

## Time Series Models

### Simulated AR Time Series

```
. set obs 500
obs was 0, now 500

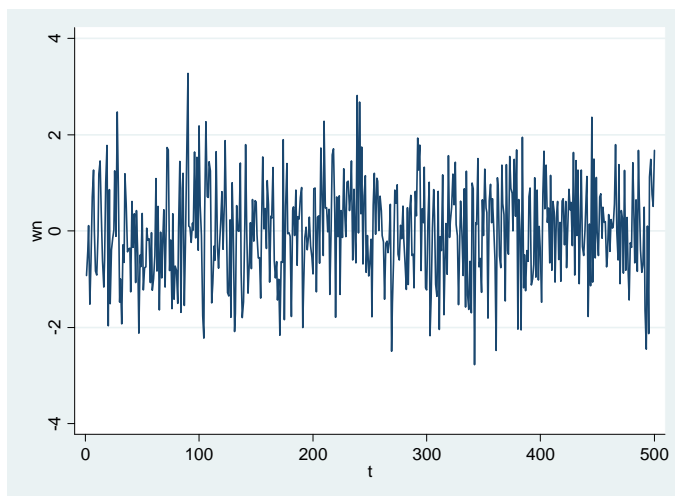
. g t=_n

. tsset t
    time variable: t, 1 to 500
    del ta: 1 uni t

. set seed 1234

. g wn=rnormal(0,1)

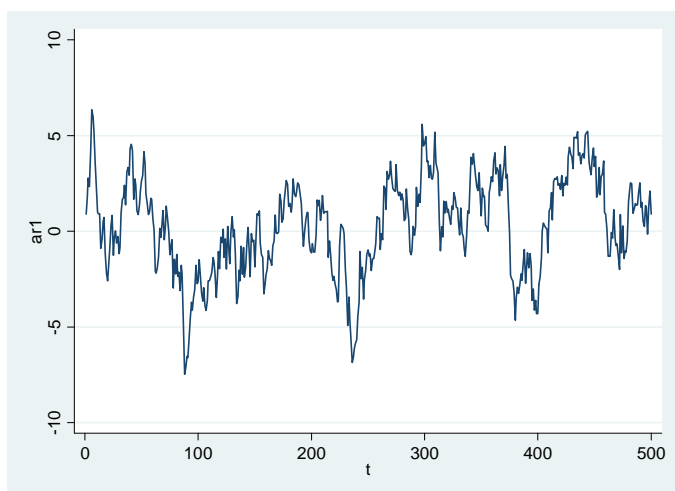
. line wn t
```



```
. g ar1=0.9 in 1
(499 missing values generated)

. replace ar1=0.9*l.ar1+rnormal(0,1) if t>1
(499 real changes made)

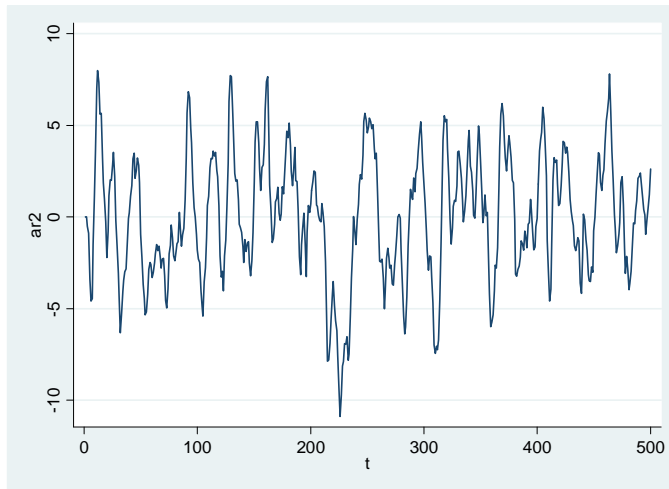
. line ar1 t
```



```
. g ar2=0 if t<3
(498 missing values generated)
```

```
. replace ar2=1.5*l.ar2-0.6*l2.ar2+rnormal(0,1) if t>2
(498 real changes made)

. line ar2 t
```



### **Random Walk without Drift:**

$$Y_{1t} = Y_{1t-1} + u_t$$

where:  $u_t$  is white noise error term with mean = 0 and variance =  $\sigma^2$

### **Random Walk with Drift:**

$$Y_{2t} = \delta + Y_{2t-1} + u_t$$

```
. set obs 200
obs was 0, now 200

. gener t=_n

. tsset t
time variable: t, 1 to 200

. gener u= rnormal(0,1)

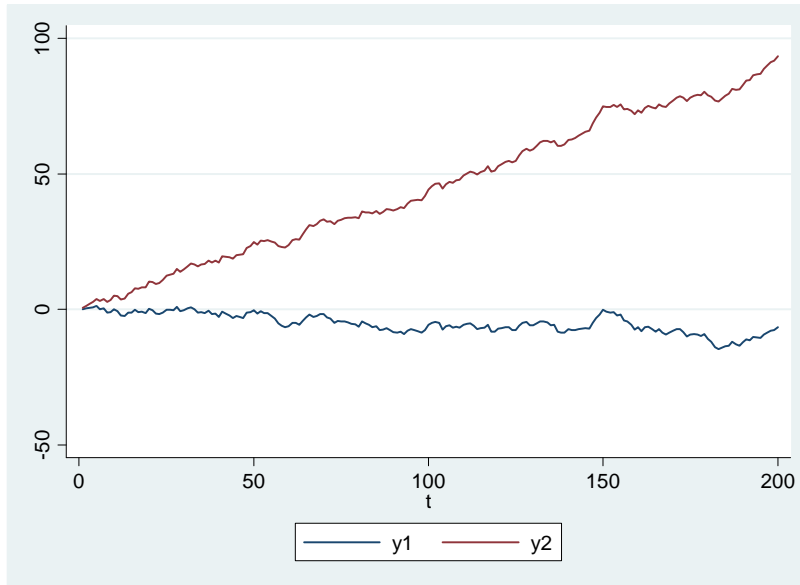
. gener y1=0 in 1
(199 missing values generated)

. gener y2=0.5 in 1
(199 missing values generated)

. replace y1=l.y1 +u if t>1
(199 real changes made)

. replace y2=0.5+l.y2 + u if t>1
(199 real changes made)

. line y1 y2 t
```



**Trend Stationary (TS) and Difference Stationary (DS)****Pure random walk**

$$\beta_1 = 0, \beta_2 = 0, \beta_3 = 1$$

$$Y_{1t} = Y_{1t-1} + u_t$$

**Random walk with drift**

$$\beta_1 \neq 0, \beta_2 = 0, \beta_3 = 1$$

$$Y_{2t} = \beta_1 + Y_{2t-1} + u_t$$

**Deterministic trend**

$$\beta_1 \neq 0, \beta_2 \neq 0, \beta_3 = 0$$

$$Y_{3t} = \beta_1 + \beta_2 t + u_t$$

**Random walk with drift and deterministic trend**

$$\beta_1 \neq 0, \beta_2 \neq 0, \beta_3 = 1$$

$$Y_{4t} = \beta_1 + \beta_2 t + Y_{4t-1} + u_t$$

**Deterministic trend with stationary AR(1) component**

$$\beta_1 \neq 0, \beta_2 \neq 0, \beta_3 < 1$$

$$Y_{5t} = \beta_1 + \beta_2 t + \beta_3 Y_{5t-1} + u_t$$

```

. set obs 200
obs was 0, now 200

. gener time=_n

. tsset time
   time variable:  time, 1 to 200

. gener y1=0 in 1
(199 missing values generated)

. gener y2=0 in 1
(199 missing values generated)

. gener y3=0 in 1
(199 missing values generated)

. gener y4=0 in 1
(199 missing values generated)

. gener y5=0 in 1
(199 missing values generated)

. gener u = rnormal(0, 1)

. gener t=_n

. replace y1=l.y1+u if time>1
(199 real changes made)

. replace y2=0.5+l.y2+u if time>1
(199 real changes made)

. replace y3=0.5+0.5*t+u if time>1
(199 real changes made)

. replace y4=0.5+0.5*t+l.y4+u if time>1
(199 real changes made)

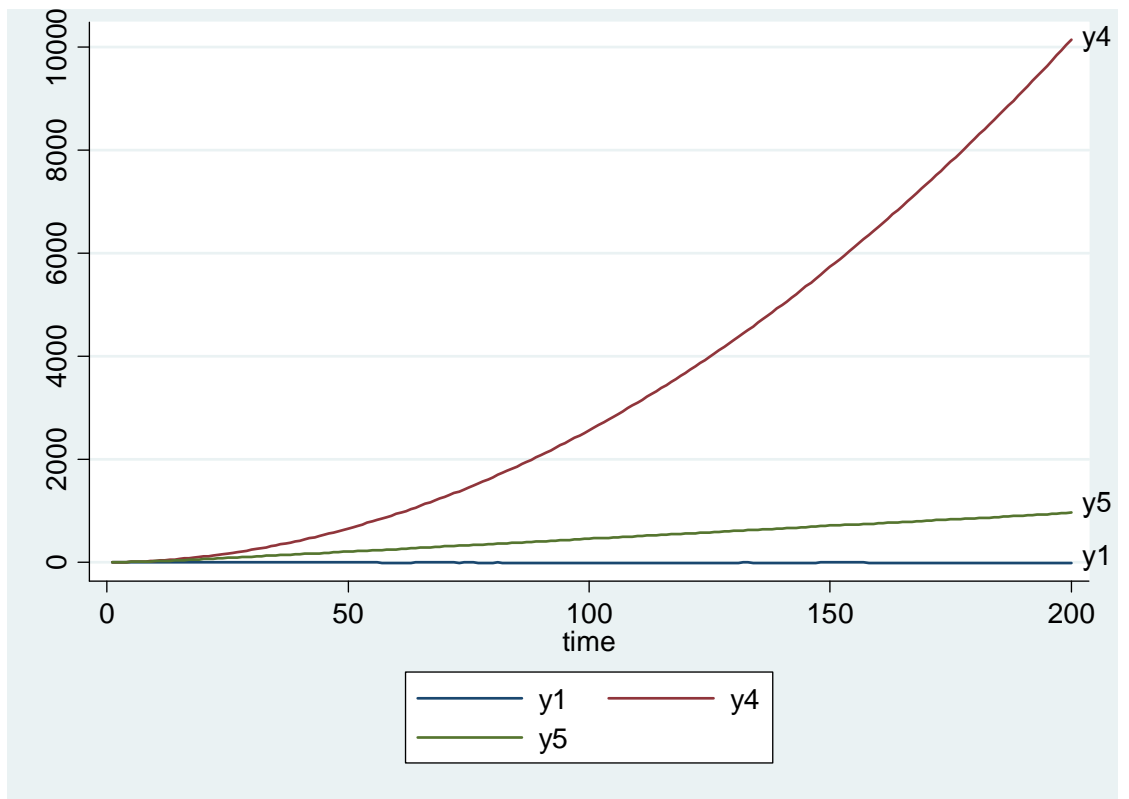
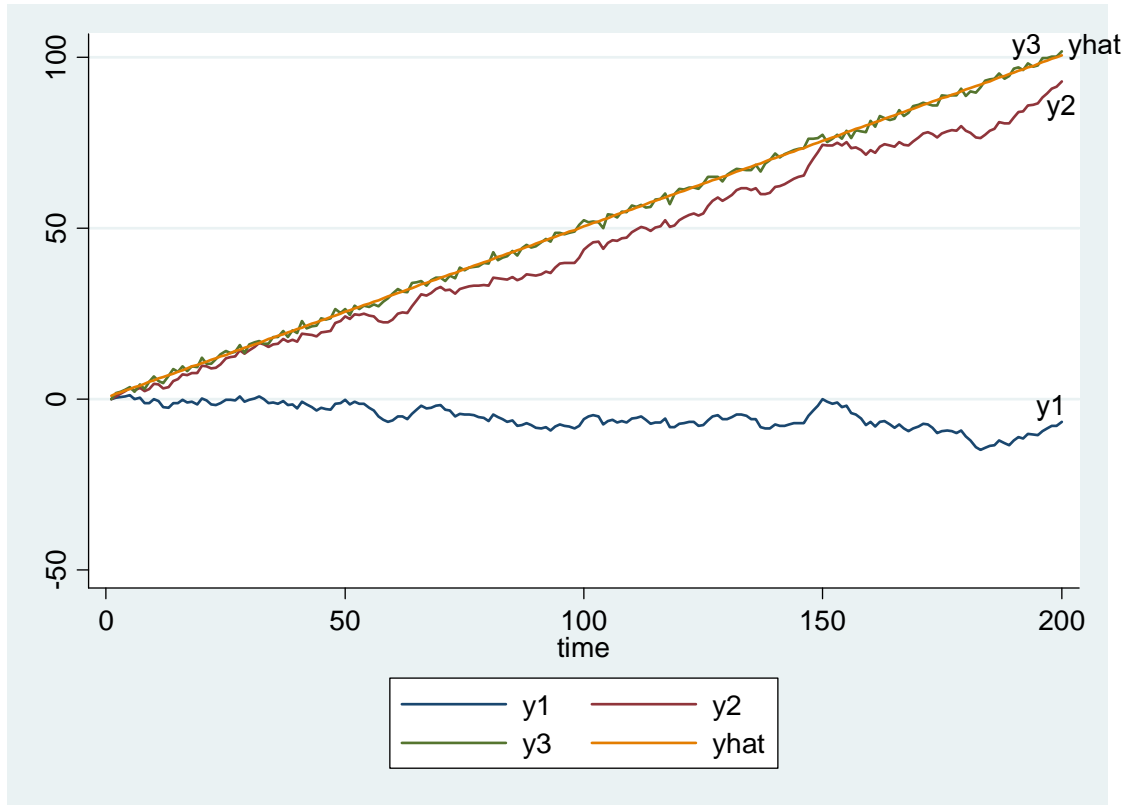
. replace y5=0.5+0.5*t+0.9*l.y5+u if time>1
(199 real changes made)

. gener yhat=0.5+0.5*t

. line y1 y2 y3 yhat time

. line y1 y4 y5 time

```



**Unit Root Problem****Autoregressive (Stationary) Series**

$$x_{1t} = 0.05 + 0.95 x_{1t-1} + u_{1t}$$

**Random Walk with Drift**

$$x_{2t} = 0.05 + x_{2t-1} + u_{2t}$$

**Explosive Series**

$$x_{3t} = 0.05 + 1.05 x_{3t-1} + u_{3t}$$

```
. set obs 500
obs was 0, now 500

. gener time=_n

. tsset time
   time variable:  time, 1 to 500

. gener x1=0 in 1
(499 missing values generated)

. gener x2=0 in 1
(499 missing values generated)

. gener x3=0 in 1
(499 missing values generated)

. gener u1= rnormal(0,1)

. gener u2= rnormal(0,1)

. gener u3= rnormal(0,1)

. replac x1=0.05+0.95*_l.x1+u1 if time>1
(499 real changes made)

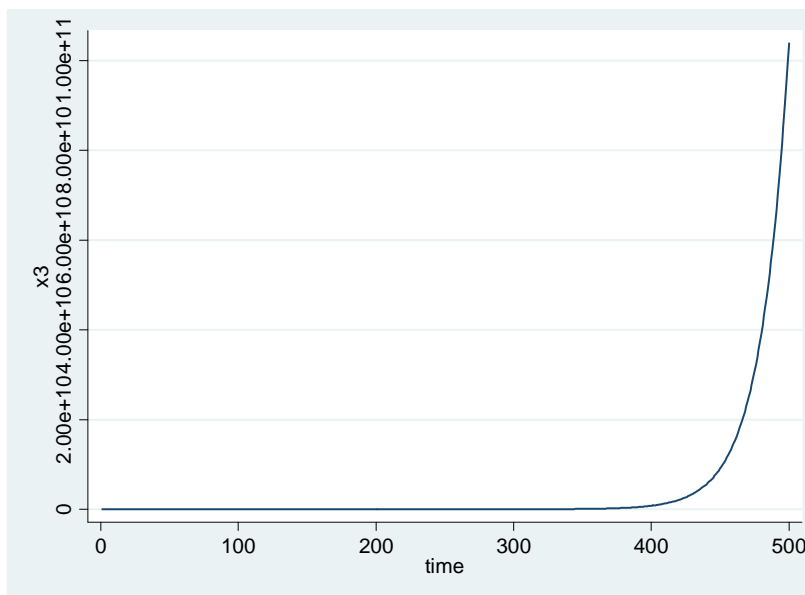
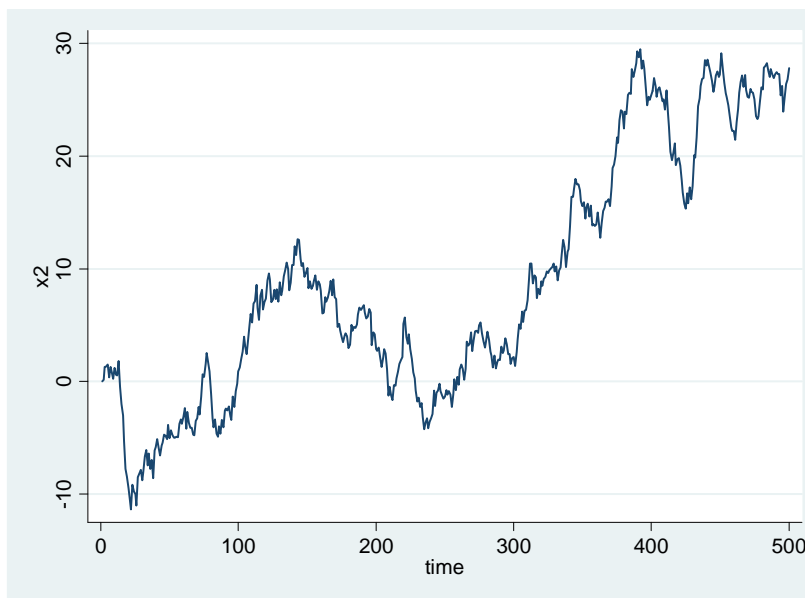
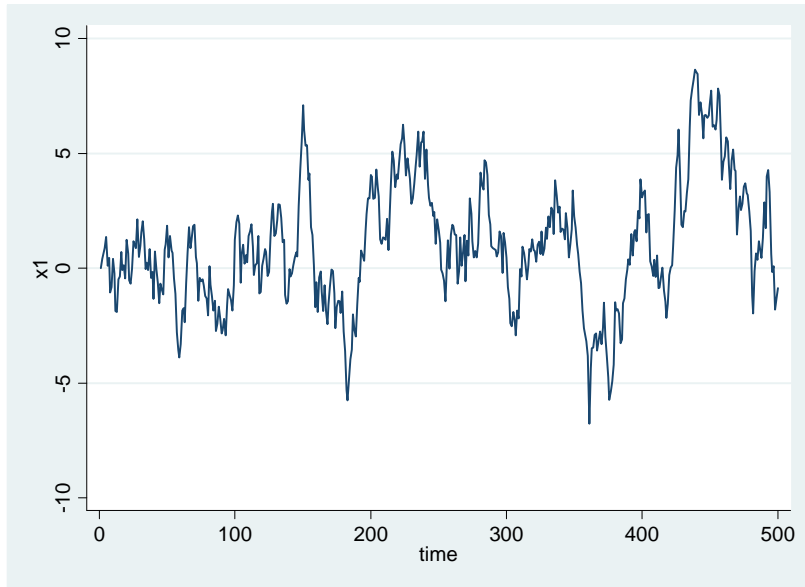
. replac x2=0.05+_l.x2+u2 if time>1
(499 real changes made)

. replac x3=0.05+1.05*_l.x3+u3 if time>1
(499 real changes made)

. line x1 time

. line x2 time

. line x3 time
```



**Spurious Regression**

$$Y_t = Y_{t-1} + u_t$$

$$X_t = X_{t-1} + v_t$$

If  $X_t$  and  $Y_t$  are uncorrelated  $I(1)$  processes, regression  $Y_t = \beta_1 + \beta_2 X_t + \varepsilon_t$  can lead to spurious problem.

```
. set obs 500
obs was 0, now 500

. gener time=_n

. tsset time
   time variable:  time, 1 to 500

. gener y=0 in 1
(499 missing values generated)

. gener x=0 in 1
(499 missing values generated)

. gener u= rnormal(0,1)

. gener v= rnormal(0,1)

. replace y=l.y+u if time>1
(499 real changes made)

. replace x=l.x+v if time>1
(499 real changes made)

. regress y x
```

| Source   | SS         | df  | MS         |                 |        |  |
|----------|------------|-----|------------|-----------------|--------|--|
| Model    | 1563.54402 | 1   | 1563.54402 | Number of obs = | 500    |  |
| Residual | 10336.0273 | 498 | 20.7550748 | F( 1, 498) =    | 75.33  |  |
| Total    | 11899.5713 | 499 | 23.8468362 | Prob > F =      | 0.0000 |  |
|          |            |     |            | R-squared =     | 0.1314 |  |
|          |            |     |            | Adj R-squared = | 0.1297 |  |
|          |            |     |            | Root MSE =      | 4.5558 |  |

| y     | Coef.     | Std. Err. | t      | P> t  | [95% Conf. Interval] |           |
|-------|-----------|-----------|--------|-------|----------------------|-----------|
| x     | -.1385236 | .0159599  | -8.68  | 0.000 | -.1698807            | -.1071665 |
| _cons | -10.1792  | .24019    | -42.38 | 0.000 | -10.65111            | -9.707289 |

```
. dwstat
```

```
Durbin-Watson d-statistic( 2, 500) = .0573669
```

## Unit Root Test

### 1. Test with all terms (intercept, trend, and lags):

```
. dfuller y, trend lags(1) regress
```

Augmented Dickey-Fuller test for unit root                      Number of obs    =            498

|      | Test<br>Statistic | -----<br>1% Critical<br>Value | Interpolated Dickey-Fuller<br>5% Critical<br>Value | -----<br>10% Critical<br>Value |
|------|-------------------|-------------------------------|--|--------------------------------|
| Z(t) | -2.049            | -3.980                        | -3.420   | -3.130                         |

MacKinnon approximate p-value for Z(t) = 0.5747

| D. y   | Coef.     | Std. Err. | t     | P> t  | [95% Conf. Interval]   |
|--------|-----------|-----------|-------|-------|------------------------|
| y      |           |           |       |       |                        |
| L1.    | -.0187253 | .0091402  | -2.05 | 0.041 | -.0366837    -.0007668 |
| LD.    | -.1694873 | .0445717  | -3.80 | 0.000 | -.2570608    -.0819138 |
| _trend | .0299962  | .0292846  | 1.02  | 0.306 | -.0275415    .0875339  |
| _cons  | -14.66152 | 9.461164  | -1.55 | 0.122 | -33.25061    3.927562  |

The result shows p-value=0.5747 (fail to reject null hypothesis of unit root). Then, check whether trend is significant and find out that p-value=0.306. Thus, there must be no trend. Go to next step:

### 2. Test with all terms (intercept, trend, and lags):

```
. dfuller y, lags(1) regress
```

Augmented Dickey-Fuller test for unit root                      Number of obs    =            498

|      | Test<br>Statistic | -----<br>1% Critical<br>Value | Interpolated Dickey-Fuller<br>5% Critical<br>Value | -----<br>10% Critical<br>Value |
|------|-------------------|-------------------------------|--|--------------------------------|
| Z(t) | -1.935            | -3.440                        | -2.870   | -2.570                         |

MacKinnon approximate p-value for Z(t) = 0.3156

| D. y  | Coef.     | Std. Err. | t     | P> t  | [95% Conf. Interval]   |
|-------|-----------|-----------|-------|-------|------------------------|
| y     |           |           |       |       |                        |
| L1.   | -.0175501 | .0090684  | -1.94 | 0.054 | -.0353673    .0002672  |
| LD.   | -.1685012 | .0445635  | -3.78 | 0.000 | -.2560582    -.0809442 |
| _cons | -6.704068 | 5.400393  | -1.24 | 0.215 | -17.31459    3.906451  |

The result shows p-value=0.3156 (fail to reject null hypothesis of unit root). Then, check whether constant term is significant and find out that p-value=0.215. Thus, there must be no constant term. Go to next step:

### 3. Test with all terms (intercept, trend, and lags):

```
. dfuller y, nocon lags(1) regress
```

Augmented Dickey-Fuller test for unit root                      Number of obs    =            498

|  | Test<br>Statistic | -----<br>1% Critical<br>Value | Interpolated Dickey-Fuller<br>5% Critical<br>Value | -----<br>10% Critical<br>Value |
|--|-------------------|-------------------------------|--|--------------------------------|
|--|-------------------|-------------------------------|--|--------------------------------|

|       |           |           |        |        |                      |
|-------|-----------|-----------|--------|--------|----------------------|
| Z(t)  | -1.484    | -2.580    | -1.950 | -1.620 |                      |
| ----- |           |           |        |        |                      |
| D. y  | Coef.     | Std. Err. | t      | P> t   | [95% Conf. Interval] |
| ----- |           |           |        |        |                      |
| L1.   | -.0104078 | .0070134  | -1.48  | 0.138  | -.0241875 .0033718   |
| LD.   | -.1723889 | .0444776  | -3.88  | 0.000  | -.2597766 -.0850012  |
| ----- |           |           |        |        |                      |

Dickey-Fuller GLS Test

. dfgls y, maxlag(10)

DF-GLS for y Number of obs = 489

| [lags] | DF-GLS tau Test Statistic | 1% Critical Value | 5% Critical Value | 10% Critical Value |
|--------|---------------------------|-------------------|-------------------|--------------------|
| 10     | -2.206                    | -3.480            | -2.849            | -2.563             |
| 9      | -2.229                    | -3.480            | -2.852            | -2.567             |
| 8      | -2.166                    | -3.480            | -2.856            | -2.570             |
| 7      | -2.508                    | -3.480            | -2.859            | -2.572             |
| 6      | -2.452                    | -3.480            | -2.862            | -2.575             |
| 5      | -2.062                    | -3.480            | -2.865            | -2.578             |
| 4      | -2.044                    | -3.480            | -2.868            | -2.581             |
| 3      | -1.894                    | -3.480            | -2.871            | -2.583             |
| 2      | -1.831                    | -3.480            | -2.874            | -2.586             |
| 1      | -1.698                    | -3.480            | -2.876            | -2.588             |

Opt Lag (Ng-Perron seq t) = 8 with RMSE 90.41123  
 Min SC = 9.089339 at lag 1 with RMSE 92.94486  
 Min MAIC = 9.062736 at lag 8 with RMSE 90.41123

. dfgls d.y, maxlag(10)

DF-GLS for D.y Number of obs = 488

| [lags] | DF-GLS tau Test Statistic | 1% Critical Value | 5% Critical Value | 10% Critical Value |
|--------|---------------------------|-------------------|-------------------|--------------------|
| 10     | -4.642                    | -3.480            | -2.849            | -2.564             |
| 9      | -4.526                    | -3.480            | -2.852            | -2.567             |
| 8      | -4.790                    | -3.480            | -2.856            | -2.570             |
| 7      | -5.308                    | -3.480            | -2.859            | -2.572             |
| 6      | -4.967                    | -3.480            | -2.862            | -2.575             |
| 5      | -5.404                    | -3.480            | -2.865            | -2.578             |
| 4      | -6.980                    | -3.480            | -2.868            | -2.581             |
| 3      | -7.989                    | -3.480            | -2.871            | -2.583             |
| 2      | -10.145                   | -3.480            | -2.874            | -2.586             |
| 1      | -13.424                   | -3.480            | -2.876            | -2.588             |

Opt Lag (Ng-Perron seq t) = 7 with RMSE 92.82793  
 Min SC = 9.149196 at lag 5 with RMSE 93.36743  
 Min MAIC = 9.550801 at lag 9 with RMSE 92.51999

Phillips-Perron Test

. pperron y, trend regress

Phillips-Perron test for unit root Number of obs = 499  
Newey-West lags = 2

|        | Test Statistic | 1% Critical Value | 5% Critical Value | 10% Critical Value |
|--------|----------------|-------------------|-------------------|--------------------|
| Z(rho) | -9.437         | -28.898           | -21.499           | -18.100            |
| Z(t)   | -2.228         | -3.980            | -3.420            | -3.130             |

Mackinnon approximate p-value for Z(t) = 0.4742

