EE325 Section 1 HW 2 Due Thursday February 20th (23:00 hr.),2020

Use 4 decimal places for numerical answers

1. In Table 1.a. X_i is total microeconomics exam point (total points are 100) and Y_i is GPA of each student.

$\bar{X} = \frac{\sum x_i}{N} = \frac{621}{8} = 77.625$

$\bar{y} = \frac{\sum y_i}{N} = \frac{25.7}{8} = 3.2125$

Table 1.a

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

$(-14.625)(-0.4125) = 6.0328$ $(-5.625)(0.188) = -1.0575$ $(0.375)(-.2125) = -0.0797$

$\sum(x_i-\bar{x})(y_i-\bar{y}) = (63-77.625)(2.8-3.2125) + (72-77.625)(3.4-3.212) + (78-77.625)(3-3.2125)$

$(3.375)(0.2875) = 0.9703$ $(9.375)(0.3875) = 3.6328$ $(-2.625)(-0.2125) = 0.5578$

$+ (81-77.625)(3.5-3.2125) + (87-77.625)(3.6-3.2125) + (75-77.625)(3-3.2125)$

$(-2.625)(-0.5125) = 1.3453$ $(12.375)(0.4875) = 6.0328$

$+(75-77.625)(2.7-3.2125) + (90-77.625)(3.7-3.2125) = 17.4346$

213.8906 31.6406 0.1406 6.8906

$\sum(x_i-\bar{x})^2 = (63-77.625)^2 + (72-77.625)^2 + (78-77.625)^2 + (75-77.625)^2$

11.3906 87.8906 6.8906 153.1406

$+ (81-77.625)^2 + (87-77.625)^2 + (75-77.625)^2 + (90-77.625)^2 = 511.8748$

1.1 Now consider the two-variable $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Use OLS to find the estimator of β_0 and β_1. (Note: *NIID* = Normally, Identically, and Independently Distributed).

$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X} = 3.2125 - (0.0341)(77.625) = 0.5655$

$\hat{\beta}_1 = \frac{\sum_i (x_i-\bar{x})(y_i-\bar{y})}{\sum_i (x_i-\bar{x})^2} = \frac{17.4346}{511.8748} = 0.0341$

$\hat{y}_i = 0.5655 + 0.0341 \hat{x}_i + \hat{u}_i$

1.2 For each observation i, find $\widehat{Y}_i$ and $\widehat{u}_i$. Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

$y_i - \hat{y}_i = y_i - (0.5655 + 0.0341 \hat{x}_i)$

Student	Y_i	X_i	$\hat{u}_i$	$\hat{y}_i$	x_i^2
1	2.8	63	.0862	2.9	3,969
2	3.4	72	.3793	3.4	5,184
3	3	78	-0.2251	3	6,084
4	3.5	81	0.1724	3.5	6561
5	3.6	87	0.0678	3.6	7,569
6	3.0	75	-0.123	3	5625
7	2.7	75	-0.423	2.7	5625
8	3.7	90	.0655	3.7	8100
total			0	25.7	48,717

from new tables

$\sum \hat{u}_i = \sum y_i - \sum \hat{y}_i$

$= 25.7 - 25.7 = 0.$

1.3 Find $var(\widehat{u_i}), var(\widehat{\beta_0}), var(\widehat{\beta_1})$

$$Var(\hat{u}_i) = \frac{\sum \hat{u}_i^2}{n-2} = \frac{(.0862)^2+(.3793)^2+(-.2253)^2+(.1724)^2+(.0678)^2+(.127)^2+(.423)^2+(.0655)^2}{6}$$

$$= \frac{0.0074 + .1439 + .0508 + .0297 + 0.0046 + .0151 + .1789 + 0.0043}{6}$$

$$= \frac{0.4347}{6} = 0.07245$$

$$Var(\hat{\beta}_1) = \frac{\hat{\sigma}^2}{SSE} = \frac{\hat{\sigma}^2}{\sum(x_i-\bar{x})^2} = \frac{0.0725}{511.8748} = 0.00014$$

$$Var(\hat{\beta}_0) = \frac{\sum x_i^2 \cdot \hat{\sigma}^2}{n\sum(x_i-\bar{x})^2} = \frac{43717(0.001)}{8(511.8748)} = 0.0119$$

2. Data is listed in the table

X_i	Y_i
10	0
12	2
14	5
16	6
18	7
22	10
24	10
26	15
28	16
30	20

$$\bar{X} = \frac{\sum x_i}{N} = \frac{200}{10} = 20$$

$$\bar{y} = \frac{\sum y_i}{N} = \frac{91}{10} = 9.1$$

$$\sum_i (x_i-\bar{x})^2 = (10-20)^2+(12-20)^2+(14-20)^2+(16-20)^2 + (18-20)^2+(22-20)^2+(24-20)^2+(26-20)^2 + (28-20)^2+(30-20)^2 = 440$$

$$\sum_i (x_i-\bar{x})(y_i-\bar{y}) = (10-20)(0-9.1)+(12-20)(2-9.1)+(14-20)(5-9.1) + (16-20)(6-9.1)+(18-20)(7-9.1)+(22-20)(10-9.1)+(24-20)(10-9.1) + (26-20)(15-9.1)+(28-20)(16-9.1)+(30-20)(20-9.1) =$$

$$= 91 + 56.8 + 24.6 + 12.4 + 4.2 + 1.8 + 3.6 + 35.4 + 55.2 + 109 = 394$$

2.1 From the simple regression model $Y_i = \beta_0 + \beta_1 X_i + u_i, u_i \sim NIID(0, \sigma^2)$. Find estimators of β_0 and β_1 from the OLS method and interpret the meaning.

$$\hat{\beta}_1 = \frac{\sum_i (x_i-\bar{x})(y_i-\bar{y})}{\sum_i (x_i-\bar{x})^2} = \frac{394}{440} \approx 0.8955$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1\bar{x} = 9.1 - (0.8955)(20) \approx -8.81$$

when x_i equal to 0, y_i will equal to -8.81 and when x_i increases by one unit y_i increases by 0.8955 units

$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 \hat{x}_i + \hat{u}_i \quad = -8.81 + 0.8955\, x_i$

2.2 Find the value of $\widehat{Y}_i$ ~~and $\widehat{u}_i$~~. Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

$\hat{u}_i = y_i - \hat{y}_i = y_i - (-8.81 + 0.8955 X_i)$

X_i	Y_i	$\hat{u}_i$	$\hat{y}_i$	X_i^2
10	0	-0.145	0	100
12	2	0.064	2	144
14	5	1.273	5	196
16	6	0.482	6	256
18	7	-0.309	7	324
22	10	-0.891	10	484
24	10	-2.682	10	576
26	15	0.527	15	676
28	16	-0.264	16	784
30	20	1.945	20	900
	total	0	91	4440

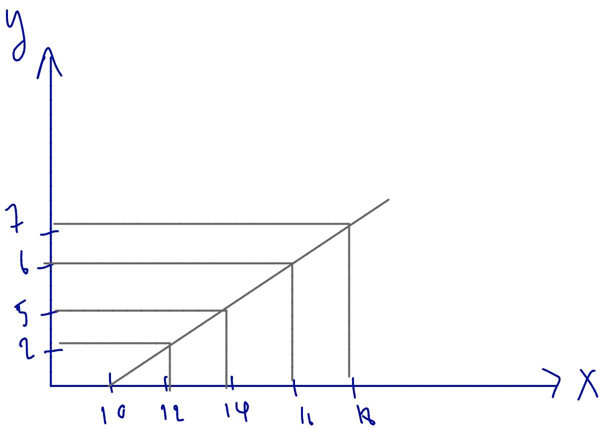
from new table

$\sum \hat{u}_i = \sum y_i - \sum \hat{y}_i$

$= 91 - 91 = 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

$\bar{y} = \hat{\beta}_0 + \hat{\beta}_1 \bar{x} = -8.81 + 0.8955(20) = 9.1$ ✓ $\rightarrow$ pas $(\bar{x}, \bar{y})$



2.4 If X_i =16, what is the predicted Y?

$\hat{y}_i = -8.81 + 0.8955(16) = 5.518$.

2.5 Find $var(\widehat{u}_i), var(\widehat{\beta_0}), var(\widehat{\beta_1})$

$$Var(\hat{u}_i) = \frac{\sum u_i^2}{n-2} = \frac{\overset{0.021}{(0.145)^2} + \overset{0.0041}{(0.064)^2} + \overset{1.6205}{(1.273)^2} + \overset{0.2323}{(0.482)^2} + \overset{0.0955}{(-0.309)^2} + \overset{0.7939}{(0.891)^2} + \overset{7.1931}{(-2.682)^2} + \overset{0.2777}{(0.527)^2} + \overset{0.0697}{(-0.264)^2} + \overset{3.783}{(1.945)^2}}{8} = \frac{14.089}{8} = 1.7611 = \sigma^2$$

$$Var(\hat{\beta}_0) = \frac{\sum x_i^2}{n \sum (x_i - \bar{x})^2} \sigma^2 = \frac{4440}{10(440)} (1.7611) = 1.7771$$

$$Var(\hat{\beta}_1) = \frac{\sigma^2}{SST} = \frac{\sigma^2}{\sum (x_i - \bar{x})^2} = \frac{1.7611}{440} = 0.004$$

3. Consider the below regression function:

$$Y_i = \beta_1 X_i + u_i \,,$$

Where $u_i \sim NIID(0, \sigma^2)$**.** Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the SLR assumption(s) when used.

OLS estimator of β_1

so $\underline{\sum_i (u_i)^2}$ minimize

$$\frac{d}{d\beta_1} \sum_i (Y_i - \beta_1 x_i)^2 = 0$$

$$\sum_i 2(Y_i - \beta_1 x_i)(-x_i) = 0$$

$$\sum_i (Y_i - \beta_1 x_i)(x_i) = 0$$

$$\sum_i (Y_i x_i - \beta_1 x_i^2) = 0$$

$$\sum_i x_i y_i - \sum_i \beta_1 x_i^2 = 0$$

$$\beta_1 = \frac{\sum_i x_i y_i}{\sum_i x_i^2}$$

PRF: substitute $y_i = \beta_1 x_i + u_i$ in $\hat{\beta}_1$ equation

$$\hat{\beta}_1 = \frac{\sum_i x_i(\beta_1 x_i + u_i)}{\sum_i x_i^2}$$

$$= \frac{\sum(\beta_1 x_i^2 + u_i x_i)}{\sum_i x_i^2}$$

$$\hat{\beta}_1 = \beta_1 + \frac{\sum_i u_i x_i}{\sum_i x_i^2}$$

SLR 4

$$E(\hat{\beta}_1) = E(\beta_1) + E\left(\frac{\sum_i u_i x_i}{\sum x_i^2}\right)$$

$E(u_i|x_i) = 0$

$$E(\hat{\beta}_1) = \beta_1$$