Lecture4: Valuing Bonds

6-3. **The following table summarizes prices of various default-free, zero-coupon bonds (expressed as a percentage of face value):**

Maturity (years)	1	2	3	4	5
Price (per $100 face value)	$95.51	$91.05	$86.38	$81.65	$76.51

a. **Compute the yield to maturity for each bond.**

b. **Plot the zero-coupon yield curve (for the first five years).**

c. **Is the yield curve upward sloping, downward sloping, or flat?**

a. Use the following equation.

$$1+\text{YTM}_n = \left(\frac{\text{FV}_n}{\text{P}}\right)^{1/n}$$

$$1+\text{YTM}_1 = \left(\frac{100}{95.51}\right)^{1/1} \Rightarrow \text{YTM}_1 = 4.70\%$$

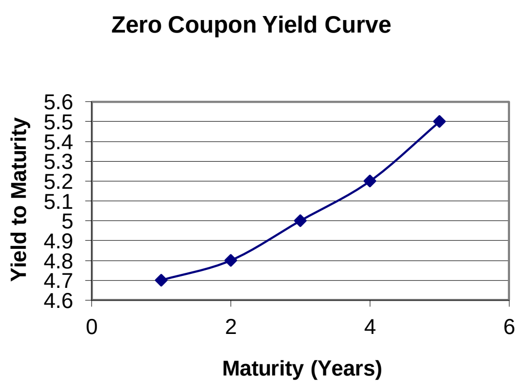
$$1+\text{YTM}_1 = \left(\frac{100}{91.05}\right)^{1/2} \Rightarrow \text{YTM}_1 = 4.80\%$$

$$1+\text{YTM}_3 = \left(\frac{100}{86.38}\right)^{1/3} \Rightarrow \text{YTM}_3 = 5.00\%$$

$$1+\text{YTM}_4 = \left(\frac{100}{81.65}\right)^{1/4} \Rightarrow \text{YTM}_4 = 5.20\%$$

$$1+\text{YTM}_5 = \left(\frac{100}{76.51}\right)^{1/5} \Rightarrow \text{YTM}_5 = 5.50\%$$

b. The yield curve is as shown below.



c. The yield curve is upward sloping.

6-6. Suppose a 10-year, $1000 bond with an 8% coupon rate and semiannual coupons is trading for a price of $1034.74.

a. What is the bond's yield to maturity (expressed as an APR with semiannual compounding)?

b. If the bond's yield to maturity changes to 9% APR, what will the bond's price be?

a. $$\$1,034.74 = \frac{40}{(1+\frac{YTM}{2})} + \frac{40}{(1+\frac{YTM}{2})^2} + \cdots + \frac{40+1000}{(1+\frac{YTM}{2})^{20}} \Rightarrow YTM = 7.5\%$$

Using the annuity spreadsheet:

	NPER	Rate	PV	PMT	FV	Excel Formula
Given:	20		–1,034.74	40	1,000	
Solve For Rate:		3.75%				=RATE(20,40,–1034.74,1000)

Therefore, YTM = 3.75% × 2 = 7.50%

b. $$PV = \frac{40}{\left(1+\frac{0.09}{2}\right)} + \frac{40}{\left(1+\frac{0.09}{2}\right)^2} + L + \frac{40+1000}{\left(1+\frac{0.09}{2}\right)^{20}} = \$934.96.$$

Using the spreadsheet

With a 9% YTM = 4.5% per 6 months, the new price is $934.96

	NPER	Rate	PV	PMT	FV	Excel Formula
Given:	20	4.50%		40	1,000	
Solve For PV:			(934.96)			=PV(0.045,20,40,1000)

6-7. Suppose a five-year, $1000 bond with annual coupons has a price of $900 and a yield to maturity of 6%. What is the bond's coupon rate?

$$900 = \frac{C}{(1+0.06)} + \frac{C}{(1+0.06)^2} + \cdots + \frac{C+1000}{(1+0.06)^5} \Rightarrow C = \$36.26, \text{ so the coupon rate is } 3.626\%.$$

We can use the annuity spreadsheet to solve for the payment.

	NPER	Rate	PV	PMT	FV	Excel Formula
Given:	5	6.00%	–900.00		1,000	
Solve For PMT:				36.26		=PMT(0.06,5,–900,1000)

Therefore, the coupon rate is 3.626%.

6-8. The prices of several bonds with face values of $1000 are summarized in the following table:

Bond	A	B	C	D
Price	$972.50	$1040.75	$1150.00	$1000.00

For each bond, state whether it trades at a discount, at par, or at a premium.

Bond A trades at a discount. Bond D trades at par. Bonds B and C trade at a premium.

6-10. Suppose a seven-year, $1000 bond with an 8% coupon rate and semiannual coupons is trading with a yield to maturity of 6.75%.

a. Is this bond currently trading at a discount, at par, or at a premium? Explain.

b. If the yield to maturity of the bond rises to 7% (APR with semiannual compounding), what price will the bond trade for?

a. Because the yield to maturity is less than the coupon rate, the bond is trading at a premium.

b. $$\frac{40}{(1+0.035)}+\frac{40}{(1+0.035)^2}+\cdots+\frac{40+1000}{(1+0.035)^{14}}=\$1,054.60$$

	NPER	Rate	PV	PMT	FV	Excel Formula
Given:	14	3.50%		40	1,000	
Solve For PV:			(1,054.60)			=PV(0.035,14,40,1000)

6-11. Suppose that General Motors Acceptance Corporation issued a bond with 10 years until maturity, a face value of $1000, and a coupon rate of 7% (annual payments). The yield to maturity on this bond when it was issued was 6%.

a. What was the price of this bond when it was issued?

b. Assuming the yield to maturity remains constant, what is the price of the bond immediately before it makes its first coupon payment?

c. Assuming the yield to maturity remains constant, what is the price of the bond immediately after it makes its first coupon payment?

a. When it was issued, the price of the bond was

$$P=\frac{70}{(1+0.06)}+\ldots+\frac{70+1,000}{(1+0.06)^{10}}=\$1,073.60.$$

b. Before the first coupon payment, the price of the bond is

$$P=70+\frac{70}{(1+0.06)}\ldots+\frac{70+1,000}{(1+0.06)^{9}}=\$1,138.02.$$

c. After the first coupon payment, the price of the bond will be

$$P=\frac{70}{(1+0.06)}\ldots+\frac{70+1,000}{(1+0.06)^{9}}=\$1,068.02.$$

6-12. Suppose you purchase a 10-year bond with 6% annual coupons. You hold the bond for four years, and sell it immediately after receiving the fourth coupon. If the bond's yield to maturity was 5% when you purchased and sold the bond,

a. What cash flows will you pay and receive from your investment in the bond per $100 face value?

b. What is the internal rate of return of your investment?

a. First, we compute the initial price of the bond by discounting its 10 annual coupons of $6 and final face value of $100 at the 5% yield to maturity.

	NPER	Rate	PV	PMT	FV	Excel Formula
Given:	10	5.00%		6	100	
Solve For PV:			(107.72)			= PV(0.05,10,6,100)

Thus, the initial price of the bond = \$107.72. (Note that the bond trades above par, as its coupon rate exceeds its yield.)

Next, we compute the price at which the bond is sold, which is the present value of the bonds cash flows when only 6 years remain until maturity.

	NPER	Rate	PV	PMT	FV	Excel Formula
Given:	6	5.00%		6	100	
Solve For PV:			(105.08)			= PV(0.05,6,6,100)

Therefore, the bond was sold for a price of \$105.08. The cash flows from the investment are therefore as shown in the following timeline.

Year	0	1	2	3	4
Purchase Bond	–\$107.72				
Receive Coupons		\$6	\$6	\$6	\$6
Sell Bond					\$105.08
Cash Flows	–\$107.72	\$6.00	\$6.00	\$6.00	\$111.08

b. We can compute the IRR of the investment using the annuity spreadsheet. The PV is the purchase price, the PMT is the coupon amount, and the FV is the sale price. The length of the investment *N* = 4 years. We then calculate the IRR of investment = 5%. Because the YTM was the same at the time of purchase and sale, the IRR of the investment matches the YTM.

	NPER	Rate	PV	PMT	FV	Excel Formula
Given:	4		–107.72	6	105.08	
Solve For Rate:		5.00%				= RATE(4,6,–107.72,105.08)

6-13. Consider the following bonds:

Bond	Coupon Rate (annual payments)	Maturity (years)
A	0%	15
B	0%	10
C	4%	15
D	8%	10

a. What is the percentage change in the price of each bond if its yield to maturity falls from 6% to 5%?

b. Which of the bonds A–D is most sensitive to a 1% drop in interest rates from 6% to 5% and why? Which bond is least sensitive? Provide an intuitive explanation for your answer.

a. We can compute the price of each bond at each YTM using Eq. 8.5. For example, with a 6% YTM, the price of bond A per \$100 face value is

$$P(\text{bond A, 6\% YTM}) = \frac{100}{1.06^{15}} = \$41.73.$$

The price of bond D is

$$P(\text{bond D, 6\% YTM}) = 8 \times \frac{1}{0.06}\left(1 - \frac{1}{1.06^{10}}\right) + \frac{100}{1.06^{10}} = \$114.72.$$

One can also use the Excel formula to compute the price: –PV(YTM, NPER, PMT, FV).

Once we compute the price of each bond for each YTM, we can compute the % price change as

$$\text{Percent change} = \frac{(\text{Price at 5\% YTM}) - (\text{Price at 6\% YTM})}{(\text{Price at 6\% YTM})}.$$

The results are shown in the table below.

Bond	Coupon Rate (annual payments)	Maturity (years)	Price at 6% YTM	Price at 5% YTM	Percentage Change
A	0%	15	$41.73	$48.10	15.3%
B	0%	10	$55.84	$61.39	9.9%
C	4%	15	$80.58	$89.62	11.2%
D	8%	10	$114.72	$123.17	7.4%

b. Bond A is most sensitive, because it has the longest maturity and no coupons. Bond D is the least sensitive. Intuitively, higher coupon rates and a shorter maturity typically lower a bond's interest rate sensitivity.

6-14. Suppose you purchase a 30-year, zero-coupon bond with a yield to maturity of 6%. You hold the bond for five years before selling it.

a. If the bond's yield to maturity is 6% when you sell it, what is the internal rate of return of your investment?

b. If the bond's yield to maturity is 7% when you sell it, what is the internal rate of return of your investment?

c. If the bond's yield to maturity is 5% when you sell it, what is the internal rate of return of your investment?

d. Even if a bond has no chance of default, is your investment risk free if you plan to sell it before it matures? Explain.

a. Purchase price = 100 / 1.06^{30} = 17.41. Sale price = 100 / 1.06^{25} = 23.30. Return = $(23.30 / 17.41)^{1/5}$ – 1 = 6.00%. I.e., since YTM is the same at purchase and sale, IRR = YTM.

b. Purchase price = 100 / 1.06^{30} = 17.41. Sale price = 100 / 1.07^{25} = 18.42. Return = $(18.42 / 17.41)^{1/5}$ – 1 = 1.13%. I.e., since YTM rises, IRR < initial YTM.

c. Purchase price = 100 / 1.06^{30} = 17.41. Sale price = 100 / 1.05^{25} = 29.53. Return = $(29.53 / 17.41)^{1/5}$ – 1 = 11.15%. I.e., since YTM falls, IRR > initial YTM.

d. Even without default, if you sell prior to maturity, you are exposed to the risk that the YTM may change.

6-24. Assume there are four default-free bonds with the following prices and future cash flows:

Bond	Price Today	Cash Flows Year 1	Year 2	Year 3
A	$934.58	1000	0	0
B	881.66	0	1000	0
C	1,118.21	100	100	1100
D	839.62	0	0	1000

Do these bonds present an arbitrage opportunity? If so, how would you take advantage of this opportunity? If not, why not?

To determine whether these bonds present an arbitrage opportunity, check whether the pricing is internally consistent. Calculate the spot rates implied by Bonds A, B, and D (the zero-coupon bonds), and use this to check Bond C. (You may alternatively compute the spot rates from Bonds A, B, and C, and check Bond D, or some other combination.)

$$934.58 = \frac{1000}{(1+YTM_1)} \Rightarrow YTM_1 = 7.0\%$$

$$881.66 = \frac{1000}{(1+YTM_2)^2} \Rightarrow YTM_2 = 6.5\%$$

$$839.62 = \frac{1000}{(1+YTM_3)^3} \Rightarrow YTM_3 = 6.0\%$$

Given the spot rates implied by Bonds A, B, and D, the price of Bond C should be $1,105.21. Its price really is $1,118.21, so it is overpriced by $13 per bond. Yes, there is an arbitrage opportunity.

To take advantage of this opportunity, you want to (short) Sell Bond C (since it is overpriced). To match future cash flows, one strategy is to sell 10 Bond Cs (it is not the only effective strategy; any multiple of this strategy is also arbitrage). This complete strategy is summarized in the table below.

	Today	1 Year	2Years	3Years
Sell Bond C	11,182.10	–1,000	–1,000	–11,000
Buy Bond A	–934.58	1,000	0	0
Buy Bond B	–881.66	0	1,000	0
Buy 11 Bond D	–9,235.82	0	0	11,000
Net Cash Flow	130.04	0	0	0

Notice that your arbitrage profit equals 10 times the mispricing on each bond (subject to rounding error).