Chapter 3

Discrete Random Variable and Their Probability Distributions

- Basic Definition
- The Probability Distribution for a Discrete Random Variable

Basic Definition

A random variable (rv) is a real-valued function defined over a sample space. Consequently, a random variable can be used to identify numerical events that are of interest in an experiment.

Such a numerical value will depend on the outcome, which in turn are governed by an underlying probability model (or distribution).

Definition 1 : A random variable Y is said to be discrete if it can assume only a finite or countably infinite number of distinct values.

For example, the event of interest in an opinion poll regarding voter preferences. Let Y = the number of voters favoring a certain candidate or issue. The observed value of this random variable must be 0 or an integer between 1 and the sample size. Thus, this random variable can take only a finite number and is said to be discrete.

The Probability Distribution for a Discrete Random Variable

Notations :

- Uppercase letter (Y) – random variables
- Lowercase letter (y) – particular values that a random variable may assume
- The expression $(Y=y)$ can be read, the set of all points in S assigned the value y by the random variable Y.
- $P(Y=y)$ = the probability that Y takes on the value y.

Definition 2 : The probability that Y takes on the value y, $P(Y=y)$, is defined as the *sum of the probabilities of all sample points in S* that are assigned the value y. We will sometimes denote $P(Y=y)$ by $p(y)$.

Definition 3 : The *probability distribution* for a discrete variable Y can be represented by a formula, a table, or a graph, which provides $p(y) = P(Y=y)$ for all y.

Examples

Example 1 : Draw 2 balls and win the sum. Fee is $2. Interested in the net gain.

- 9 balls of zero
- 2 balls of three
- 1 ball of five

Let Y = net gain (in $)   - random variable

fee

Then Y = sum of two numbers drawn – 2

(i) Determine all possible values of Y

(ii) Corresponding probabilities associated with each value of Y.

Note :

The previous table which includes a listing of all distinct values of a random variable (in this case Y) and their corresponding probabilities is called a probability distribution for Y.

Examples

Example 2 : A supervisor of a manufacturing plant has three men and three women working for him. He wants to choose two workers for a special job. Not wishing to show any biases in his selection, he decides to select the two workers at random. Let Y denote the number of women in his selection. Find the probability distribution for Y.

Solution :

- Let Y = number of women in his selection
- The number of ways of selection two workers from six workers is
- The values for Y are
- The number of ways of selecting Y = 0 women is

<u>Theorem 1</u> : For any discrete probability distribution, the following must be true :

(1) $0 \le \mathrm{p(y)} \le 1$ for all y.

(2) $\sum_{y} \mathrm{p(y)} = 1$, where the summation is over all values of y with nonzero probability.

The Expected Value of a Random Variable or a Function of a Random Variable

In this section, we are interested in the “mean” or average of a discrete random variable which is commonly referred to as “Expected value”.

Definition : Let *Y* be a discrete random variable with the probability function $p(y) = P(Y=y)$. Then the *expected value* of *Y*, *E(Y)*, is defined to be

$$\mu = E(Y) = \sum_{y} y p(y)$$

Example

Example 3 : Refer to Example 1. We are interested in determining if the game is fair.

(For this we need to compute the expected net gain . Ie. We want E(Y).)

probability distribution

Y = y	-2	1	3	4	6
P(y)	72/132	36/132	18/132	2/132	4/132

More generally, we are interested in not only the expected value of a given random variable, but also the expected value of a function of a random variable.

Most notable example of this is the variance of a random variable.

Theorem 2 : Let Y be a discrete random variable with probability function $p(y)$ and $g(Y)$ be a real-valued function of Y. Then the expected value of $g(Y)$ is given by

$$E[g(Y)] = \sum_y g(y)p(y).$$

Definition 5 : The variance of a random variable Y is defined to be the expected value of $(Y-\mu)^2$. That is,

$$V(Y) = E\left((Y-\mu)^2\right).$$

The standard deviation of Y is the positive square root of $V(Y)$.

Example

Example 4 : The probability distribution for a random variable Y is given below. Find the mean, variance, and standard deviation of Y.

y	2	4	6
p(y)	.2	.3	.5

<u>Theorem 3</u> : Let Y be a discrete random variable with probability function p(y) and c be a constant. Then E(c) = c.

<u>Proof</u>

Theorem 4 : Let Y be a discrete random variable with probability function p(y), g(Y) be a function of Y, and let c be a constant. Then

$$E\left[cg(Y)\right]=cE\left[g(Y)\right].$$

Proof :

Theorem 5 : Let Y be a discrete random variable with probability function $p(y)$ and $g_1(Y)$, $g_2(Y)$, …, $g_k(Y)$ be k functions of Y. Then

$$E\left[g_1(Y)+g_2(Y)+\ldots+g_k(Y)\right]=E\left[g_1(Y)\right]+E\left[g_2(Y)\right]+\ldots+E\left[g_k(Y)\right].$$

Proof :

Theorem 6 : Let Y be a discrete random variable with probability function p(y); then

$$\sigma^2 = V(Y) = E\left((Y-\mu)^2\right) = E(Y)^2 - \mu^2 \ .$$

Proof :

Examples

Example 5 : Refer to Example 1. Draw 2 balls and win the sum. Fee is \$2. Interested in the net gain.

- 9 balls of zero
- 2 balls of three
- 1 ball of five

Let Y = net gain (in \$)

Then Y = sum of two numbers drawn – 2

Calculate V(Y) for this example.

Solution :

The probability distribution is

Y = y	-2	1	3	4	6
P(y)	72/132	36/132	18/132	2/132	4/132

Example 6 : Refer to Example 4. The probability distribution for a random variable Y is given below. Find variance of Y by using Theorem 3.6 .

y	2	4	6
p(y)	.2	.3	.5

Solution :

<u>Example 7</u>: A box contains 7 red balls, 3 black balls, 2 balls are randomly selected without replacement.

(a) Find the probability distribution for the number of red balls.

(b) Find the mean of the random variable represented by the number of red balls.

(c) Find the standard deviation for the number of red balls.

Solution : Let X = # of red balls

The Binomial Probability Distribution

Review

Recall that a binomial experiment satisfies :

(1) Fixed number (n) of trials.

(2) Each trial has only 2 possible outcomes.

Success (S)

Failure (F)

(3) $P(S) = p$ => same on every trial

$P(F) = 1 - p = q$.

(4) the trials are independent.

(5) The random variable of interest is Y, the number of successes observed during the n trials.

Binomial Distribution

Definition 7 : A random variable Y is said to have a *binomial distribution* based on *n* trials with success probability *p* if and only if

$$p(y) = \binom{n}{y} p^y q^{n-y} \quad , \qquad y = 0, 1, 2, \ldots, n \text{ and } 0 \le p \le 1$$

- call Y a binomial random variable

Binomial Expansion

Recall the Binomial expansion Theorem.

$$(x+y)^n = \binom{n}{0}x^n + \binom{n}{1}x^{n-1}y^1 + \binom{n}{2}x^{n-2}y^2 + \ldots + \binom{n}{n-1}x^1 y^{n-1} + \binom{n}{n}y^n$$

Observe that

$$\sum_{y=0}^{n} p(y) = \sum_{y=0}^{n} \binom{n}{y} p^y q^{n-y} = (p+q)^n = (p+1-p)^n = 1$$

sum to 1

Theorem 7 : Let Y be a binomial random variable based on n trials and success probability p. Then

$$\mu = E(Y) = np \quad \text{and} \quad \sigma^2 = V(Y) = npq$$

- proof is in the text, but is omitted here.

Examples

Example 8 : Experience has shown that 30% of all persons afflicted by a certain illness recover. A drug company has developed a new medication. Ten people with the illness were selected at random and received the medication; nine recovered shortly thereafter. Suppose that the medication was absolutely worthless. What is the probability that at least nine of ten receiving the medication will recover?

Solution :

- Let Y denote the number of people who recover.
- n =
- P(success) = probability that a single ill person will recover =

Want : P (Y ≥ 9) =?

Example 9 : A multiple-choice examination has 15 questions, each with five possible answers, only one of which is correct. Suppose that one of the students who takes the examination answers each of the questions with an independent random guess. What is the probability that he answers at least ten questions correctly?

Solution :

Example 10 : Vaccinate children for a disease, and the life time immunity is found to be 70%. If 3 children are vaccinated.

(1) Find the probability that the first child immunized but the next two did not.

(2) Find the probability that the only second child immunized.

(3) Find the probability that exactly 3 children immunized.

(4) Find the probability that at least 2 children immunized.

(5) If 10 children are vaccinated. Find the expected number to immunize and the standard deviation of number of children immunized.

Solution : Let

X = total # of children immunized

The Geometric Probability Distribution

Setting is similar to Binomial

(i) Independent trials (# of trial are no longer fixed)

(ii) 2 outcomes (S, F)

(iii) P(S) = p -- constant

But instead of the # of successes in n trials, we are interested in

Y = # of trials until the first success is observed.

- In the case the # of trials is unknown and the possible values of Y is infinite (countable)

$Y = 1$, success on first trial , $P(Y=1) = p$

$Y = 2$, FS → $P(Y=2) = qp$

$Y = 3$, FFS → $P(Y=3) = q^2p$

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$Y = y$, $\underbrace{FF\cdots F}_{y-1}S$ → $P\ (Y=y) = q^{y-1}p$

Definition : A random variable Y is said to have a *geometric probability distribution* if and only if

$$p(y) = q^{y-1}p, \qquad y = 1,2,3,\dots, \; 0 \le p \le 1$$

Example 11 :

Suppose that 30 % of the applicants for a certain industrial job possess advanced training in computer programming. Applicants are interviewed sequentially and are selected at random from the pool. Find the probability that the first applicant with advanced training in programming is found on the fifth interviewed.

Solution :

- Let Y = # of interviews required to find an applicant with advanced training.

- Geometric random variables often model distributions of lengths of waiting times.
- They also get their name from the geometric series.
- $1+a+a^2+a^3+\ldots.. = \dfrac{1}{1-a} \qquad 0 \le a < 1$

$$\begin{array}{r l}
 & \;1+a \\
1-a \big) & \overline{1+0} \\
 & \underline{1\text{-}a} \\
 & \quad +a \\
 & \quad \underline{a\text{-}a^2} \\
 & \qquad +a^2
\end{array}$$

← long division

- consider the product $(1\text{-}a)(1+a+a^2+\ldots) = 1$.

Geometric series

In particular,

$$\sum_{y=1}^{\infty} P(Y=y) = \sum_{y=1}^{\infty} q^{y-1} p = p \sum_{y=1}^{\infty} q^{y-1}$$

$$= p\left(\frac{1}{1-q}\right) = \frac{p}{p} = 1$$

Theorem 3.8 : If Y is a random variable with a geometric distribution,

$$\mu = E(Y) = \frac{1}{p} \quad \text{and} \quad \sigma^2 = V(Y) = \frac{1-p}{p^2}.$$

Proof :

Examples

<u>Example 12</u> : Refer to Example 11. What is the expected number of applicants who need to be interviewed in order to find the first one with advanced training?

<u>Solution</u> : Recall

- Let Y = # of interviews required to find an applicant with advanced training.

Example 13 : A certified public accountant (CPA) has found that nine of ten company audits contain substantial errors. If the CPA audits a series of company accounts, what is the probability that

(a) the first account containing substantial errors is the third one to be audited?

Let Y = # of the first account containing substantial error.

(b) the first account containing substantial errors will occur on or after the third audited account?

Example 14 : Refer to Example 13. What are the mean and standard deviation of the number of accounts that must be examined to find the first one with substantial errors?

Solution :

The Negative Binomial Probability Distribution

Similar setting as in Binomial experiment.

(i) 2 outcomes (S, F)

(ii) P(S) = p

(iii) Independent trials

However, in this case we are interested in the number of trials until the r^{th} success ($r \geq 1$).

(Geometric insisted $r = 1$)

Y = # of trials until r^{th} success

Let us select fixed values for r and y and consider events A and B, where

A = { the first (y-1) trials contain (r-1) successes }

B = { trial y results in a success }.

Because we assume that the trials are independent, it follows that A and B are independent events, and previous assumptions imply that $P(B) = p$. Therefore,

$$p(y) = p(Y=y) = P(A \cap B) = P(A) \cdot P(B).$$

Note that if $(y-1) < (r-1)$ or $y < r$, then $P(A) = 0$.

If $y \geq r$, our previous work with the binomial distribution implies that

$$P(A) = \binom{y-1}{r-1} p^{r-1} q^{(y-1)-(r-1)} \quad .$$

Finally,

$$p(y) = P(A)P(B) = \binom{y-1}{r-1} p^{r-1} q^{y-r}\, p$$

$$= \binom{y-1}{r-1} p^{r} q^{y-r} \quad , y = r, r+1, r+2, \ldots$$

Examples

Example 15 : The telephone line serving an airline reservation office are all busy about 60% of the time.

(a) If you are calling this office, what is the probability that you will complete your call on the first try? The second try? The third try?

Solution :

Let Y = # of attempts until you complete your call

(b) If you and a friend must both complete calls to this office, what is the probability that a total of four tries will be necessary for both of you to get through?

Solution :

Let Y = # attempts until both calls are completed

<u>Theorem 3.9</u> : If Y is a random variable with a negative binomial distribution,

$$\mu = E(Y) = \frac{r}{p} \quad \text{and} \quad \sigma^2 = V(Y) = \frac{r(1-p)}{p^2}$$

(Proof is omitted here)

<u>Example 16</u> : Refer to Example 15. What is E(Y) in case (b) ?

Examples

Example 16 : A geological study indicates that an exploratory oil well should strike oil with probability .2. What is the probability that

(a) the first strike comes on the third well drilled?

Let Y = # wells drilled until the first strike of oil.

(b) the third strike comes on the seventh well drilled?

Let Y = # of wells drilled until the third strike of oil.

(c) What assumption did you make to obtain the answer to (a) and (b) ?

(d) Find the mean and variance of the number of wells that must be drilled if the company wants to set up three producing wells.

Let Y = # of wells drilled until three producing wells are found.

The Hypergeometric Probability Distribution

The Hypergeometric Probability Distribution

To be more precise suppose we are sampling from a shipment of N games that contains r defective and N-r not defective games. If we randomly draw n games from the lot and let Y denote the number of defective games drawn. Then the random variable Y has a probability distribution that depends on whether draws are made with or without replacement.

The Hypergeometric Probability Distribution

- Case 1 (with replacement) : If after each draw we check whether or not it is defective and then return it back, then the draws are independent, so Y ~ Bin (*n* , *p* = *r/N*). Hence

$$P(Y=y)=\binom{n}{y}\left(\frac{r}{N}\right)^{y}\left(\frac{N-r}{N}\right)^{n-y} \quad \text{for} \quad y=0,1,2,\dots,n$$

and

$$E(Y)=np=n\frac{r}{N}$$

The Hypergeometric Probability Distribution

- Case 2 (without replacement, must have n ≤ N)

  Now the trials are dependent.

  So Y $\nsim$ Bin (n , p). Then

$$n(S) = \binom{N}{n} \quad \leftarrow \text{ select a sample of n from N}$$

P(Y=y) = P(sample of n games contains y defective games)

$$= \binom{r}{y}\binom{N-r}{n-y} \Big/ \binom{N}{n} \qquad \text{y = 0,1,2,..., n}$$

Notation : $\binom{n}{r} = 0$ if $r > n$

In this case Y is said to have a Hypergeometric distribution with parameters *n, r, N*, for short Y~ Hypergeometric (*n, r, N*).

As before,

$$E(Y) = n\left(\frac{r}{N}\right) \quad .$$

and

$$\sigma^2 = V(Y) = n\left(\frac{r}{N}\right)\left(\frac{N-r}{N}\right)\left(\frac{N-n}{N-1}\right)$$

Examples

Example 17 : Used photocopy machines are returned to the supplier, cleaned, and then sent back out on lease agreements. Major repairs are not made, however, and as a result, some customers receive malfunctioning machines. Among eight used photocopiers available today, three are malfunctioning. A customer wants to lease four machines immediately. To meet the customer’s deadline, four of the eight machines are randomly selected and, without further checking, shipped to the customer.

(a) What is the probability that the customer receives no malfunctioning machines?

Let Y = # of malfunctioning copiers selected.

(b) What is the probability that the customer receives at least one malfunctioning machine?

Example 18 : Industrial product is shipped in lots of 20. Sample 5 items from each lot and reject lot if more than one defective is observed. If a lot contains four defectives,

(a) What is the probability that it will be rejected?

Let Y = # of defective found in the sample of 5 selected item

(b) What is the expected number of defective that will be found?

Let Y = # of defective found in the sample of 5 selected item

The Poisson Probability Distribution

Poisson Probability Distribution

Definition : A random variable Y is said to have a Poisson probability distribution if and only if

$$P(Y = y) = p(y) = \frac{\lambda^y}{y!} e^{-\lambda} \quad , \quad y = 0,1,2,\ldots$$

$$\lambda > 0 \qquad e = 2.718\ldots$$

Remarks :

(1) Certainly $P(Y = y) \geq 0$, since $\lambda^y > 0$, $e^{-\lambda} > 0$

(2) Taylor Series for e^y :

$$e^y = \sum_{n=0}^{\infty} \frac{y^n}{n!} \qquad \text{for all } y$$

(3) Claim : $\sum_{y=0}^{\infty} P(Y = y) = 1$

$$\sum_{y=0}^{\infty} P(Y = y) = \sum_{y=0}^{\infty} \frac{\lambda^y e^{-\lambda}}{y!} = e^{-\lambda} \sum_{y=0}^{\infty} \frac{\lambda^y}{y!}$$

$$= \left(e^{-\lambda}\right)\left(e^{\lambda}\right) = 1$$

The Poisson probability distribution often provides a good model for the probability distribution of the number Y of rare events that occur in space, time, volume, or any other dimension, where λ is the average value of Y. The examples of random variables with approximate Poisson distributions are

(1) the # of telephone calls handled by a switchboard in a time interval,

(2) the #of radioactive particles that decay in a particular time period,

(3) the # of errors a typist makes in typing a page,

(4) the # of automobiles using a freeway access ramp in a 10-minute interval.

etc.

Theorem 11 : If Y is a random variable possessing a Poisson distribution with parameter λ, then

$$\mu = E(Y) = \lambda \text{ and } \sigma^2 = V(Y) = \lambda$$

Proof :

$$E(Y(Y-1)) = \sum_{y=0}^{\infty} y(y-1)p(y)$$

$$= \sum_{y=0}^{\infty} y(y-1)\frac{\lambda^y e^{-\lambda}}{y!} = \sum_{y=2}^{\infty} \frac{\lambda^y e^{-\lambda}}{(y-2)!}$$

$$= \lambda^2 \sum_{y=2}^{\infty} \frac{\lambda^{y-2} e^{-\lambda}}{(y-2)!} = \lambda^2(1) = \lambda^2$$

$$E(Y(Y-1)) = E(Y^2 - Y) = E(Y^2) - E(Y) = E(Y^2) - \lambda$$

$$\Rightarrow E(Y^2) = \lambda^2 + \lambda$$

So $V(Y) = E(Y^2) - \left(E(Y)\right)^2 = \lambda^2 + \lambda - \lambda^2 = \lambda$

Important Fact :

If Y = total # of happenings that occur randomly and independently in a time (area, volume) window of size t units and with at the average rate of δ per unit time (area, volume), then Y is a Poisson random variable with parameter $\lambda=\delta t$

Examples

Example 19 : Suppose heart attack emergencies at a local hospital occur at random times at the average rate of 2 per day. Let Y = total # of heart attacks emergencies at the hospital during a certain 5-day period. Find

(a) $P(Y \geq 15)$.

(b) $P(Y \leq 5 \mid Y \leq 9)$

(c) P(exactly one of the 5 days will have no heart attack emergencies)

Example 20 : Suppose a .001 (mm^3) sample of blood from a patient with white cells count 6000 per mm^3 is analyzed. Let Y = total # of white cells in the blood sample. Assuming white cells are randomly distributed in blood, compute :

(a) E(Y) , V(Y)

(b) $P(5 \leq Y \leq 7)$

(c) $P(Y = 8)$

(d) $P(Y \leq 8 \mid Y \geq 2)$

Example 21 : Customers arrive at a checkout counter in a department store according to a Poisson distribution at an average of seven per hour. During a given hour, what are the probabilities that

(a) no more than three customers arrive?

Let Y = # of customers arriving.

(b) at least two customers arrive?

(c) Exactly five customers arrive?

Example 21 : Refer to Example 20. If it takes approximately 10 minutes to serve each customer, find the mean and variance of the total service time for customers arriving during a 1-hour period. (Assume that a sufficient number of servers are available so that no customer must wait for service.)

Let Y = # of customers arriving.

Example 21 (con’t) : Is it likely that the total service time will exceed 2.5 hours?

Example 22 : Refer to Example 21. Find the probability that exactly two customers arrive in the 2-hour period of time :

(a) Between 2.00 P.M. and 4.00 P.M. (one continuous 2-hour period).

Let Y = # of customer that arrive in a given 2-hour period of time.

(b) Between 1.00 P.M. and 2.00 P.M. or between 3.00 P.M. and 4.00 P.M. (two separate 1-hour periods that total 2 hours).

Let Y_1 = # of customers that arrive between 1.00 p.m. and 2.00 p.m.

Y_2 = # of customers that arrive between 3.00 p.m. and 4.00 p.m.

Moments and Moment–Generating Functions

Moments and Moment Generating Function

Goal :

Look at $E(Y)$, $E(Y^2)$, $V(Y)$ in more detail as they all turn out to be special cases of a more general notion – called moments.

Question:

What do we need to uniquely describe a probability distribution, if we do not readily have the actual probability function $p(y)$?

Moments and Moment-Generating Function

Definition : (Moment) The k^{th} moment of a random variable Y taken about the origin is defined to be $E(Y^k)$ and is denoted by μ_k'.

eg : First moment : $\mu_1' = \mu = E(Y)$ - expected value

2nd moment : $\mu_2' = E(Y^2)$

$$\Rightarrow V(Y) = E(Y^2) - \left(E(Y)\right)^2 = E\left(\left(Y - \mu\right)^2\right)$$

Moments and Moment-Generating Function

Definition : The k^{th} moment of a random variable Y taken about its mean, or the k^{th} central moment of Y, is defined to be $E[(Y-\mu)^k]$ and is denoted by μ_k .

$$\text{eg}\ :\text{First moment}: \mu_1 = 0$$

$$\text{2nd moment}\ : \mu_2 = V(Y)$$

Moments and Moment-Generating Function

Useful Fact :

If Y, Z are two random variables with moments $\mu'_{\mathrm{k,Y}}$ and $\mu'_{\mathrm{k,Z}}$ respectively, and $\mu'_{\mathrm{k,Y}} = \mu'_{\mathrm{k,Z}}$ for all k, then Y and Z have the same probability distribution.

Moments and Moment-Generating Function

Definition : The moment-generating function , $m(t)$, for a random variable Y is defined to be

$$m(t) = E(e^{tY}) \,.$$

We say that a moment-generating function for Y exists if there exists a positive constant b such that $m(t)$ is finite for $|\, t \,| \le b$.

Why “ moment-generating” ?

$$E(e^{tY}) = ?$$

1. $e^{tY} = \sum_{n=0}^{\infty} \frac{(ty)^n}{n!}$ - always exists ← Taylor Series

2. $E(e^{tY}) = \sum_{y} e^{ty} P(Y=y) \qquad = \sum_{y} \left(\sum_{n=0}^{\infty} \frac{(ty)^n}{n!} \right) P(Y=y)$

$$= \sum_{y} \left(1 + ty + \frac{(ty)^2}{2!} + \dots \right) P(Y=y)$$

$$= \sum_{y} P(Y=y) + t \sum_{y} y P(Y=y) + \frac{t^2}{2!} \sum_{y} y^2 P(Y=y)$$

$$= \underbrace{1 + t\mu_1' + \frac{t^2}{2!}\mu_2' + \frac{t^3}{3!}\mu_3'}_{\text{generates } \mu_k'} + \dots$$

Why " moment-generating" ?

Note : Coefficient $t^k/k!$ is exactly μ'_k

$$\Rightarrow m(t) = E(e^{tY}) = \sum_{k=0}^{\infty} (\mu'_k) \frac{t^k}{k!}$$

Thus $E(e^{tY})$ is a function of all the moments μ'_k about the origin, for k = 1,2,3,… .

Moments and Moment-Generating Function

Theorem 12 : If $m(t)$ exist, then for any positive integer k,

$$\left.\frac{d^k m(t)}{dt^k}\right]_{t=0} = m^{(k)}(0) = \mu_k' \ .$$

Proof:

Examples

Example 23 : Find the moment-generating function $m(t)$ for a Poisson distributed random variable with parameter λ.

Example 24 : Using $m(t)$ from the previous example to find the mean and variance for the Poisson random variable.

Example 25 : Find $m(t)$ for a geometric random variable Y with parameter p.

Example 26 : Using the $m(t)$ from Example 25 to find the mean for the geometric random variable.

Example 27 : Suppose Y is a r.v. with moment generating function

(1) $m(t) = e^{5(e^t - 1)}$

(2) $m(t) = \dfrac{.3}{e^{-t} - .7}$

What is the distribution of Y?

Solution :