

## Assignment 2

### Part I

The study on bankruptcy firm employs the following regression model.

$$z_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i} + \beta_4 x_{4i} \quad (1)$$

The log-likelihood function of this model is as follows:

$$\ln L = \begin{cases} \ln \Phi(z_i) & \text{if } y_i = 1 \\ \ln \Phi(-z_i) & \text{if } y_i = 0 \end{cases} \quad (2)$$

where:  $y_i$  = 1 for bankruptcy firm and 0 otherwise.

$x_{1i}$  = Debt coverage ratio of firm i

$x_{2i}$  = Liquidity ratio of firm i

$x_{3i}$  = Profitability index of firm i

$x_{4i}$  = Solidity ratio of firm i

$\Phi(\cdot)$  = Logistic probability distribution function.  $\Phi(z_i) = \frac{1}{1 + e^{-z_i}}$

From the given data set (assign02-1.dta):

1. Estimate the above models using MLE with Newton-Ralphson algorithm.
2. Perform hypothesis testing whether  $\beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$  using LR-test and Wald test.
3. Estimate the above models using MLE with BHHH algorithm, make comparison of the estimated result with the result from (1), and give explanation why are they different?

### Part II

In the study of interest rate structure, the continuous time model can be specified as:

$$r_{t+\Delta t} - r_t = (\alpha + \beta r_t) \Delta t + \varepsilon_{t+\Delta t} \quad (3)$$

where:  $E[\varepsilon_{t+\Delta t}] = 0$  and  $E[\varepsilon_{t+\Delta t}^2] = \sigma^2 r_t^{2\gamma} \Delta t$

Then, the model can be transformed to be discrete time model by setting  $\Delta t = 1$ . The discrete time model can be stated as:

$$r_{t+1} - r_t = \Delta r_t = \alpha + \beta r_t + \varepsilon_{t+1} \quad (4)$$

where:  $E[\varepsilon_{t+1}] = 0$  and  $E[\varepsilon_{t+1}^2] = \sigma^2 r_t^{2\gamma}$

The above model consists of four parameters including  $\alpha$ ,  $\beta$ ,  $\sigma^2$ ,  $\gamma$ . The model can be estimate using GMM. Four moment condition equations can be stated as:

Zero mean condition:  $E(\varepsilon_{t+1}) = 0$

Orthogonality condition:  $E(\varepsilon_{t+1} r_t) = 0$

Variance condition:  $E(\varepsilon_{t+1}^2 - \sigma^2 r_t^{2\gamma}) = 0$

Zero covariance condition:  $E((\varepsilon_{t+1}^2 - \sigma^2 r_t^{2\gamma})r_t) = 0$

The above model can be claimed as unrestricted model for other eight interest rate structure models which can be indicated as follows:

Model	$\alpha$	$\beta$	$\sigma^2$	$\gamma$
(1) Unrestricted				
(2) Merton		0		0
(3) Vasicek				0
(4) CIR SR				0.5
(5) Dothan	0	0		1
(6) GBM	0			1
(7) Brennan & Schwartz				1
(8) CIR VR	0	0		1.5
(9) CEV	0			

From the given data set (assign02-2.dta):

4. Estimate the interest rate structure models applying all 9 models using GMM.
5. Determine the most appropriated model using Wald Test.

### Part III

From the model: 
$$y_i = \alpha + \beta x_i + u_i \quad (5)$$

where:  $y_i$  is dependent variable

$x_i$  is explanatory variable

$u_i$  is stochastic error term

$E(u_i) = 0$  but  $E(x_i u_i) \neq 0$ .

From the given data set (assign02-3.dta):

6. Estimate model (5) using OLS.
7. Based on  $z_1, z_2, z_3, z_4$ , determine the best set of instrumental variables, then, estimate model (5) using GMM.
8. Determine whether OLS estimated results in (6) or GMM estimated results in (7) is more appropriate using the Hansen's J chi2 statistic test.