

Example 1.C (cont.): National income model

- From the example 1.B, it is straightforward to solve for all the endogenous equilibrium solutions, Y^* , C^* , Y_d^* .

$$Y_d^* = Y^* - T_0$$

$$Y^* = C^* + I_0 + G_0$$

$$C^* = c + bY_d^*$$

$$Y^* = c + b(Y^* - T_0) + I_0 + G_0$$

$$C^* = c + b(Y^* - T_0)$$

- Numerically, if $a = 1$, $T_0 = \$0$, $I_0 = \$1$, $G_0 = \$1$ and $b = 0.5$, this yields us,

$$Y^* = c + b(Y^* - T_0) + I_0 + G_0$$

$$C^* = a + b(Y^* - T_0)$$

$$Y^* = c + bY^* - bT_0 + I_0 + G_0$$

$$= 1 + 0.5(1 - 0)$$

$$= 4$$

$$Y^*(1 - b) = c - bT_0 + I_0 + G_0$$

$$Y_d^* = Y^* - T_0$$

$$Y = \frac{c - bT_0 + I_0 + G_0}{1 - b}$$

$$= 4 - 0$$

$$= 4$$

$$= \frac{-1 - 0 + 1 + 1}{1 - 0.5} = \frac{3}{0.5} = 6$$

Exercise 2.A:

2.A.1) Given a demand function by $p = a - bQ$, derive the formula for the elasticity of demand, and show that the third property holds

2.A.2) Given the market supply $p = c + dQ$ where $d \geq 0$, show that

(i) elasticity of supply is always greater than 1 if $c > 0$,

(ii) elasticity of supply is always equal to 1 if $c = 0$,

(iii) elasticity of supply is always less than 1 if $c < 0$.

$$\begin{aligned} 1.) \quad E_p &= \frac{\Delta Q^d}{\Delta p} \cdot \frac{p}{Q^d} \\ &= -b \cdot \frac{p}{Q^d} \end{aligned}$$

- 2.)
- i.) elastic
 - ii.) unit elastic
 - iii.) inelastic

Example 2.I: A monopolist firm faces the market demand given by $P = 10 - Q$. Consider the following questions if the cost function $C(Q) = 4Q$.

- What is the revenue-maximizing level of output?

$$\begin{aligned} R_{\text{revenue}} &= P(Q) = (10 - Q)Q \\ &= 10Q - Q^2 \end{aligned}$$

- What is the break-even output?

$$\begin{aligned} \text{Total Revenue} &= \text{Total Cost} \\ 10Q - Q^2 &= 4Q \end{aligned}$$

- What is the profit-maximizing level of output?

$$\begin{aligned} \pi &= (10 - Q)Q - 4Q \\ \pi &= 10Q - Q^2 - 4Q \\ \pi &= 6Q - Q^2 \end{aligned}$$

Exercise 2B. Consider a function that relates tax revenues R , in billions of dollars, to the average tax rate t such that $R = 350t - 500t^2$.

(a) What tax rate(s) is consistent with raising tax revenues equal to \$60 billion?

(b) What tax rate(s) is consistent with raising tax revenues equal to \$61.25 billion?

(c) What tax rate is consistent with the maximum tax revenue?

$$a.) \quad 60 = 350t - 500t^2$$

$$500t^2 - 350t + 60 = 0$$

$$50t^2 - 35t + 6 = 0$$

$$50t^2 - 15t - 20t + 6 = 0$$

$$5t(10t - 3) - 2(10t - 3) = 0$$

$$(10t - 3)(5t - 2) = 0$$

$$t = 3/10, 2/5$$

$$b.) \quad 61.25 = 350t - 500t^2$$

$$500t^2 - 350t + 61.25 = 0$$

$$500t^2 - 350t + \frac{245}{4} = 0$$

$$2000t^2 - 1400t + 245 = 0$$

$$400t^2 - 280t + 49 = 0$$

$$(20t - 7)(20t - 7)$$

$$t = 7/20$$

$$c.) \quad \pi = TR - TC$$

$$R = 350t - 500t^2$$

$$\frac{dR}{dt} = 350 - 1000t$$

$$t = \frac{35}{1000} = \frac{7}{200}$$