B.E. International Program

Faculty of Economics, Thammasat University

Semester: 1/2015

EE 425 Econometrics 1

Homework #3

---

1. Given the following data

Y	X_2	X_3
1	1	2
3	2	1
8	3	-3
10	4	-5

Estimate the coefficients for the following equation

(a) $Y_i = \alpha_1 + \alpha_2 X_{2i} + u_{1i}$

(b) $Y_i = \lambda_1 + \lambda_3 X_{3i} + u_{2i}$

(c) $Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + v_i$

Is $\hat{\alpha}_2 = \hat{\beta}_2$, $\hat{\lambda}_3 = \hat{\beta}_3$? Why or why not. Offer you explanation.

2. The following table gives quarterly data on these variables.

Y = quantity of roses sold, dozens

X_2 = average wholesale price of roses, $/dozen

X_3 = average wholesale price of carnations, $/dozen

X_4 = average weekly family disposable income, $/week

X_5 = the trend variable taking values of 1,2, and so on, for the period 2001-III to 2005-II in the Detroit metropolitan area.

Year and quarter	Y	X_2	X_3	X_4	X_5
2001-III	11,484	2.26	3.49	158.11	1
-IV	9,348	2.54	2.85	173.36	2
2002-I	8,429	3.07	4.06	165.26	3
-II	10,079	2.91	3.64	172.92	4
-III	9,240	2.73	3.21	178.46	5
-IV	8,862	2.77	3.66	198.62	6
2003-I	6,216	3.59	3.76	186.28	7
-II	8,253	3.23	3.49	188.98	8
-III	8,038	2.60	3.13	180.49	9
-IV	7,476	2.89	3.20	183.33	10
2004-I	5,911	3.77	3.65	181.87	11
-II	7,950	3.64	3.60	185.00	12
-III	6,134	2.82	2.94	184.00	13
-IV	5,868	2.96	3.12	188.20	14
2005-I	3,160	4.24	3.58	175.67	15
-II	5,872	3.69	3.53	188.00	16

You are asked to consider the following demand functions:

$$Y_t = \alpha_1 + \alpha_2 X_{2t} + \alpha_3 X_{3t} + \alpha_4 X_{4t} + \alpha_5 X_{5t} + u_t$$

$$\ln Y_t = \beta_1 + \beta_2 \ln X_{2t} + \beta_3 \ln X_{3t} + \beta_4 \ln X_{4t} + \beta_5 \ln X_{5t} + u_t$$

(a) Estimate the parameters of the linear model and interpret the results.
(b) Estimate the parameters of the log-linear model and interpret the results.
(c) β_2, β_3, and β_4 give, respectively, the *own-price*, *cross-price*, and *income elasticities* of demand. What are their a priori signs? Do the results concur with the a priori expectations?
(d) Compute the *own-price*, *cross-price*, and *income elasticities* of demand for the linear model, using the mean values.
(e) Can you compare R^2 of the two models? Why or why not?

3. The following table gives data on output and total cost of production of a commodity in the short run.

Output	Total Cost, $
1	193
2	226
3	240
4	244
5	257
6	260
7	274
8	297
9	350
10	420

To test whether the preceding data suggest the U-shaped average and marginal cost curves typically encountered in the short run, one can use the following model:

$$Y_i = \beta_1 + \beta_2 X_i + \beta_3 X_i^2 + \beta_4 X_i^3 + u_i$$

where Y=total cost and X=output. The additional explanatory variables X_i^2 and X_i^3 are derived from X.

a) Express the data in the deviation form and obtain (X'X) and (X'Y).

b) Estimate β_2, β_3, and β_4.

c) Estimate the var-cov matrix of $\hat{\beta}$.

d) Estimate β_1. Interpret $\hat{\beta}_1$ in the context of the problem.

e) Obtain R^2 and $\bar{R}^2$.

f) A priori, what are the signs of β_2, β_3, and β_4? Why?

g) From the total cost function given previously obtain expressions for the marginal and average cost functions.

4. In an application of the Cobb-Douglas production function the following results were obtained:

$$\widehat{lnY_i} = 2.3542 + 0.9576\, lnX_{2i} + 0.8242\, lnX_{3i}$$

(0.3022) (0.3571)

R^2=0.8432 df=12

where Y=output, X_2=labor input, and X_3=capital input, and where the figures in parentheses are the estimate standard errors.

a) The coefficients of the labor and capital inputs in the preceding equation give the elasticities of output with respect to labor and capital. Test the hypothesis that these elasticites are individually equal to unity.

b) Test the hypothesis that the labor and capital elasticities are equal, assuming (i) the covariance between the estimated labor and capital coefficients is zero, and (ii) it is-0.0972.

c) Test the overall significance of the preceding regression equation.

5. In order to study the labor force participation of urban poor families (families earning less than $3,943 in 1969), the data in Table was obtained from the 1970 Census of Population.

Table Labor Force Participation Experience of the Urban Poor: Census Tracts, New York City, 1970

Tract No.	% in Labor Force, Y*	Mean Family Income, X_2**	Mean Family Size, X_3	Unemployment Rate, X_4***
137	64.3	1,998	2.95	4.4
139	45.4	1,114	3.40	3.4
141	26.6	1,942	3.72	1.1
142	87.5	1,998	4.43	3.1
143	71.3	2,026	3.82	7.7
145	82.4	1,853	3.90	5.0
147	26.3	1,666	3.32	6.2
149	61.6	1,434	3.80	5.4
151	52.9	1,513	3.49	12.2
153	64.7	2,008	3.85	4.8
155	64.9	1,704	4.69	2.9
157	70.5	1,525	3.89	4.8
159	87.2	1,842	3.53	3.9
161	81.2	1,735	4.96	7.2
163	67.9	1,639	3.68	3.6

Y* = family heads under 65 years old.
X_2** = dollars.
X_4*** = percent of civilian labor force unemployed.
Source: Census Tracts: New York, Bureau of the Census, U.S. Department of Commerce, 1970

a) Using the regression model

   (i) $Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$

   (ii) $Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + \beta_4 X_{4i} + u_i$, obtain the estimates of the regression coefficients and interpret your results.

b) A priori, what are the expected signs of the regression coefficients in the preceding model and why?

c) How would you test the hypothesis that the unemployment rate has no effect on the labor force participation of the urban poor in the census tracts given in the accompanying table?

d) How would you test the hypothesis that the **incremental contribution** of the unemployment rate is statistically significant? Show your ANOVA table.

e) Should any variables be dropped from the preceding model? Why?