

Problem Set 1 (Macroeconomics)

1) Consider the problem of an oil company that wants to maximize profits from an oil well. Revenue at time t is given as, $R_t = p_t \cdot u_t$. Where p_t is the price of oil at t , and u_t is the amount of oil that is extracted and sold. The company's cost function is quadratic in the amount of oil that is extracted,

$$C_t = 0.05 u_t^2 .$$

The amount of oil remaining in the well follows the recursion or transition equation,

$$x_{t+1} = x_t - u_t,$$

where x_t is known as state variable in dynamic programming language. The company applies the discount factor $\beta = 0.9$ for profit streams that occur in the future. We also assume that the company intends to have the oil well depleted in 3 years and let the initial date be $t = 0$, which means that $x_3=0$. Therefore, the above problem can also be stated as follows,

$$\text{Maximize} \quad \sum_{t=0}^2 \beta^t (p_t u_t - 0.05 u_t^2),$$

$$\text{Subject to,} \quad x_{t+1} = x_t - u_t, \quad \text{for } t = 0, 1, 2., \quad \text{and} \quad x_3 = 0$$

Then, solve the above problem for optimal policy rules, u_t for $t = 0, 1$ and 2 , by using Bellman equations.