

16.7 Index Numbers

Chapter 16 has presented various methods for forecasting time-series data. In this section, index numbers are used to compare a value of a time series relative to another value of a time series. **Index numbers** measure the value of an item (or group of items) at a particular point in time, as a percentage of the value of an item (or group of items) at another point in time. They are commonly used in business and economics as indicators of changing business or economic activity. There are many kinds of index numbers, including price indices, quantity indexes, value indexes, and sociological indexes. In this section, only the price index is considered. In addition to allowing comparison of prices at different points in time, price indices are also used to deflate the effect of inflation on a time series in order to compare values in *real* dollars instead of *actual* dollars.

Price Indices

A **price index** compares the price of a commodity in a given period of time to the price paid for that commodity at a particular point in time in the past. A **simple price index** tracks the price of a single commodity. An **aggregate price index** tracks the prices for a group of commodities (called a market basket) at a given period of time to the price paid for that group of commodities at a particular point in time in the past. The **base period** is the point in time in the past against which all comparisons are made. In selecting the base period for a particular index, if possible, you select a period of economic stability rather than one at or near the peak of an expanding economy or the bottom of a recession or declining economy. In addition, the base period should be relatively recent so that comparisons are not greatly affected by changing technology and consumer attitudes and habits. Equation (16.21) defines the simple price index.

SIMPLE PRICE INDEX

$$I_i = \frac{P_i}{P_{base}} \times 100 \quad (16.21)$$

where

I_i = price index for year i

P_i = price for year i

P_{base} = price for the base year

As an example of the simple price index, consider the price per gallon of unleaded gasoline in the United States from 1980 to 2015. Table 16.4 on the next page and stored in [Gasoline](#), presents the prices plus two sets of index numbers.

To illustrate the calculation of the simple price index for 1981, using 1980 as the base year, from Equation (16.21) and Table 16.4,

$$I_{1981} = \frac{P_{1981}}{P_{1980}} \times 100 = \frac{1.38}{1.25} \times 100 = 110.4$$

This index value means that the price per gallon of unleaded gasoline in the United States in 1981 was 10.4% higher than in 1980.

TABLE 16.4

Price per Gallon of Unleaded Gasoline in the United States and Simple Price Index, with 1980 and 1995 as the Base Years (1980–2015)

Year	Gasoline Price	Price Index—1980	Price Index—1995
1980	1.25	100.0	108.7
1981	1.38	110.4	120.0
1982	1.30	104.0	113.0
1983	1.24	99.2	107.8
1984	1.21	96.8	105.2
1985	1.20	96.0	104.3
1986	0.93	74.4	80.9
1987	0.95	76.0	82.6
1988	0.95	76.0	82.6
1989	1.02	81.6	88.7
1990	1.16	92.8	100.9
1991	1.14	91.2	99.1
1992	1.14	91.2	99.1
1993	1.11	88.8	96.5
1994	1.11	88.8	96.5
1995	1.15	92.0	100.0
1996	1.23	98.4	107.0
1997	1.23	98.4	107.0
1998	1.06	84.8	92.2
1999	1.17	93.6	101.7
2000	1.51	120.8	131.3
2001	1.46	116.8	127.0
2002	1.36	108.8	118.3
2003	1.59	127.2	138.3
2004	1.88	150.4	163.5
2005	2.30	184.0	200.0
2006	2.59	207.2	225.2
2007	2.40	192.0	208.7
2008	2.77	221.6	240.9
2009	2.61	208.8	227.0
2010	2.99	239.2	260.0
2011	3.28	262.4	285.2
2012	3.33	266.4	289.6
2013	3.28	262.40	285.22
2014	2.56	204.80	222.61
2015	2.19	175.20	190.43

Source: Bureau of Labor Statistics, U.S. Department of Labor, www.bls.gov.

To compute the simple price index for 2008,

$$I_{2008} = \frac{P_{2008}}{P_{1980}} \times 100 = \frac{2.77}{1.25} \times 100 = 221.6$$

This index value means that the price per gallon of unleaded gasoline in the United States in 2008 was 121.2% higher than in 1980.

An examination of the price indices for 1980 to 2008 in Table 16.4 indicates that the price of unleaded gasoline increased in 1981 and 1982 over the base year of 1980 but then was below the 1980 price every year until 2000. Because the base period for the index numbers in Table 16.4

is 1980, one should choose a base year closer to the present. The price remained fairly constant from 1990 to 1995; thus, it is appropriate to use 1995 as a base year. Index numbers with a new base can be calculated using Equation (16.22).

SHIFTING THE BASE FOR A SIMPLE PRICE INDEX

$$I_{new} = \frac{I_{old}}{I_{new\ base}} \times 100 \quad (16.22)$$

where

I_{new} = new price index

I_{old} = old price index

$I_{new\ base}$ = value of the old price index for the new base year

To change the base year to 1995, $I_{new\ base} = 92.0$. Using Equation (16.22) to find the new price index for 2008,

$$I_{new} = \frac{I_{old}}{I_{new\ base}} \times 100 = \frac{221.6}{92.0} \times 100 = 240.9$$

Thus, the 2008 price for unleaded gasoline in the United States was almost 2.5 times the price that it was in 1995.

Aggregate Price Indices

An aggregate price index consists of a group of commodities taken together. The group of commodities under consideration is often called a *market basket*. There are two types of aggregate price indices: unweighted aggregate price indices and weighted aggregate price indices. An **unweighted aggregate price index**, defined in Equation (16.23), places equal weight on all the items in the market basket.

UNWEIGHTED AGGREGATE PRICE INDEX

$$I_U^{(t)} = \frac{\sum_{i=1}^n P_i^{(t)}}{\sum_{i=1}^n P_i^{(0)}} \times 100 \quad (16.23)$$

where

t = time period (0, 1, 2, . . .)

i = item (1, 2, . . . , n)

n = total number of items under consideration

$\sum_{i=1}^n P_i^{(t)}$ = sum of the prices paid for each of the n commodities at time period t

$\sum_{i=1}^n P_i^{(0)}$ = sum of the prices paid for each of the n commodities at time period 0

$I_U^{(t)}$ = value of the unweighted price index at time period t

Table 16.5 presents the mean prices for three fruit items for selected periods from 1980 to 2010 (stored in [Fruit](#)).

TABLE 16.5

Prices (in Dollars per Pound)
for Three Fruit Items

Fruit	Year						
	1980	1985	1990	1995	2000	2005	2010
Apples	$P_i^{(0)}$	$P_i^{(1)}$	$P_i^{(2)}$	$P_i^{(3)}$	$P_i^{(4)}$	$P_i^{(5)}$	$P_i^{(6)}$
Apples	0.692	0.684	0.719	0.835	0.927	0.966	1.141
Bananas	0.342	0.367	0.463	0.490	0.509	0.485	0.586
Oranges	0.365	0.533	0.570	0.625	0.638	0.838	0.899

Source: Bureau of Labor Statistics, U.S. Department of Labor, www.bls.gov.

To calculate the unweighted aggregate price index for the various years, using Equation (16.23) and 1980 as the base period:

$$1980: I_U^{(0)} = \frac{\sum_{i=1}^3 P_i^{(0)}}{\sum_{i=1}^3 P_i^{(0)}} \times 100 = \frac{0.692 + 0.342 + 0.365}{0.692 + 0.342 + 0.365} \times 100 = \frac{1.399}{1.399} \times 100 = 100.0$$

$$1985: I_U^{(1)} = \frac{\sum_{i=1}^3 P_i^{(1)}}{\sum_{i=1}^3 P_i^{(0)}} \times 100 = \frac{0.684 + 0.367 + 0.533}{0.692 + 0.342 + 0.365} \times 100 = \frac{1.584}{1.399} \times 100 = 113.2$$

$$1990: I_U^{(2)} = \frac{\sum_{i=1}^3 P_i^{(2)}}{\sum_{i=1}^3 P_i^{(0)}} \times 100 = \frac{0.719 + 0.463 + 0.570}{0.692 + 0.342 + 0.365} \times 100 = \frac{1.752}{1.399} \times 100 = 125.2$$

$$1995: I_U^{(3)} = \frac{\sum_{i=1}^3 P_i^{(3)}}{\sum_{i=1}^3 P_i^{(0)}} \times 100 = \frac{0.835 + 0.490 + 0.625}{0.692 + 0.342 + 0.365} \times 100 = \frac{1.950}{1.399} \times 100 = 139.4$$

$$2000: I_U^{(4)} = \frac{\sum_{i=1}^3 P_i^{(4)}}{\sum_{i=1}^3 P_i^{(0)}} \times 100 = \frac{0.927 + 0.509 + 0.638}{0.692 + 0.342 + 0.365} \times 100 = \frac{2.074}{1.399} \times 100 = 148.2$$

$$2005: I_U^{(5)} = \frac{\sum_{i=1}^3 P_i^{(5)}}{\sum_{i=1}^3 P_i^{(0)}} \times 100 = \frac{0.966 + 0.485 + 0.838}{0.692 + 0.342 + 0.365} \times 100 = \frac{2.289}{1.399} \times 100 = 163.6$$

$$2010: I_U^{(6)} = \frac{\sum_{i=1}^3 P_i^{(6)}}{\sum_{i=1}^3 P_i^{(0)}} \times 100 = \frac{1.141 + 0.586 + 0.899}{0.692 + 0.342 + 0.365} \times 100 = \frac{2.625}{1.399} \times 100 = 187.7$$

Thus, in 2010, the combined price of a pound of apples, a pound of bananas, and a pound of oranges was 87.7% more than it was in 1980.

An unweighted aggregate price index represents the changes in prices, over time, for an entire group of commodities. However, an unweighted aggregate price index has two shortcomings. First, this index considers each commodity in the group as equally important. Thus, the most expensive commodities per unit are overly influential. Second, not all the commodities are consumed at the same rate. In an unweighted index, changes in the price of the least-consumed commodities are overly influential.

Weighted Aggregate Price Indices

Due to the shortcomings of unweighted aggregate price indices, weighted aggregate price indices are generally preferable. **Weighted aggregate price indices** account for differences in the magnitude of prices per unit and differences in the consumption levels of the items in the market basket. Two types of weighted aggregate price indices are commonly used in business and economics: the Laspeyres price index and the Paasche price index. Equation (16.24) defines the **Laspeyres price index**, which uses the consumption quantities associated with the base year in the calculation of all price indices in the series.

LASPEYRES PRICE INDEX

$$I_L^{(t)} = \frac{\sum_{i=1}^n P_i^{(t)} Q_i^{(0)}}{\sum_{i=1}^n P_i^{(0)} Q_i^{(0)}} \times 100 \quad (16.24)$$

where

t = time period (0, 1, 2, . . .)

i = item (1, 2, . . . , n)

n = total number of items under consideration

$Q_i^{(0)}$ = quantity of item i at time period 0

$I_L^{(t)}$ = value of the Laspeyres price index at time t

$P_i^{(t)}$ = price paid for commodity i at time period t

$P_i^{(0)}$ = price paid for commodity i at time period 0

TABLE 16.6

Prices (in Dollars per Pound) and Quantities (Annual per Capita Consumption, in Pounds) for Three Fruit Items

Fruit	Year						
	1980	1985	1990	1995	2000	2005	2010
	$P_i^{(0)}, Q_i^{(0)}$	$P_i^{(1)}, Q_i^{(1)}$	$P_i^{(2)}, Q_i^{(2)}$	$P_i^{(3)}, Q_i^{(3)}$	$P_i^{(4)}, Q_i^{(4)}$	$P_i^{(5)}, Q_i^{(5)}$	$P_i^{(6)}, Q_i^{(6)}$
Apples	0.692, 19.2	0.684, 17.3	0.719, 19.6	0.835, 18.9	0.927, 17.5	0.966, 16.0	1.141, 15.6
Bananas	0.342, 20.2	0.367, 23.5	0.463, 24.4	0.490, 27.4	0.509, 28.5	0.485, 26.8	0.586, 25.6
Oranges	0.365, 14.3	0.533, 11.6	0.570, 12.4	0.625, 12.0	0.638, 11.7	0.838, 10.6	0.899, 9.7

Sources: Bureau of Labor Statistics, U.S. Department of Labor, www.bls.gov, the Statistical Abstract of the United States, U.S. Census Bureau, www.census.gov, and the U.S. Department of Agriculture Economic Research Service, www.ers.usda.gov

Table 16.6 gives the price and per capita consumption, in pounds, for the three fruit items comprising the market basket of interest (and stored in **Fruit**).

Using 1980 as the base year, you calculate the Laspeyres price index for 2010 ($t = 5$) using Equation (16.24):

$$I_L^{(6)} = \frac{\sum_{i=1}^3 P_i^{(6)} Q_i^{(0)}}{\sum_{i=1}^3 P_i^{(0)} Q_i^{(0)}} \times 100 = \frac{(1.141 \times 19.2) + (0.586 \times 20.2) + (0.899 \times 14.3)}{(0.692 \times 19.2) + (0.342 \times 20.2) + (0.365 \times 14.3)} \times 100$$

$$= \frac{46.6001}{25.4143} \times 100 = 183.4$$

Thus, the Laspeyres price index is 183.4, indicating that the cost of purchasing these three items in 2010 was 83.4% more than in 1980. This index is slightly less than the unweighted index, 187.7, because the most-purchased items, apples and bananas, increased in price slightly less than oranges in the time span. In other words, in the unweighted index, the least-purchased commodity (oranges) is slightly more influential.

The **Paasche price index** uses the consumption quantities in the year of interest instead of using the initial quantities. Thus, the Paasche index is a more accurate reflection of total consumption costs at that point in time. However, there are two major drawbacks of the Paasche index. First, accurate consumption values for current purchases are often difficult to obtain. Thus, many important indices, such as the consumer price index (CPI), use the Laspeyres method. Second, if a particular product increases greatly in price compared to the other items in the market basket, consumers will avoid the high-priced item out of necessity, not because of changes in what they might prefer to purchase. Equation (16.25) defines the Paasche price index.

PAASCHE PRICE INDEX

$$I_P^{(t)} = \frac{\sum_{i=1}^n P_i^{(t)} Q_i^{(t)}}{\sum_{i=1}^n P_i^{(0)} Q_i^{(t)}} \times 100 \quad (16.25)$$

where

- t = time period (0, 1, 2, . . .)
- i = item (1, 2, . . . , n)
- n = total number of items under consideration
- $Q_i^{(t)}$ = quantity of item i at time period t
- $I_P^{(t)}$ = value of the Paasche price index at time period t
- $P_i^{(t)}$ = price paid for commodity i at time period t
- $P_i^{(0)}$ = price paid for commodity i at time period 0

To calculate the Paasche price index in 2010, using 1980 as a base year, you use $t = 6$ in Equation (16.25):

$$I_L^{(6)} = \frac{\sum_{i=1}^3 P_i^{(6)} Q_i^{(6)}}{\sum_{i=1}^3 P_i^{(0)} Q_i^{(6)}} \times 100 = \frac{(1.141 \times 15.6) + (0.586 \times 25.6) + (0.899 \times 9.7)}{(0.692 \times 15.6) + (0.342 \times 25.6) + (0.365 \times 9.7)} \times 100$$

$$= \frac{41.5215}{23.0909} \times 100 = 179.8$$

The Paasche price index for this market basket is 179.8. Thus, the cost of these three fruit items in 2010 was 79.8% higher in 2010 than in 1980, when using 2010 quantities.

Some Common Price Indices

Various price indices are commonly used in business and economics. The CPI is the most familiar index in the United States. This index is officially referred to as the CPI-U to reflect that it measures the prices “urban” residents are subject to, but it is commonly referred to as the CPI. The CPI, published monthly by the U.S. Bureau of Labor Statistics, is the primary measure of changes in the cost of living in the United States.

The CPI is a weighted aggregate price index, using the Laspeyres method, for 400 commonly purchased food, clothing, transportation, medical, and housing items. Currently computed using 1982–1984 averages as a base year, the CPI was 236.7 in 2014. (The file [CPI-U](#) contains the CPI-U for 1965–2014.)

Another important price index published by the U.S. Bureau of Labor Statistics is the producer price index (PPI). The PPI is a weighted aggregate price index that also uses the Laspeyres method for prices of commodities sold by wholesalers. The PPI is considered a leading indicator of the CPI. In other words, increases in the PPI tend to precede increases in the CPI, and, similarly, decreases in the PPI tend to precede decreases in the CPI.

Financial indices such as the DJIA Index, the S&P 500 Index, and the NASDAQ Index are price indices for different sets of stocks in the United States. Many indices measure the performance of international stock markets, including the Nikkei Index for Japan, the Dax 30 for Germany, and the SSE Composite for China.

PROBLEMS FOR SECTION 16.7

LEARNING THE BASICS

16.65 The simple price index for a commodity in 2015, using 1995 as the base year, is 175. Interpret this index number.

16.66 Prices for a commodity from 2013 to 2015 were as follows: 2013, \$5; 2014, \$8; and 2015, \$7.

- Calculate the simple price indices for 2013 to 2015, using 2013 as the base year.
- Calculate the simple price indices for 2013 to 2015, using 2014 as the base year.

16.67 The following are prices and consumption quantities for three commodities in 1995 and 2015:

COMMODITY	YEAR	
	1995	2015
	Price, Quantity	Price, Quantity
A	\$2, 20	\$3, 21
B	\$18, 3	\$36, 2
C	\$3, 18	\$4, 23

- Calculate the unweighted aggregate price index for 2015, using 1995 as the base year.
- Calculate the Laspeyres aggregate price index for 2015, using 1995 as the base year.
- Calculate the Paasche aggregate price index for 2015, using 1995 as the base year.

APPLYING THE CONCEPTS

16.68 The file [DJIA](#) contains the closing values of the DJIA from 1979 to 2014.

- Calculate the price index for the DJIA, using 1979 as the base year.

- Shift the base of the DJIA to 2000 and recalculate the price index.
- Compare the results of (a) and (b). Which price index do you think is more useful in understanding the changes in the DJIA? Explain.

16.69 The file [Coffee Prices](#) contains the mean price per pound of coffee in the United States from 1980 to 2014.

Source: Bureau of Labor Statistics, U.S. Department of Labor, www.bls.gov.

- Calculate the simple price indices for 1980 to 2014, using 1980 as the base year.
- Interpret the simple price index for 2014, using 1980 as the base year.
- Recalculate the simple price indices found in (a), using Equation (16.22), with 1990 as the base year.
- Interpret the simple price index for 2014, using 1990 as the base year.
- Would it be a good idea to use 1995 as the base year? Explain.
- Describe the trends in coffee costs from 1980 to 2014.

16.70 The file [Tomato Prices](#) contains the mean prices per pound of fresh tomatoes in the United States from 1980 to 2014.

Source: Bureau of Labor Statistics, U.S. Department of Labor, www.bls.gov.

- Calculate the simple price indices for 1980 to 2014, using 1980 as the base year.
- Interpret the simple price index for 2014, using 1980 as the base year.
- Recalculate the simple price indices found in (a), using Equation (16.22), with 1990 as the base year.
- Interpret the simple price index for 2014, using 1990 as the base year.
- Describe the trends in the cost of fresh tomatoes from 1980 to 2014.

16.71 The file **Energy Prices** contains the mean price for three types of energy products in the United States from 1992 to 2015: electricity (dollars per 500 kWh), natural gas (dollars per 40 therms), and fuel oil (dollars per gallon).

Source: Bureau of Labor Statistics, U.S. Department of Labor, www.bls.gov.

- a. Calculate the 1992–2015 simple price indices for electricity, natural gas, and fuel oil, using 1992 as the base year.
- b. Recalculate the price indices in (a), using 1996 as the base year.
- c. Calculate the 1992–2015 unweighted aggregate price indices for the group of three energy items.
- d. Calculate the 2015 Laspeyres price index for the group of three energy items for a family that consumed 5,000 kWh of electricity (10 units), 960 therms of natural gas (24 units), and 400 gallons of fuel oil (400 units) in 1992.
- e. Calculate the 2015 Laspeyres price index for the group of three energy items for a family that consumed 6,500 kWh of electricity, 1,040 therms of natural gas, and 235 gallons of fuel oil in 1992.

EG16.8 INDEX NUMBERS

Use the **Unweighted_Aggregate_PI** and the **Weighted_Aggregate_PI worksheets** of the **Index Numbers workbook** to compute index numbers. Open to the **Unweighted_Aggregate_FORMULAS** and the **Weighted_Aggregate_FORMULAS worksheets** to examine the details of all formulas used in the workbook.