Lecture 7: Intertemporal Choice

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1 Introduction

We next want to examine an application of the consumer choice model to a situation where the consumer chooses consumption today versus consumption tomorrow. This application provides a number of useful insights, some of them similar to insights from the labor supply application.

The intertemporal choice model is also the foundation for a large fraction of modern macroeconomics and much of modern finance. We will mention a few of these practical applications here, and you should find this material helpful in later courses. In addition, this material provides a good illustration and review of the techniques we have developed so far.

Consider a consumer who thinks about how much to consume of some composite good in each of two periods:

$$(c_1, c_2)$$

The price of consumption in each period is fixed at 1.

The consumer receives an endowment of consumption goods (manna from heaven) in each period:

$$(\omega_1, \omega_2).$$

The key issue is specifying how the consumer can transfer endowments across time. Essentially, we want to understand what factors determine how much the consumer chooses to consume this period versus next period.

In all the examples below, we assume the consumer has a utility function that depends on the amount of consumption this period and the amount of consumption next period:

$$u(c_1, c_2).$$

This is the most general utility function we could specify for this situation. Often it is useful to choose a more restricted utility function; for example, we might want to make $u()$ additively separable in consumption today versus consumption tomorrow:

$$u(c_{1,}c_2) = v(c_1) + w(c_2).$$

In many cases, we impose a further restriction and write

$$u(c_1, c_2) = v(c_1) + \beta v\,(c_2)$$

$$0 < \beta < 1\ .$$

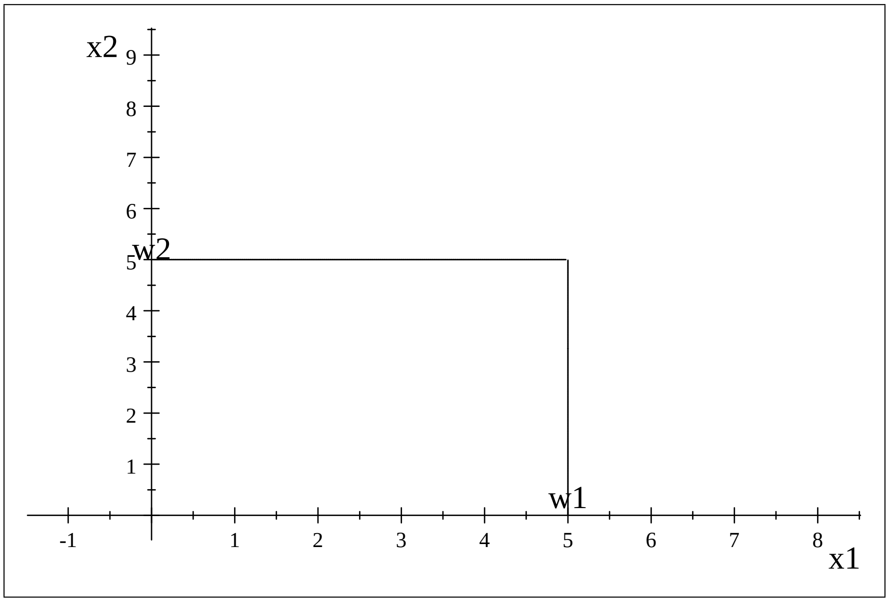
This says that the consumer has the same within period utility function in each period, but that viewed from this period, getting consumption next period is not as good as getting consumption now; the consumer discounts future consumption at the rate β. Thus, β is known as the discount factor.

2 Intertemporal Consumption Choice

Given this framework, the first case to consider is one where the consumption good is non-storable; for example, the good might be food that only lasts one period and then becomes inedible. In addition, we assume that no asset markets exist, so the consumer cannot borrow or lend or buy stocks or own anything that has durable value between periods 1 and 2.

In this case, what does the budget set look like?

Graph: Budget Set for Two Period Problem, No Storage
$x_1 = 5,\ for\ 0 \leq x_2 \leq 5$
$x_2 = 5,\ for\ 0 \leq x_1 \leq 5$



With algebra, we can write this as

$$c_1 \leq \omega_1$$
$$c_2 \leq \omega_2$$

So, the budget set is not a triangle; it is a square.

The consumer just consumes the endowments that arrive each period (or, if satiated, something inside the box).

Now, this first set of assumptions is too restrictive for many settings. Even in primitive societies, many kinds of food are storable.

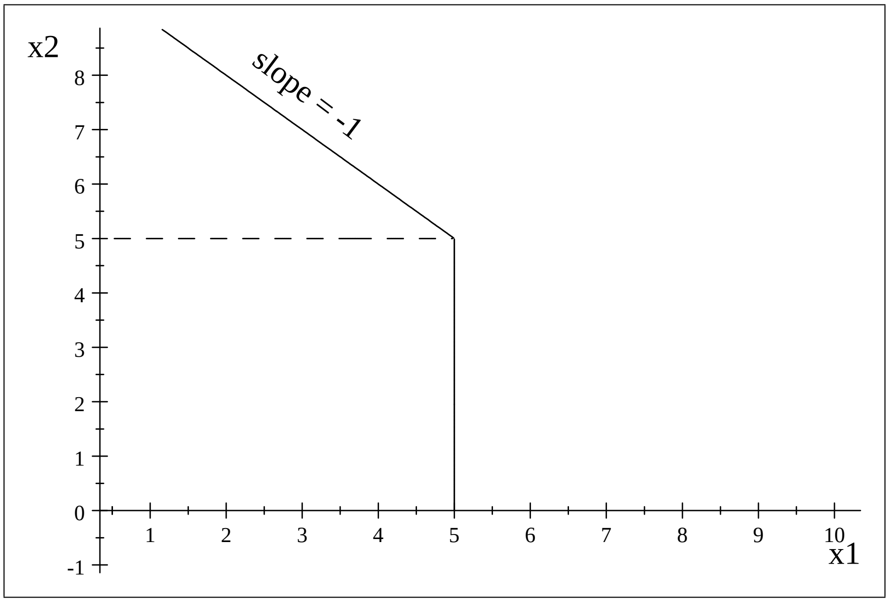
So, what does the budget set look like in that case? The consumer now has more options. He can go hungry in the first period and eat some of his first period endowments in the second period. The budget set is strictly bigger than it was under the first scenario.

The algebra looks like this:

$$c_1 \leq \omega_1$$
$$c_2 \leq \omega_1 - c_1 + \omega_2$$

and the graph like this:

Graph: Budget Set with Storable Output
$x_2 = 10 - x_1$
$x_1 = 5$

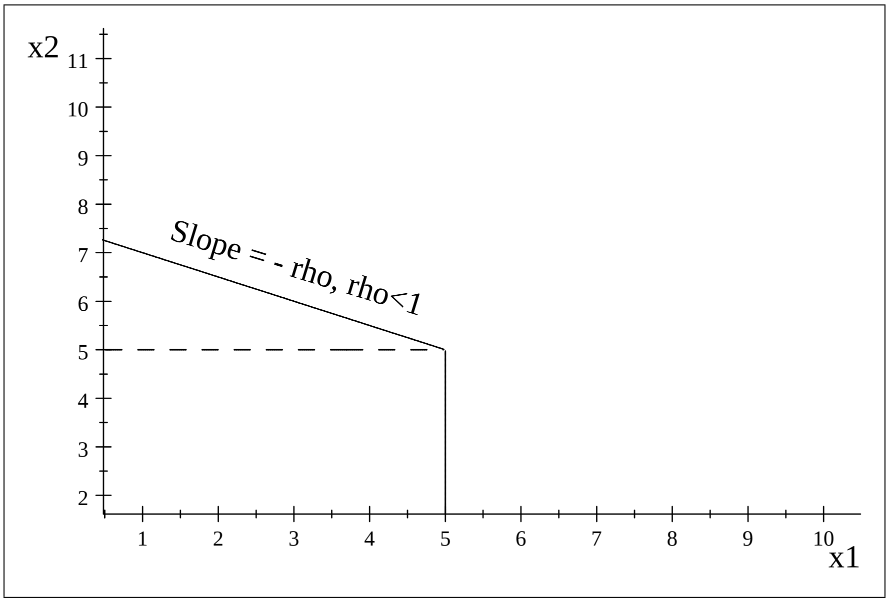


A third possibility is that the endowment lasts but decays; some fraction evaporates, or rots, or gets eaten by mice, or whatever. Then the constraints are:

$$c_1 \leq \omega_1$$
$$c_2 \leq \rho(\omega_1 - c_1) + \omega_2$$

where ρ is the fraction of the first period endowment that lasts from period 1 to period 2. Graphically this looks like:

Graph: Budget Constraint with Rotting Food
$x_2 = 7.5 - 0.5x_1$
$x_1 = 5$



Still a fourth possibility is that the consumer can lend out ω_1 and earn some return r. Then we have

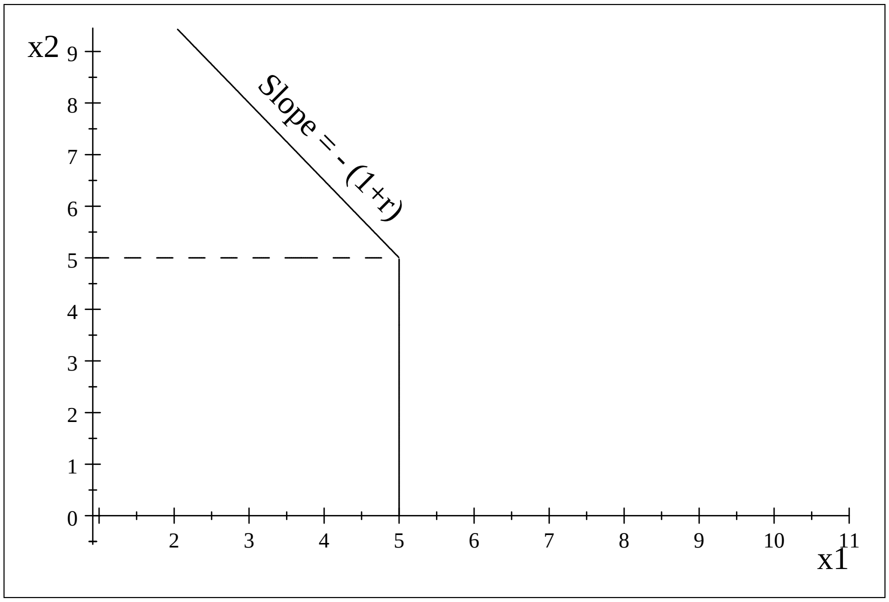
$$c_1 \leq \omega_1$$
$$c_2 \leq (\omega_1 - c_1)(1 + r) + \omega_2$$

and we get this graph:

Graph: Budget Set in Two-Period Model with Lending but not Borrowing

$x_2 = 12.5 - 1.5x_1$

$x_1 = 5$



A final possibility (for now) is that the consumer can borrow against future endowments, at least up to the value of those endowments. Then we have

$$c_1 + \frac{c_2}{1+r} = \omega_1 + \frac{\omega_2}{1+r}$$

Why is this the right constraint?

If the consumer consumes less than the endowment in period 1, i.e., the consumer is a saver:

$$c_2 = \omega_2 + (\omega_1 - c_1) + r(\omega_1 - c_1)$$

This says that second period consumption equals the second period endowment, plus any part of the first period endowment that was not consumerd, plus the interest earned on this amount since it was loaned out for one period.

If the consumer consumes more than the endowment in period 1, i.e., is a borrower, then

$$c_2 = \omega_2 - (c_1 - \omega_1) - r(c_1 - \omega_1)$$

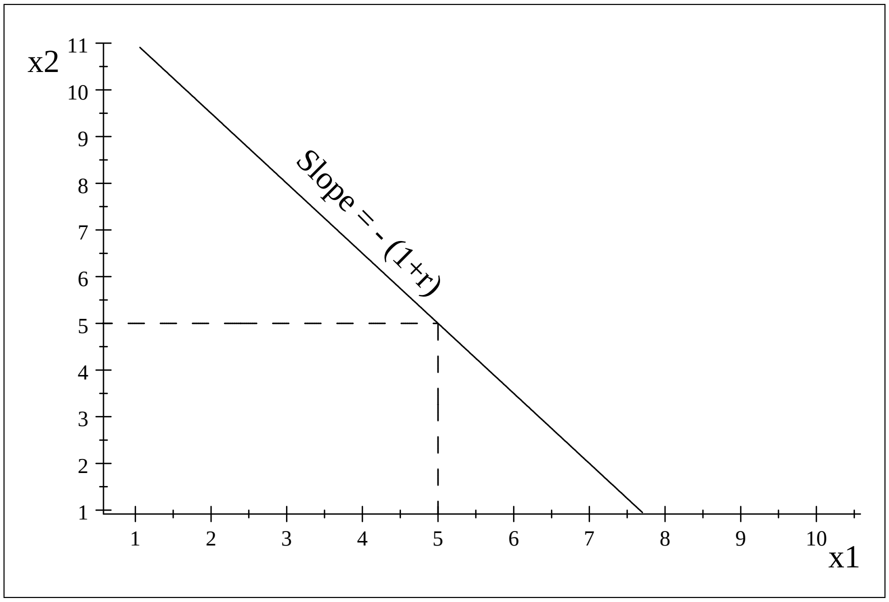
This says that second period consumption equals the second period endowment, minus the repayment of the principal on the loan the consumer took out to finance a level of first period consumption in excess of the first period endowment, minus the interest paid on this loan.

The equations are the same for a borrower and a saver, so they apply to everyone, and after rearranging they imply the above constraint as written.

The graph for this scenario is therefore:

Graph: Budget Constraint for Two-Period Problem with Borrowing and Lending

$x_2 = 12.5 - 1.5x_1$
$x_2 = 5$
$x_1 = 5$



Additional possibilities allowing both borrowing and lending but at different interest rates; that will be on a problem set.

3 Comparative Statics: Borrowers versus Lenders

The analysis so far has shown what the budget sets look like under various assumptions about the opportunities a consumer has for transferring goods across time.

The next issue is to determine how a change in the economic environment affects the consumer's consumption and savings decisions.

We can look at this in two steps. For the moment, we will just examine whether a change in the interest rate can make a borrower into a lender or a lender into a borrower.

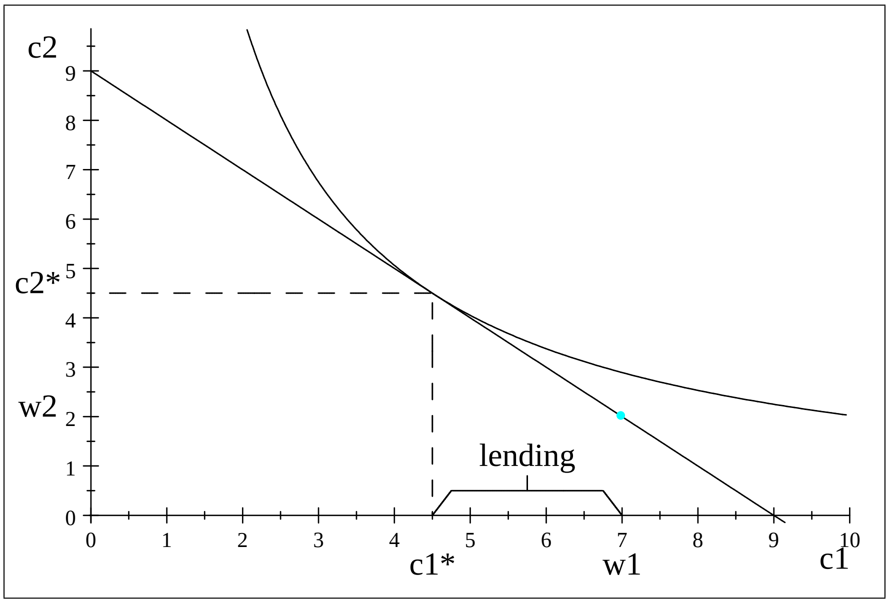
Consider first the case where the consumer's optimal plan yields

$$c_1 < \omega_1,$$

meaning this consumer is a lender (has positive savings) at the initial interest rate.

Graph: Consumer is Initially a Lender

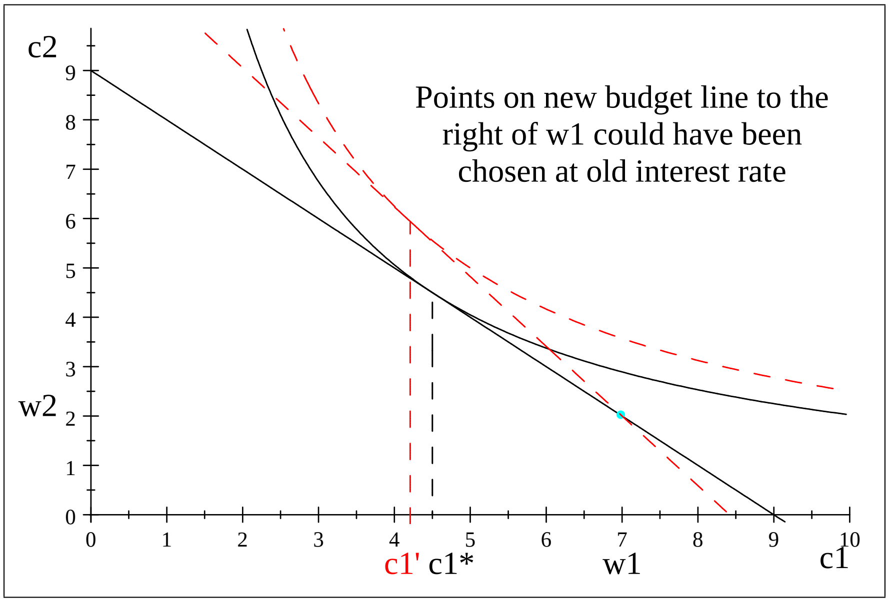
$y = 9 - x$



How does the consumer react to a change in the interest rate?

Increasing the interest rate tilts the budget line to make it steeper: for a given reduction in c_1 you get more c_2 when r is higher:

Graph: An Initial Lender Must Remain a Lender
$y = 9 - x$



Therefore, an initial lender remains a lender. The reason is that all points to the right of ω_1, the points at which this consumer would be a borrower, were affordable at the initial (lower) interest rate but were not chosen.

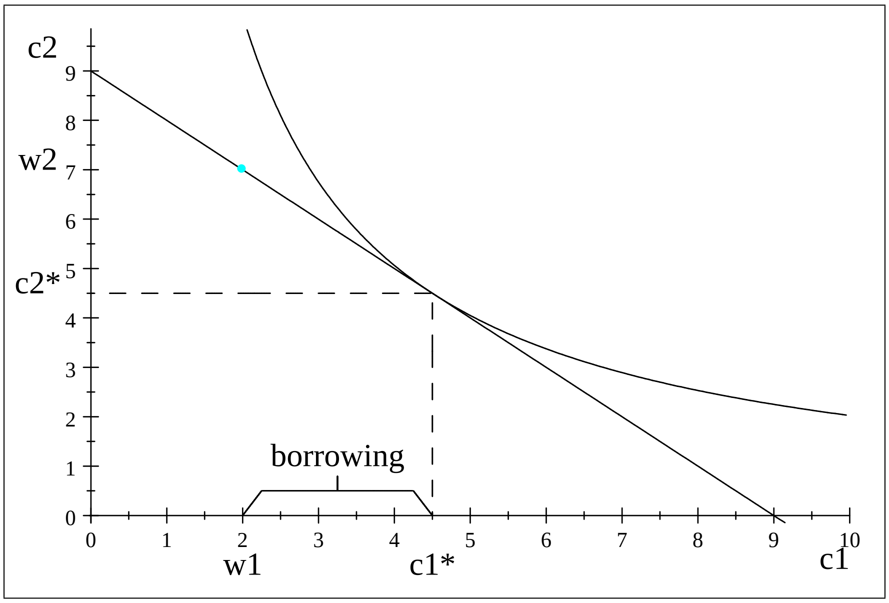
The second case to consider is where

$$c_1 > \omega_1,$$

meaning the consumer is a borrower (has negative savings) at the initial interest rate.

Graph: Consumer is Initially a Borrower

$y = 9 - x$



In this case analogous reasoning shows that if the consumer is initially a borrower, and the interest rate *declines*, the consumer must remain a borrower; becoming a lender would violate revealed preference.

You should draw graphs that illustrate the remaining two cases: an initial lender faces a decline in the interest rate, and an initial borrower faces an increase in the interest rate. Show that in these situations a lender might or might not become a borrower and a borrower might or might not become a lender.

4 The Interest Rate and Savings

The discussion above has illustrated what an interest rate change means for borrowers or lenders. We now want to ask a more specific question: what happens to the amount of consumption and savings in the first period if the interest rates rises?

An increase in the interest rate is a change in the price of consumption in period 1 relative to period 2. As with any price change, two effects operate.

The substitution effect, as always, operates in the opposite direction from the price change. An increase in the interest rate is equivalent to an increase in the price of first period consumption. At a higher interest rate, you give up more second period consumption in order to have a unit of first period consumption.

So the substitution effect says the consumer should consume less and save more in the first period if the interest rate increases.

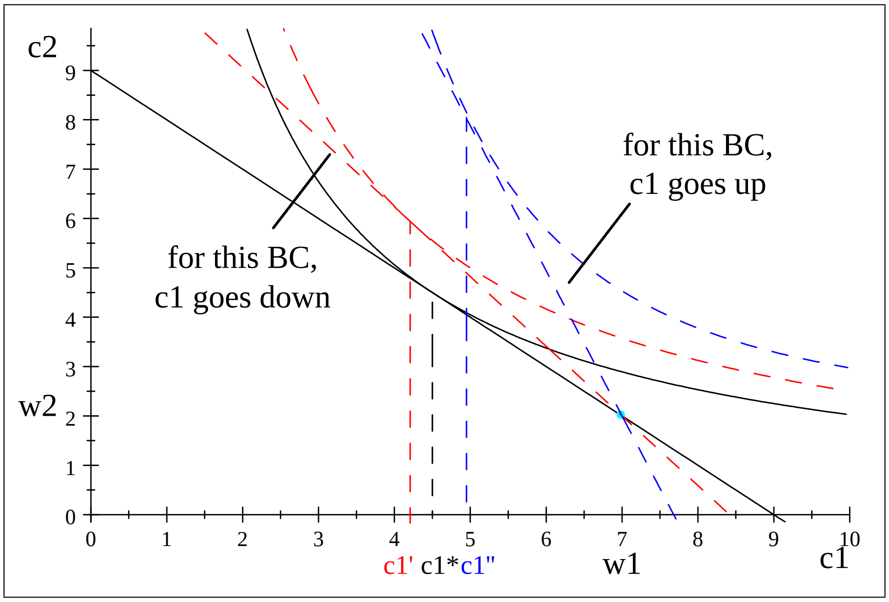
What about the income effect? Assume that consumption in each period is a normal good.

The effect of the interest rate increase for an initial lender is to raise the value of the consumer's initial savings because the consumer earns a higher return on this savings. This implies a positive income effect, meaning higher first period consumption.

Thus, the overall effect of an interest rate increase for an initial lender depends on the magnitude of the income effect. relative to the substitution effect.

Graph: Ambiguous Effect of an Interest Rate Increase on c_1, s_1 for an Initial Lender

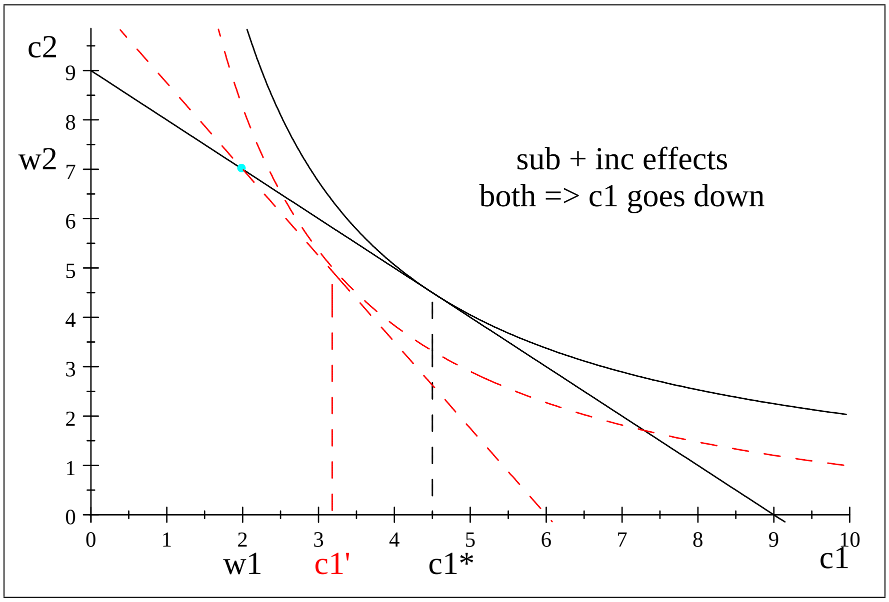
$y = 9 - x$



For a consumer who is an initial borrower, the income effect goes in the same direction as the substitution effect (a higher interest rate means less first period consumption). The interest rate increase makes borrowing more expensive, so the consumer is poorer and chooses less first period consumption.

Graph: Unambiguous Effect of an Interest Rate Increase for an Initial Borrower

$y = 9 - x$



Thus the effect of an interest rate increase for an initial borrower is to reduce first period consumption and increase savings.

The overall effect of the interest rate on savings is thus ambiguous, assuming the economy has some borrowers and some lenders. This is consistent with the evidence.

5 Inflation and Real versus Nominal Interest Rates

So far we have assumed that the price of consumption goods is constant (and = 1) across periods; that is, the general price level does not change. This is the same as assuming no inflation.

This assumption is easy to modify.

Let p_1 and p_2 be the prices of consumption good in the two periods. Then the budget constraint is

$$p_1 c_1 + \frac{p_2 c_2}{(1+r)} = p_1 \omega_1 + \frac{p_2 \omega_2}{(1+r)}$$

Dividing through by p_1, we get

$$c_1 + \frac{(p_2/p_1)c_2}{(1+r)} = \omega_1 + \frac{(p_2/p_1)\omega_2}{(1+r)}$$

Define inflation, π, to be the rate of growth of prices:

$$\pi = \frac{p_2 - p_1}{p_1}$$

Then we can re-write the budget constraint in a way that allows for inflation as

$$c_1 + \frac{(1+\pi)c_2}{(1+r)} = \omega_1 + \frac{(1+\pi)\omega_2}{(1+r)}$$

Now define the real interest rate by

$$1 + \rho = \frac{1+r}{1+\pi}$$

This allows us to re-write the budget constraint in terms of the real interest rate as

$$c_1 + \frac{c_2}{(1+\rho)} = \omega_1 + \frac{\omega_2}{(1+\rho)}$$

This is a correct formula whether or not there is inflation. If there is no inflation, then the budget constraint can also be written correctly in terms of the nominal rate. The version that uses the real rate is more general, however, and less likely to generate confusion.

Note that, if you take logs of both sides of the equation that defines the real rate, you get

$$\ln(1+\rho) = \ln(1+r) - \ln(1+\pi)$$

Since for "small" x,

$$\ln(1+x) \approx x$$

We have as an approximation that

$$\rho = r - \pi.$$

In words, the real interest rate is just the nominal interest rate minus the inflation rate.