ENDOGENOUS GROWTH MODELS

EE 462 Development Macroeconomics

Semester 1/2019

Topics

- Transitional Dynamics in Solow Model
- The AK Model
- The Lucas Model
- Romer’s Model

Transitional Dynamics (Solow Model)

- Recall from Solow model, capital accumulation can be written as:

$$\Delta k = sy - (n+d)k$$

- Rewrite in terms of the rate of change in k:

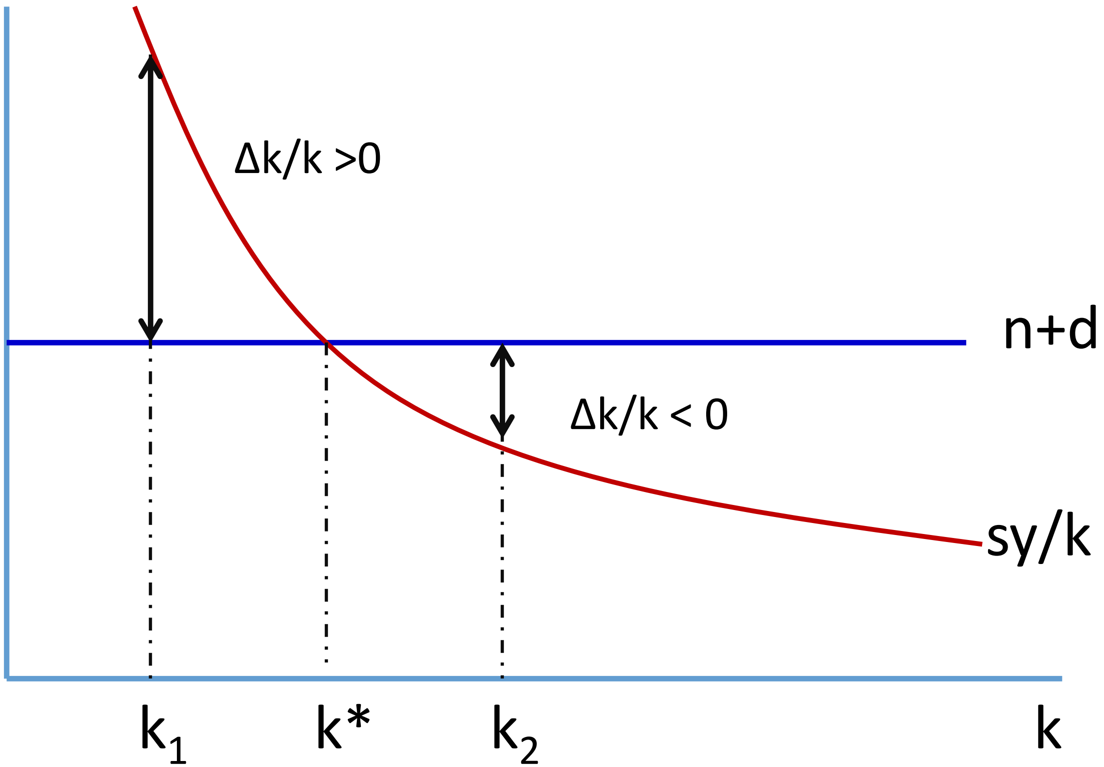
$$\Delta k/k = sy/k - (n+d)$$

- Given the production function: y = f(k) = k^{α}. Then,

➔ $$\Delta k/k = sk^{\alpha-1} - (n+d)$$

  - Countries with high levels of *k* $(k > k_{ss})$ will have negative growth rates of *k*.
  - Countries with low levels of *k* $(k < k_{ss})$ will have a positive growth rates of *k*.

Graph: Transitional Dynamics in Solow Model



Endogenous Growth Theory

- The *endogenous growth theory* or *new growth theory* focuses on understanding the economic forces underlying technological progress.
- By endogenous growth, it means persistent income growth that is determined by the system governing the production process rather than forces outside that system.
- The main differences from neoclassical models:
  - Discard the assumption of diminishing marginal returns to capital
  - Allow increasing returns to scale in aggregate production
  - Try to explain the divergent long-term growth patterns among countries
  - Often focus on the role of externalities and human capital

The AK Model

- Assume a production function without diminishing returns:

  $Y = AK$ where $A > 0$

- Output per capita can be written as:

  $y = Ak$

  where $y = Y/L$ and $k = K/L$

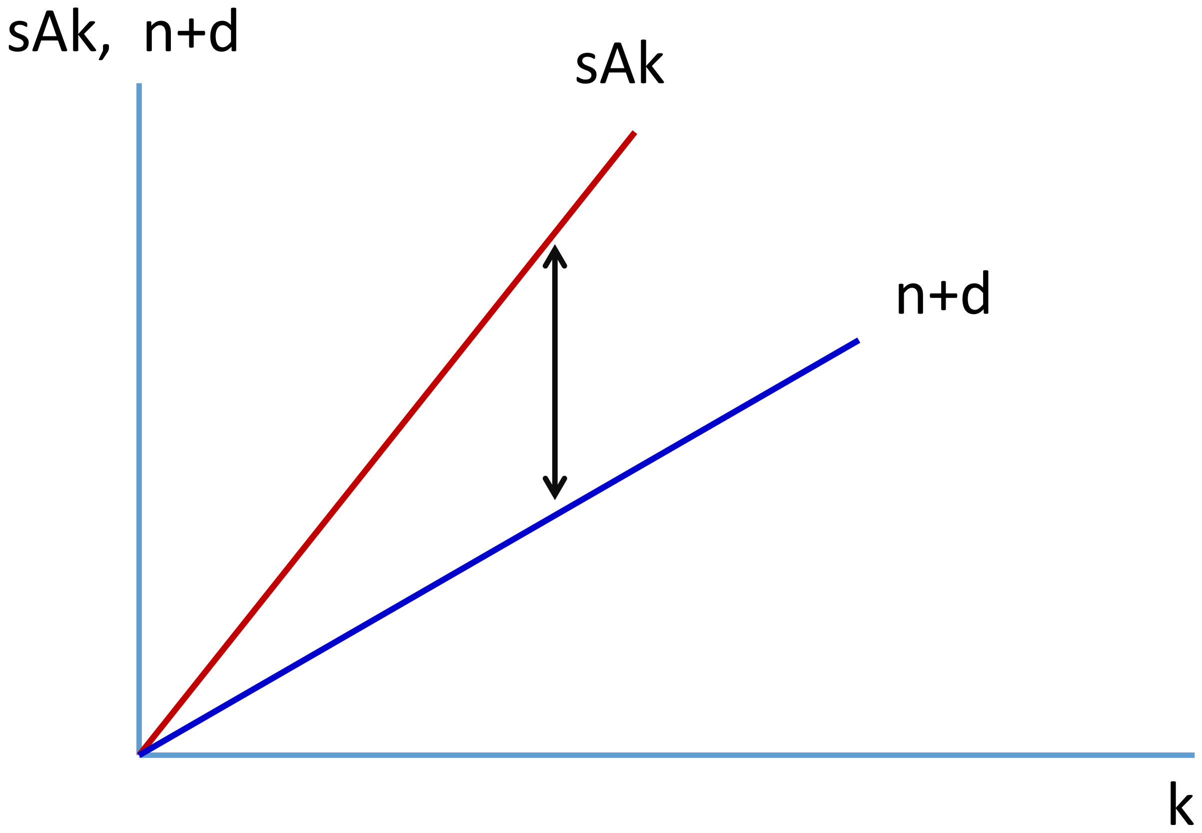
- The accumulation of capital per worker can be written as:

  ➔ $\Delta k/k = sA - (n+d)$

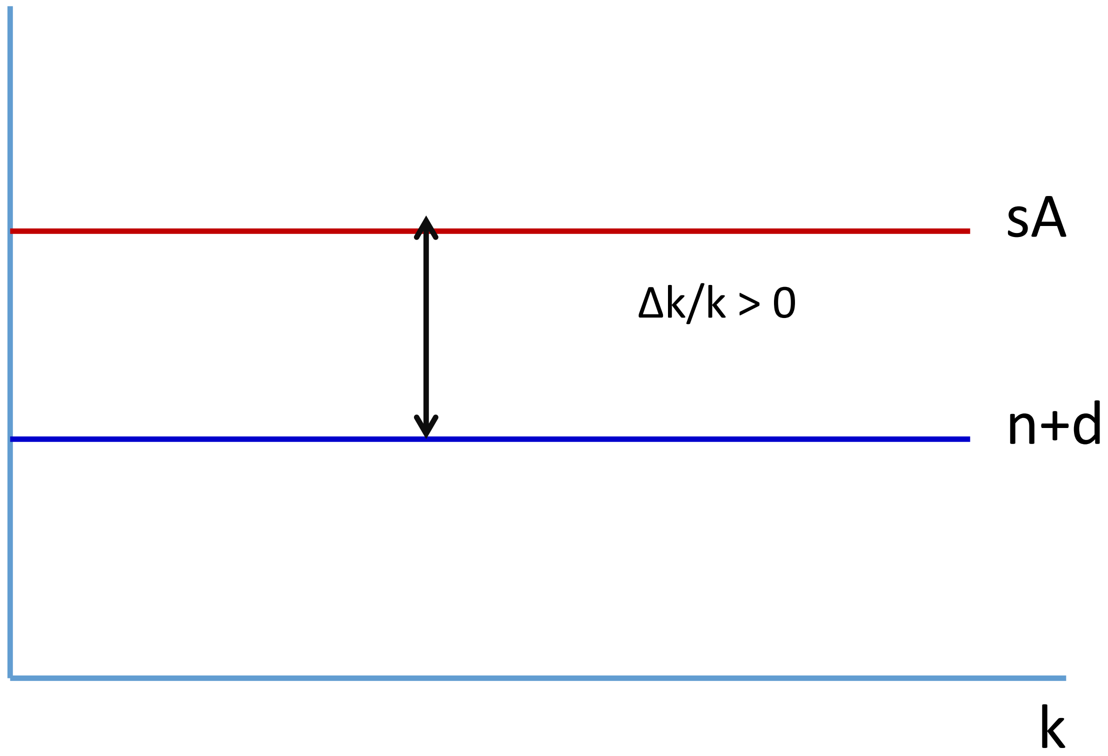
  ➔ $\Delta k = sAk - (n+d)k$

- There is no convergence in the long run.
- What's the growth rate in per capita output?

Graph: The AK Model



Transitional Dynamics: AK Model



The Lucas (1988) Model

- Lucas (JME, 1988) defines human capital as the skill embodied in workers.
- The production function is given by:

$$Y = AK^{\alpha}(uhN)^{1-\alpha} = AK^{\alpha}(L)^{1-\alpha}$$

where N = total workers

h = stock of human capital (of each worker)

u = proportion of total labor spent on producing output

L = uhN = effective worker

- Human capital grows at a constant rate:

$$\Delta h/h = a(1-u)$$

Where *a* reflects quality or efficiency of education

Lucas Model (cont'd)

- Physical capital accumulation is defined as usual: $\Delta K = sY + d.$
- Rewrite the production function in terms of per effective worker:

$$y_e = Ak_e^{\alpha} \qquad (A > 0)$$

- The growth rate of physical capital per effective worker is:

$$\frac{\Delta k_e}{k_e} = \frac{\Delta K}{K} - \frac{\Delta u}{u} - \frac{\Delta h}{h} - \frac{\Delta N}{N}$$

$$\frac{\Delta k_e}{k_e} = sAk_e^{\alpha-1} - d - a(1-u) - n$$

➔ $$\Delta k_e = sy_e - [d + a(1-u) + n]k_e$$

- At the equilibrium in the long run,

➢ Capital per effective worker is: $k_e^* = [s/(d + a(1-u) + n)]^{1/(1-\alpha)}$

➢ Output per effective worker is: $y_e^* = A[s/(d + a(1-u) + n)]^{\alpha/(1-\alpha)}$

- Note: This simplified version of Lucas model is similar to the Solow Model with augmented labor technology.

Lucas Model (cont'd)

- In the long run, there is no change in physical capital per effective worker. Thus, the growth rate of physical capital per worker can be written as:

$$\frac{\Delta k_e}{k_e} = 0 \rightarrow 0 = \frac{\Delta K}{K} - \frac{\Delta N}{N} - \frac{\Delta h}{h}$$

$$\frac{\Delta k}{k} = \frac{\Delta K}{K} - \frac{\Delta N}{N} = \frac{\Delta h}{h} = a(1-u)$$

$$\rightarrow \frac{\Delta k}{k} = a(1-u)$$

- Main driver for growth is the growth rate of human capital.
  - Policies that promote knowledge accumulate will result in higher growth rate of output.
- 1-u can be thought of as the production of knowledge.

Romer Model

- The Romer model endogenizes technological progress by introducing the search for new ideas by researchers interested in profiting from their inventions.
- This model is aimed to explain mainly why and how the advance countries exhibit sustained growth.

- A firm’s R&D raises its profits, but also create a positive externality on other firm (lead to increasing returns overall).
- Thus, technological progress is driven by profit-maximizing firms or investors’ own interests.

Romer Model: Basic Elements (1)

- Let's start with a model of firms that invest in R&D.
- The production function is given by:

$$Y = K^{\alpha}(AL_Y)^{1-\alpha}, \qquad (0 < \alpha < 1) \qquad \text{-- (1)}$$

where A = a given level (or stock) of knowledge

L_Y = labor used in the production sector

L = total labor: $L = L_Y + L_A$ (L_A = labor used in R&D)

➢ The production function exhibits CRTS w.r.t. K and L, but exhibits IRTS w.r.t. all three inputs (K, L, and A).

- Capital accumulations as people forego consumption at rate s_K.

$$\Delta K = s_K Y - dK \qquad \text{-- (2)}$$

Romer Model: Basic Elements (2)

- Labor (equivalent to population) grows at a constant rate *n*:

$$\frac{\Delta L}{L} = n \quad \text{-- (3)}$$

- Romer assumes that the number of new ideas (ΔA) at any given point in time is can be derived from:

$$\Delta A = \bar{\delta} L_A, \quad \text{-- (4)}$$

where L_A is the number of people attempting to discover new ideas, and $\bar{\delta}$ is the rate at which new ideas are discovered.

- $\bar{\delta}$ could a function of A: $\bar{\delta} = \delta A^{\phi}$ -- (5)
  - $\phi > 0$ → Productivity of research increases with the stock of knowledge
  - $\phi < 0$ → the "fishing out" case
  - $\phi = 0$ → Productivity of research is independent of the stock of knowledge

Romer Model: Basic Elements (3)

- Also, it is possible that the average productivity of research depends on the number people searching for new ideas.
- The general production function for ideas can be written as:

$$\Delta A = \delta A^{\phi} {L_A}^{\lambda} \qquad \text{-- (6)}$$

where δ = a constant

ϕ = knowledge spillover (ϕ = 0: no new ideas),

λ = originality of idea (i.e. λ = 1: no duplication)

- Suppose further that $L_A/L = s_R$ ➔ $L_Y/L = 1 - s_R$ -- (7)

Growth in Romer Model

- Along the balance growth path, one can show that

$$g_y = g_k = g_A \qquad \text{-- (8)}$$

- Question: What determine the growth rate of A (i.e. growth rate of ideas)?
- Define the *growth rate of ideas* as $g_A = \Delta A/A$.
- Recall that $\Delta A = \delta A^{\phi} {L_A}^{\lambda}$. Hence,

$$\frac{\Delta A}{A} = \delta A^{\phi-1} {L_A}^{\lambda}. \qquad \text{-- (9)}$$

- Along a balanced growth path, $\Delta A/A \equiv g_A$ is constant. Taking logs and derivatives of both sides of equation (9) gives:

$$g_A = \frac{\lambda n}{1-\phi}. \qquad \text{-- (10)}$$

Implications

- Population growth is the *only source of* technological progress and any long-run growth in per capita income.
  - A higher population growth in the Romer model means a *higher long-run economic growth rate.*
- Government cannot change the growth rate of per capita income by *any* policy <u>except</u> a policy to increase the fertility rate.
- For instance, a policy of increasing the R&D share of labor s_R cannot change the long-run growth rate of the economy. Why?
  - They only increase the growth rate during the transition period (but not in the long run).

Comparative Statics: A Permanent Increase in R&D Share (Optional)

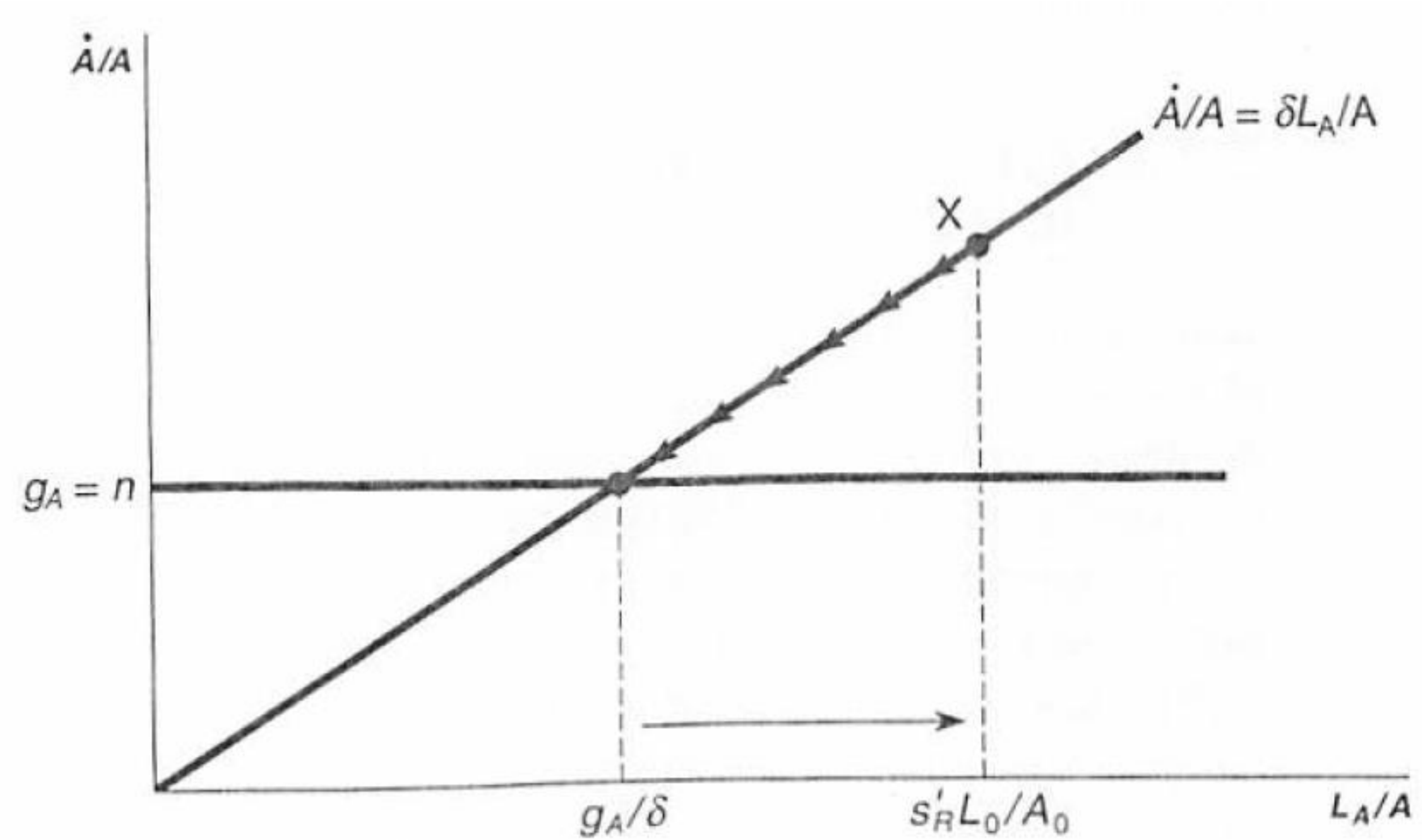
- For simplicity, assume $\lambda = 1$ and $\phi = 0$. Equation (9) can be rewritten as:

$$\frac{\Delta A}{A} = \delta \frac{S_R L}{A}, \qquad \text{-- (11)}$$

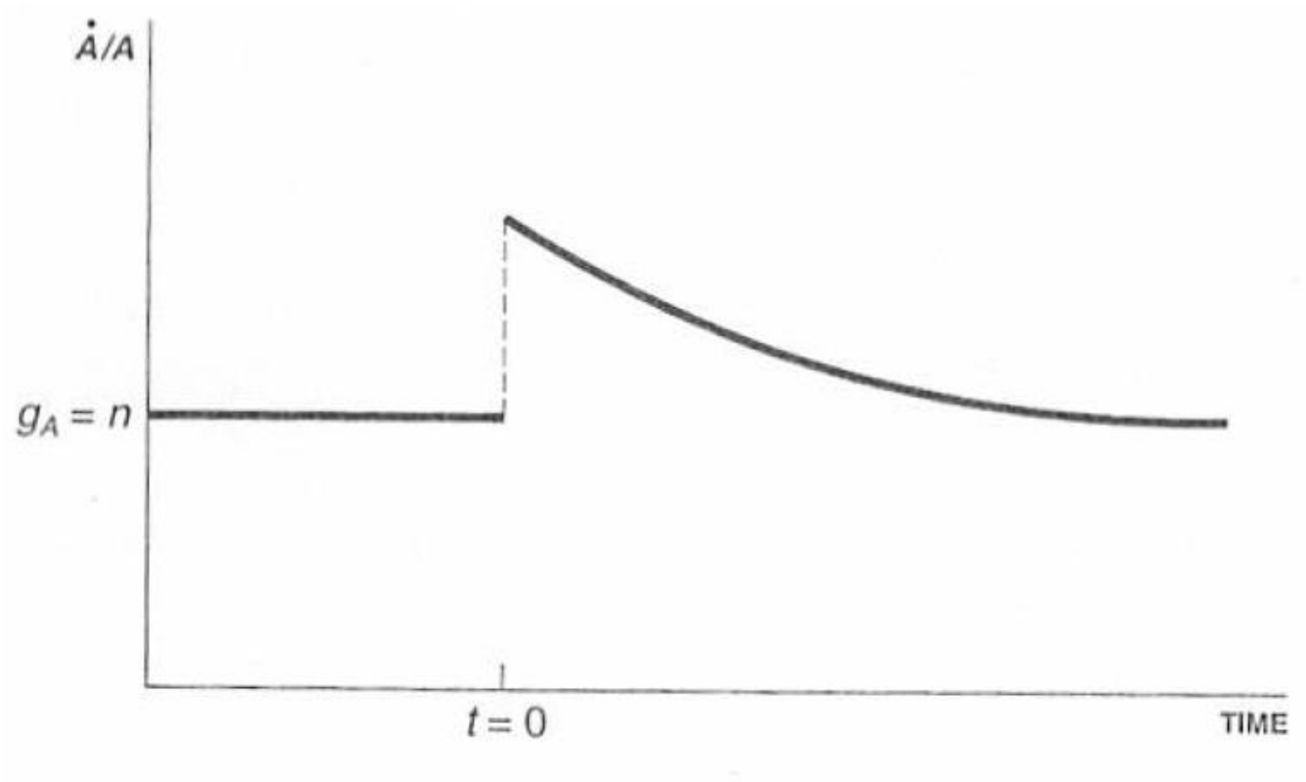
where S_R is the share of labor engaged in R&D.

- Question: What happens to the per capita output (y) if S_R increases?
  - Need to know the impact of an increase in R&D share on technological progress.
  - Use a Solow framework to derive output per effective labor and output per worker along a balanced growth path.

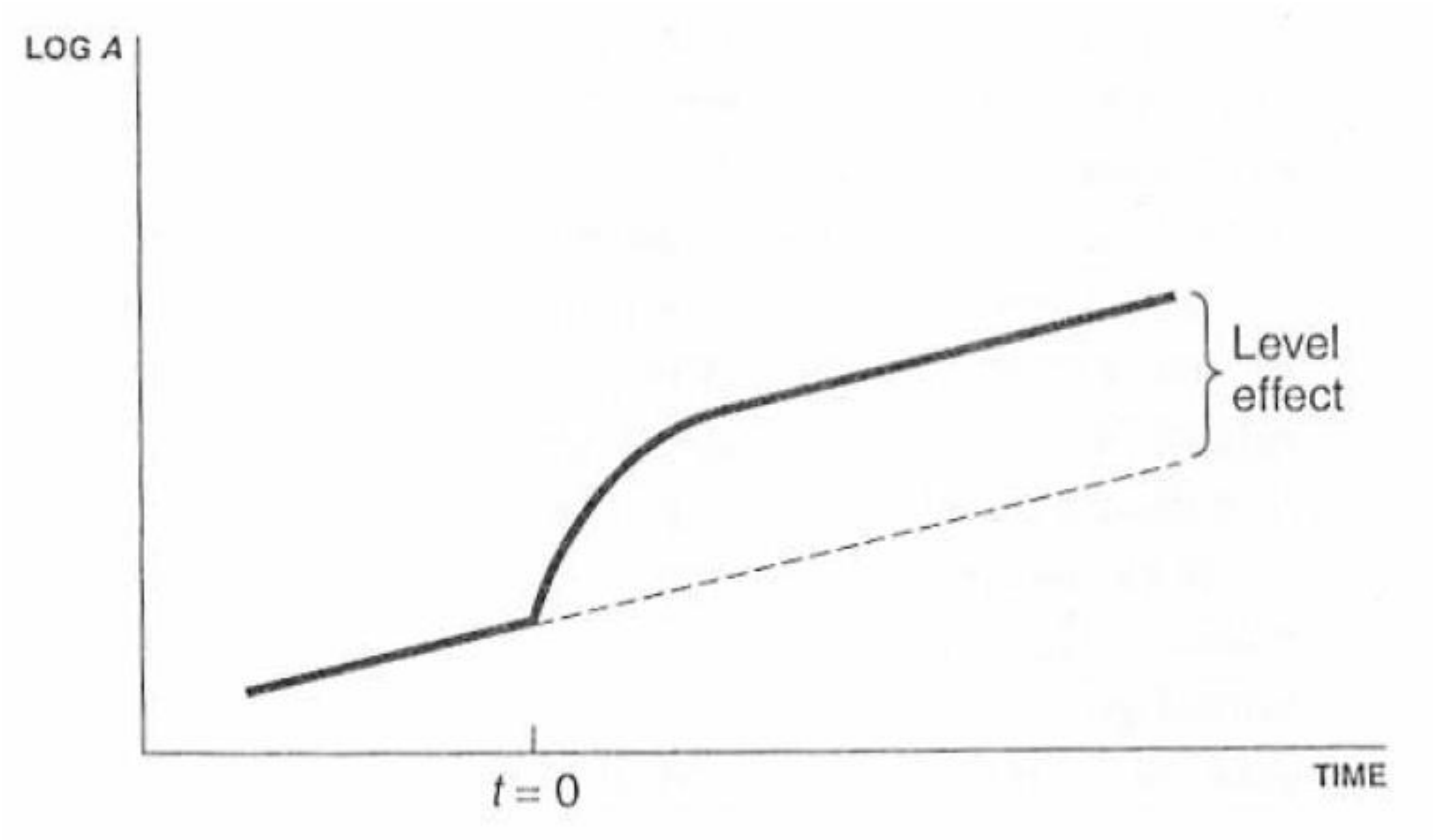
Technological Progress: An Increase in The R&D Share (Optional)



Technological Progress Over Time (Optional)



The Level of Technology Over Time(Optional)



Steady-State Output Per Capita (Optional)

- Based on the Solow framework, the ratio *y/A* along a balanced growth path is constant and is given by:

$$\left(\frac{y}{A}\right)^* = \left(\frac{s_K}{n+g_A+d}\right)^{\alpha/(1-\alpha)} (1 - s_R) \qquad \text{-- (12)}$$

- Also, along a balanced growth path, from equation (11), the level of *A* can be solved in terms of the labor force:

$$A = \delta \frac{S_R L}{g_A} \qquad \text{-- (13)}$$

- Based on the above information, we can derive:

$$y^* = \left(\frac{s_K}{n+g_A+d}\right)^{\alpha/(1-\alpha)} (1 - s_R) \frac{\delta S_R}{g_A} L \qquad \text{-- (14)}$$

   - S_R affects y* both negatively (more researchers=fewer workers producing output) and positively (more researchers = more ideas).