

**EE 325**

1. Find the answers following questions

a.  $\sum_{i=1}^5 (a + bx_i)$

b.  $\sum_{y=0}^5 f(x + y)$

c.  $\sum_{i=1}^{10} i^2$

d.  $\sum_{x=1}^2 \sum_{y=2}^3 (2x + y)$

2. Given  $X$  is discrete random variable. The probability distribution function (PDF) of this variable is shown in the table

$X$	-2	-1	0	1	2	3	4
$f(x)$	0.5b	b	2.25b	2b	1.5b	0.5b	0.25b

when  $b$  is constant number

- Find the value of  $b$
  - Find the answer for  $P(X \leq 2)$
  - Find the answer for  $P(-2 \leq X \leq 3)$
  - Find the answer for  $P(X \geq 1)$
3. Given  $X$  is continuous random variable. The probability distribution function (PDF) of this variable is

$$f(x) = -\frac{1}{9}x + \frac{6}{9}, 0 \leq x \leq 3$$

- Plot graph for  $f(x)$
- Find the answer for  $P(1 \leq X \leq 3)$
- Find the answer for  $P(X \geq 2)$
- Find the expected value of  $X$

4. Let random variable  $X$  be the outcome of throwing one dice and random variable  $Y$  be the outcome of tossing one coin. Coin has two sided that has valued 1 and 0.
- Construct the joint probability distribution function (PDF) table of  $X$  and  $Y$
  - Find the marginal probability distribution function (PDF) of  $X$
  - Find the marginal probability distribution function (PDF) of  $Y$
  - Find the conditional probability distribution function (PDF) of  $X$  given  $Y$  is equal to 1
  - Find the expected value of  $X$  given  $Y$  is equal to 1
  - Find the variance of  $X$  given  $Y$  is equal to 1

5. If  $X_1, X_2, X_3$  is a random sample from a population with mean  $\mu$  and variance  $\sigma^2$ .  $X_1, X_2, X_3$  are not independent

$$\text{Cov}(X_1, X_2) = \text{Cov}(X_1, X_3) = \text{Cov}(X_2, X_3) = \frac{1}{4}\sigma^2$$

$$\bar{X} \text{ is estimator used to estimate mean value. } \bar{X} = \frac{1}{3}(X_1 + X_2 + X_3)$$

Find  $E(\bar{X})$  and  $\text{var}(\bar{X})$

6. Given  $X_1, X_2, X_3, X_4$  are independent identically distributed random variables from population with mean  $\mu$  and variance  $\sigma^2$ .  $\bar{X}$  is estimator used to estimate mean value.  $\bar{X} = \frac{1}{4}(X_1 + X_2 + X_3 + X_4)$

- Find  $E(\bar{X})$  and  $\text{var}(\bar{X})$  in term of  $\mu$  and  $\sigma$
- Given  $\tilde{X} = \frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4$  is another estimator of  $\mu$ . Show that  $\tilde{X}$  is an unbiased estimator of  $\mu$
- Between  $\bar{X}$  and  $\tilde{X}$ , which one is the better estimator for  $\mu$ ? Why?