Simultaneous Equations Models

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Part 3

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SEMs with time series

- Define a simple Keynesian model of aggregate demand in a closed economy
  $C_t = \beta_0 + \beta_1(Y_t - T_t) + \beta_2 r_t + u_{t1}$ (1)
  $I_t = \gamma_0 + \gamma_1 r_t + u_{t2}$ (2)
  $Y_t \equiv C_t + I_t + G_t$ (3)
- Equation (3) is an identity: it holds by definition, without error.
- How many endogenous variables?
- What are exogenous variables?

Aggregate demand system

- Model according to equations (1) to (3) is completely static. We can allow for dynamics by adding lagged income.
- Equation (2) become
  $I_t = \gamma_0 + \gamma_1 r_t + \gamma_2 Y_{t-1} + u_{t2}$ (4)
  - Lagged endogenous variable is treated as a pre-determined variable, hence exogenous
  - Assume u_{t2} is uncorrelated with current exogenous variables and all past endogenous and exogenous variables
  - Here, we can estimate (4) by OLS

Aggregate demand system

- Similarly, if we add lagged consumption, C_{t-1}, to equation (1), we can treat C_{t-1} as exogenous.
- However, current disposable income is still endogenous in $C_t = \beta_0 + \beta_1(Y_t - T_t) + \beta_2 r_t + \beta_3 C_{t-1} + u_{t1}$ (5)
- We could estimate (5) by 2SLS using instruments (T_t, r_t, G_t, C_{t-1})
- If investment is determined by (4), Y_{t-1} should be added to the instrument list.
  - Why? find the reduced form

SEMs with panel data

- Consider labor supply and wage offer equations over a given period of time, we allow for unobserved effects, e.g. unobserved taste for leisure not change over time, in each equation
- With panel data, estimating SEMs involves 2 steps:
  - eliminate the unobserved effects using the fixed effects or first differencing transformation
  - find instrumental variables (time varying) for the endogenous variables in the transformed equation, then estimate transformed equation by pooled 2SLS

SEMs with panel data

- SEM for panel data:
  $y_{1it} = \alpha_1 y_{2it} + \mathbf{z_{1it}}\beta_1 + a_{1i} + u_{1it}$ (6)
  $y_{2it} = \alpha_2 y_{1it} + \mathbf{z_{2it}}\beta_2 + a_{2i} + u_{2it}$ (7)
- The unobserved effects, a_{1i} and a_{2i}, can be correlated with all explanatory variables
- Assume the idiosyncratic structural errors, u_{1it} and u_{2it}, are uncorrelated with the z in both equations and across all time periods >> z are exogenous.

SEMs with panel data

- First, suppose we difference over time to remove a_{1i}:
  $\triangle y_{1it} = \alpha_1 \triangle y_{2it} + \triangle z_{1it}\beta_1 + \triangle u_{1it}$ (8)
  - $cov(\triangle u_{1it}, \triangle z_{1it})$=0, by assumption (since they are uncorrelated even in the level term)
  - $\triangle y_{2it}$ and $\triangle u_{1it}$ are possibly correlated. So, we need IVs for $\triangle y_{2it}$
- Second, use time-varying variables in $\mathbf{z_{2it}}$ that are not in $\mathbf{z_{1it}}$ as IVs. Then, apply pooled 2SLS.