Anyamanu Lapprattaug 604640781

1. In Table 1. X_i is total econometrics exam point (total points are 100) and Y_i is GPA of each BE student.

Table 1

Student	Y_i	X_i	X_iY_i	X_i^2	$X_i - \bar{X}$
1	2.8	63	176.4	3963	-14.625
2	3.4	72	244.8	5184	-5.625
3	3.0	78	234	6084	0.375
4	3.5	81	283.5	6561	3.375
5	3.6	87	313.2	7569	9.375
6	3.0	75	225	5625	-2.625
7	2.7	75	202.5	5625	-2.625
8	3.7	90	333	8100	12.375
SUM	25.7	621	2012.4	48,717	

$\bar{y} = 3.2125$, $\bar{x} = 7.7625$

$E(y|X_i) = \beta_1 + \beta_2 X_i$

1.1 Now consider the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$

Use OLS to find the estimator of $\beta_1 and \beta_2$. Interpret the regression.

Find $\hat{\beta}_1$ & $\hat{\beta}_2$; $\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$, $\hat{\beta}_2 = \dfrac{n\sum_{i=1}^{n} x_iy_i - \sum_{i=1}^{n} y_i \sum_{i=1}^{n} x_i}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}$; $n = 8$

$$\hat{\beta}_2 = \frac{8(2012.4) - 25.7(621)}{8(48{,}717)^2 - (621)^2}$$

$$= \frac{16099.2 - 15959.7}{389{,}736 - 385{,}641}$$

$\hat{\beta}_2 = 0.0341$ #

so $\hat{\beta}_1 = 3.2125 - (0.0341)(7.7625)$

$= 0.5655$ #

$\therefore \hat{y}_i = \hat{\beta}_1 + \hat{\beta}_2 x_i$

$\hat{y}_i = 0.5655 + 0.0342(X_i)$

so every econometrics exam point increase by 1, the GPA will increase by 0.0341 #

1.2 Find $\hat{Y}_i$ and $\hat{u}_i$ and show that $\sum_{i=1}^{n} \hat{u}_i \approx 0$

$\hat{Y}_i = \hat{\beta}_1 - \hat{\beta}_2 X_i$, $\hat{u}_i = y_i - \hat{y}_i$; $\hat{\beta}_1 = 0.5655$, $\hat{\beta}_2 = 0.0341$

i	$\hat{y}_i$	$\hat{u}_i$	$\hat{u}_i^2$
1	2.7138	0.0862	0.0074
2	3.0207	0.3793	0.1439
3	3.2253	-0.2253	0.0508
4	3.3276	0.1724	0.0297
5	3.5322	0.0678	0.0046
6	3.123	-0.123	0.0151
7	3.123	-0.423	0.1789
8	3.6345	0.0655	0.0043

$\sum_{i=1}^{8} \hat{u}_i = 0.0862 + 0.3793 - 0.2253 + 0.1724 + 0.0678 - 0.123 - 0.423 + 0.0655$

$= -0.0001 \rightarrow \approx 0$

1.3 Find $var(\hat{u}_i), var(\hat{\beta}_1), and\ var(\hat{\beta}_2)$

$$var(\hat{u}_i) = \hat{\sigma}^2 = \frac{\sum \hat{u}_i^2}{n-2} = \frac{0.43472587}{8-2} = 0.0725$$

$$var(\hat{\beta}_1) = \frac{\sigma^2 \sum x_i^2}{n \sum (x_i - \bar{x})^2} = \frac{0.0725(48717)}{8(571.875)} = 0.8625 \#$$

$$var(\hat{\beta}_2) = \frac{\hat{\sigma}^2}{\sum (x_i - \bar{x})^2} = \frac{0.0725}{571.875} = 0.000142 \#$$

2. Data is listed in the table

	X_i	Y_i	$X_i Y_i$	X_i^2	$X_i - \bar{X}$
	10	0	0	100	-10
	12	2	24	144	-8
	14	5	70	196	-6
	16	6	96	236	-4
	18	7	126	324	-2
	22	10	220	484	2
	24	10	240	576	4
	26	15	390	676	6
	28	16	448	784	8
	30	20	600	900	10
Σ	200	91	2214	4440	0
mean	$\frac{200}{10} = 20$ $(\bar{x})$	9.1 $(\bar{y})$	221.4	444	0

$(\sum x_i)^2 = 40{,}000$

2.1 From the simple regression model $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$

Find estimators of β_1 and β_2 from the OLS method and interpret the meaning.

Find $\hat{\beta}_1$ & $\hat{\beta}_2$; $\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$, $\hat{\beta}_2 = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} y_i \sum_{i=1}^{n} x_i}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}$; $n = 10$, $\bar{x} = 20$ $\bar{y} = 9.1$

$$\hat{\beta}_2 = \frac{10(2214) - 200(91)}{10(4440) - (200)^2} = \frac{22{,}140 - 18200}{44{,}400 - 40{,}000} = \frac{3940}{4400} = \frac{197}{220} = 0.8955 \#$$

$$\hat{\beta}_1 = 9.1 - (0.8955)(20) = -8.81 \#$$

$\therefore y_i = -8.81 + 0.8955 X_i$

then when x increase 1 unit y will increase 0.8955 unit.

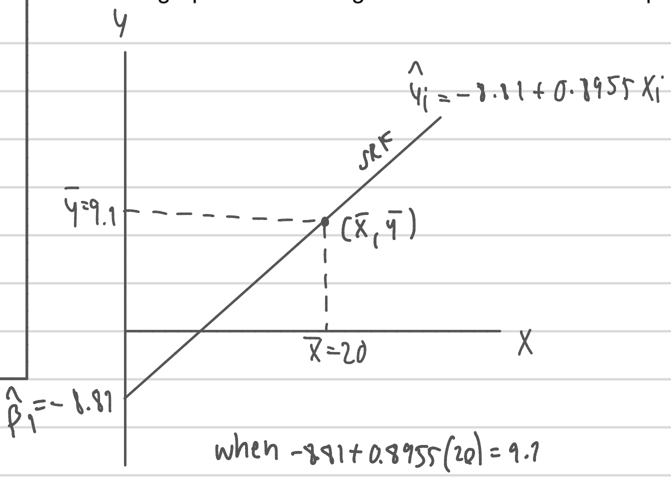
2.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum \hat{u}_i \approx 0$

$\hat{y}_i = \hat{\beta}_1 - \hat{\beta}_2 x_i$, $\hat{u}_i = y_i - \hat{y}_i$; $\hat{\beta}_1 = -8.81$, $\hat{\beta}_2 = 0.8955$

i	$\hat{y}_i$	$\hat{u}_i$	$\hat{u}_i^2$
1	0.145	-0.145	0.021
2	.936	0.064	0.0041
3	3.727	1.273	1.6205
4	5.518	0.482	0.2323
5	7.309	-0.309	0.0955
6	10.891	-0.891	0.7939
7	12.682	-2.682	7.1931
8	14.473	0.527	0.2777
9	16.264	-0.264	0.0697
10	18.055	1.945	3.7830

$\sum \hat{u}_i = 0.145 \approx 0 \#$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?



when $-8.81 + 0.8955(20) = 9.1$

$\therefore \hat{y}_i = \bar{y}$ so this line pass $(\bar{x}, \bar{y})$

(be a member of SRF)

2.4 If $X_i = 18$, what is the predicted Y?

$$X_i = 18,\quad \hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i$$
$$= -8.81 + (0.8955)(18)$$
$$= 7.309 \#$$

2.5 Find $var(\hat{u}_i)$, $var(\hat{\beta}_1)$, $var(\hat{\beta}_2)$

$$var(\hat{u}_i) = \frac{\sum \hat{u}_i^2}{n-2} = \frac{14.0908}{10-2} = 1.7614$$

$$var(\hat{\beta}_2) = \frac{\hat{\sigma}^2}{\sum (x_i - \bar{X})^2} = \frac{1.7614}{440} = 0.004$$

$$var(\hat{\beta}_1) = \frac{\sigma^2 \sum X_i^2}{n \sum (X_i - \bar{X})^2} = \frac{1.7614(4440)}{10(440)} = 1.7774$$

3. Consider the below regression function: consider the two-variable model

$$Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID(0, \sigma^2)$$

Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

Find $\hat{\beta}_2$ that unbiased $\rightarrow E(\hat{\beta}_2) = \beta_2$

from $\hat{\beta}_2 = \sum_{i=1}^{n} k_i y_i$

$= \sum k_i (\beta_1 + \beta_2 X_i + u_i)$

$= \sum (k_i \beta_1 + \beta_2 k_i X_i + k_i u_i)$

$= \beta_1 \underbrace{\sum k_i}_{0} + \beta_2 \underbrace{\sum k_i x_i}_{1} + \sum k_i u_i$

$\hat{\beta}_2 = \beta_2 + \sum_{i=1}^{n} k_i u_i \qquad E(u_i | x_i) = 0$

$E(\hat{\beta}_2) = E(\beta_2) + E(\sum k_i u_i)$

$= \beta_2 \#$

so $\hat{\beta}_2$ is unbiased estimature of true β_2

from $\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$

so, $E(\hat{\beta}_1) = E(\bar{Y} - \hat{\beta}_2 \bar{X})$

$= E(\bar{Y}) - \bar{X} E(\hat{\beta}_2)$; $E(\bar{Y}) = E\left(\frac{\sum y_i}{n}\right)$

$\therefore E(\hat{\beta}_1) = \beta_1 + \beta_2 \bar{X} - \bar{X} E(\hat{\beta}_2)$; $E(\hat{\beta}_2) = \beta_2$

$= \beta_1 + \cancel{\beta_2 \bar{X} - \beta_2 \bar{X}}$

Then $E(\hat{\beta}_1) = \beta_1 \rightarrow$ unbiased!

$\bar{Y} = \frac{\sum y_i}{n}$

$= \frac{1}{n} E\left(\frac{\sum y_i}{n}\right)$

$= \frac{1}{n} \sum \beta_1 + \beta_2 x_i$

$= \frac{n\beta_1}{n} + \frac{\beta_2 \sum x_i}{n}$

$= \beta_1 + \beta_2 \bar{X}$