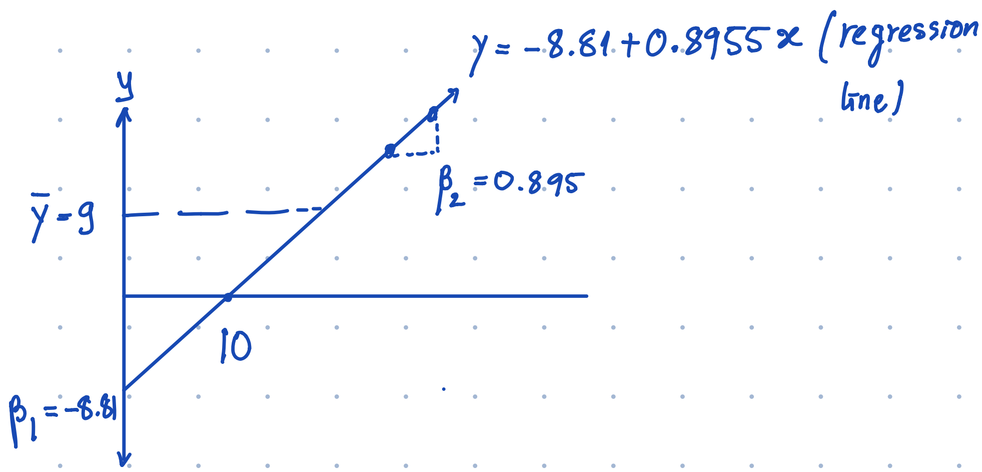
$$\sum_{i=1}^{n} Y_i = 91$$

$$\sum_{i=1}^{n} x_i / n = \frac{200}{10} = 20$$

$$\sum_{i=1}^{n} Y_i / n = \frac{91}{10} = 9.1$$

$$\boxed{\begin{aligned} Y_i &= \hat{Y} + \hat{\mu}_i \\ \hat{\mu} &= Y_i - (\hat{\beta}_1 + \hat{\beta}_2 x_i) \end{aligned}}$$

$\boxed{2.3}$ Plot graph and draw a regression line



Given $\bar{X} = 20$, $\bar{Y} = 9.1$ → substitute $\bar{X}$ in to the function

$$Y = -8.81 + 0.8955(20)$$
$$= 9.1 = \bar{Y}$$

∴ So, the regression line passed $(\bar{X}, \bar{Y})$

$\boxed{2.4}$

$$\hat{Y}_i = \beta_1 + \beta_2 + X_i$$

$$\hat{Y}_P = -8.81 + 0.8955(18)$$

$$= 7.30909$$

$\boxed{2.5}$

$$Var(\hat{\mu}_i) = \frac{\sum_{i=1}^{n} \hat{\mu}_i^2}{n-2} = \frac{14.0909}{10-2} = 1.76136$$

- $$\mathrm{Var}(\hat{\beta}_1) = \frac{\sum_{i=1}^{n} X_i^2}{n \sum_{i=1}^{n} x_i^2}(\sigma^2) = \frac{4440}{10 \times 440} \times 1.76136$$
$$\boxed{= 1.76136}$$

- $$\mathrm{Var}(\hat{\beta}_2) = \frac{\sigma^2}{\sum_{i=1}^{n} x_i^2} = \frac{1.76136}{440} = \boxed{0.004003}$$

$\boxed{3}$

$$Y_i = \beta_1 + \beta_2 X_i + u_i \; ; \quad u_i \sim NIID(0, \sigma^2)$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} X_i^2 \sum_{i=1}^{n} Y_i - \sum_{i=1}^{n} X_i \sum_{i=1}^{n} X_i Y_i}{n \sum_{i=1}^{n} X_i^2 - \left(\sum_{i=1}^{n} X_i\right)^2} = \boxed{\bar{Y} - \hat{\beta}_2 \bar{X}}$$

Proof that this is an unbiased estimator:

$$E(\hat{\beta}_1) = E(\bar{y} - \hat{\beta}_2 \bar{X})$$
$$= E(\bar{Y}) - \hat{\beta}_2 E(\bar{X})$$
$$= \beta_1 + \beta_2 \bar{X} - \bar{X} \beta_2$$
$$\therefore E(\hat{\beta}_1) = \beta_1$$

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