

Example 1.C (cont.): National income model

- From the example 1.B, it is straightforward to solve for **all the endogenous equilibrium solutions**, Y^* , C^* , Y_d^* . 1st step: find Y^* use Eq. 1. cons: $Y = AE = C + I + G$

$$Y = AE = C + I + G$$

From $C = a + bY_d$; $Y = a + bY_d + I_0 + G_0$

From $Y_d = Y - T$; $Y = a + b(Y - T) + I_0 + G_0$

$$Y = a + bY - bT + I_0 + G_0$$

$$Y - bY = a - bT + I_0 + G_0$$

$$(1 - b)Y = a - bT + I_0 + G_0$$

$$Y = \frac{a - bT + I_0 + G_0}{1 - b} \quad \#$$

- Numerically, if $a = 1$, $T_0 = \$0$, $I_0 = \$1$, $G_0 = \$1$ and $b = 0.5$, this yields us,

$$\begin{aligned}
 Y &= a + bY - bT + I_0 + G_0 \\
 Y &= 1 + 0.5Y - 0.5(0) + 1 + 1 \\
 Y - 0.5Y &= 3 \\
 0.5Y &= 3 \\
 Y^* &= \frac{3}{0.5} = 6 \quad \#
 \end{aligned}$$

$$\begin{aligned}
 Y_d^* &= Y^* - T \\
 Y_d^* &= 6 - 0 \\
 Y_d^* &= 6 \quad \#
 \end{aligned}$$

$$\begin{aligned}
 C^* &= a + bY_d^* \\
 C^* &= 1 + 0.5(6) \\
 C^* &= 4 \quad \#
 \end{aligned}$$

Exercise 2.A:

2.A.1) Given a demand function by $p = a - bQ$, derive the formula for the elasticity of demand, and show that the third property holds

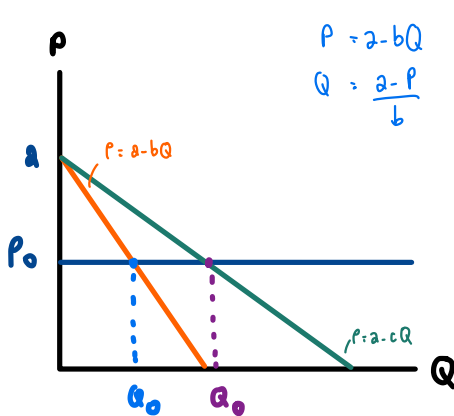
2.A.2) Given the market supply $p = c + dQ$ where $d \geq 0$, show that

(i) ^{PES} elasticity of supply is always greater than 1 if $c > 0$, $\rightarrow \frac{c}{dQ} > 1$

(ii) elasticity of supply is always equal to 1 if $c = 0$, $\rightarrow \frac{c}{dQ} = 1$

(iii) elasticity of supply is always less than 1 if $c < 0$, $\rightarrow \frac{c}{dQ} < 1$

2.A.1



$$\begin{aligned} \bullet \text{ PED} &= \frac{\Delta Q}{\Delta P} \times \frac{P_0}{Q_0} \\ &= -\frac{1}{b} \times \left(\frac{P_0}{\frac{a - P_0}{b}} \right) \\ &= -\frac{P_0}{a - P_0} \end{aligned}$$

$$\begin{aligned} \bullet \text{ PED} &= \frac{\Delta Q}{\Delta P} \times \frac{P_0}{Q_0} \\ &= -\frac{1}{c} \times \left(\frac{P_0}{\frac{a - P_0}{c}} \right) \\ &= -\frac{P_0}{a - P_0} \end{aligned}$$

#

2.A.2

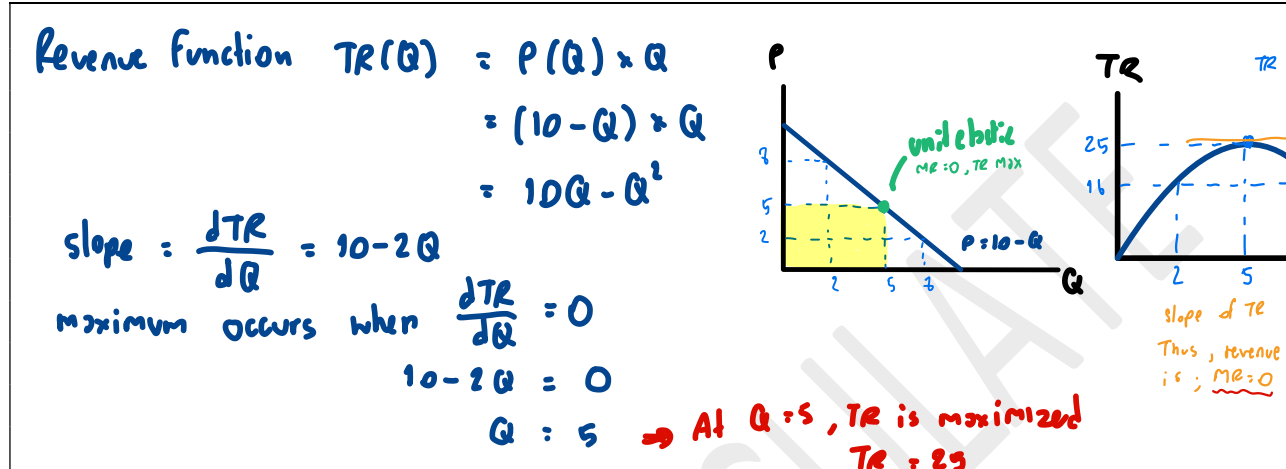
$$\begin{aligned} \text{PES} &= \frac{\Delta Q}{\Delta P} \times \frac{P_0}{Q_0} \\ &= \frac{1}{d} \times \left(\frac{c + dQ}{Q} \right) \\ &= \frac{c}{dQ} + \frac{dQ}{dQ} \\ &= \frac{c}{dQ} + 1 \end{aligned}$$

#

Example 2.1: A monopolist firm faces the market demand given by $P = 10 - Q$. Consider the following questions if the cost function $C(Q) = 4Q$. - find revenue function

- o Find Q that maximize TR
 o What is the revenue-maximizing level of output?

Max Rev : 25



- o What is the break-even output?

$$\pi = 0 \rightarrow TR = TC$$

$$\begin{aligned} TR &= TC \\ 10Q - Q^2 &= 4Q \\ 6Q &= Q^2 \\ 6 &= Q \end{aligned}$$

#

- o What is the profit-maximizing level of output?

1) $MC = MR$

$$\begin{aligned} MC &= \frac{\Delta TR}{\Delta Q} & MC &= MR \\ MC &= 4 & 4 &= 10 - 2Q \\ & & 2Q &= 6 \\ & & Q &= \frac{6}{2} = 3 \end{aligned}$$

2) $\pi = TR - TC \quad \frac{d\pi}{dQ} = 0 \rightarrow Q^*$

$$\begin{aligned} \pi &= TR - TC \\ \pi &= 10Q - Q^2 - 4Q = 6Q - Q^2 \\ \frac{d\pi}{dQ} &= 6Q - 2Q^2 = 0 \\ 6 - 2Q &= 0 \\ Q &= \frac{6}{2} = 3 \end{aligned}$$

Exercise 2B. Consider a function that relates tax revenues R , in billions of dollars, to the average tax rate t such that $R = 350t - 500t^2$.

(a) What tax rate(s) is consistent with raising tax revenues equal to \$60 billion?

(b) What tax rate(s) is consistent with raising tax revenues equal to \$61.25 billion?

(c) What tax rate is consistent with the maximum tax revenue?

2

$$R = 350t - 500t^2$$

$$60 = 350t - 500t^2$$

$$500t^2 - 350t + 60 = 0$$

$$50t^2 - 35t + 6 = 0$$

$$t = 0.3, 0.4 \quad \text{or} \quad 30\% \text{ and } 40\%$$

\therefore 30% and 40% tax rates #

6

$$61.25 = 350t - 500t^2$$

$$500t^2 - 350t + 61.25 = 0$$

$$t = 0.35 \quad \text{or} \quad 35\%$$

\therefore 35% tax rates #

c

$$\frac{dR}{dt} = 350 - 1000t$$

$$0 = 350 - 1000t$$

$$1000t = 350$$

$$t = 0.35 \quad \text{or} \quad 35\%$$

\therefore 35% tax rates #