



# B.E. International Program

Faculty of Economics, Thammasat University



EE 320 Introductory Mathematical Economics (Semester 2/2013)

## Practice Problem 10 (Integrations and Its Applications)<sup>1</sup>

1. In the manufacture of a product, the marginal cost of producing  $x$  units is  $C'(x)$  and fixed cost are  $C(0)$ . Find the total cost function  $C(x)$  when:

(a)  $C'(x) = 3x + 4$ ,  $C(0) = 40$ .

(b)  $C'(x) = ax + b$ ,  $C(0) = C_0$ .

2. (a) Find  $F(x)$  if  $F'(x) = \frac{1}{2}e^x - 2x$  and  $F(0) = \frac{1}{2}$ .

(b) Find  $F(x)$  if  $F'(x) = x(1 - x^2)$  and  $F(0) = \frac{5}{12}$ .

3. Compute the area  $A$  bounded by the graph of  $f(x) = \frac{1}{x^3}$ , the  $x$ -axis, and the two lines  $x = -2$  and  $x = -1$ . Make a drawing. (Hint:  $f(x) < 0$  in  $[-2, -1]$ .)

4. Compute the area  $A$  bounded by the graph of  $f(x) = \frac{1}{2}(e^x - e^{-x})$ , the  $x$ -axis, and the two lines  $x = -1$  and  $x = 1$ .

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<sup>1</sup> All questions are from Sydsaeter and Hammond, 2008.

5. (a) The profit of a firm as a function of its output  $x$  is given by

$$f(x) = 4000 - x - \frac{3000000}{x}, \quad x > 0$$

Find the level of output that maximizes profit. Sketch the graph of  $f$ .

(b) The actual output varies between 1000 and 3000 units. Compute the average profit

$$I = \frac{1}{2000} \int_{1000}^{3000} f(x) dx.$$

6. Let  $K(t)$  denote the capital stock of an economy at time  $t$ . Then net investment at time  $t$ , denoted by  $I(t)$ , is given by the rate of increase  $\dot{K}(t)$  of  $K(t)$ . [Note:  $\dot{K}(t) = \frac{dK}{dt}$ .]

(a) If  $I(t) = 3t^2 + 2t + 5, t \geq 0$ , what is the total increase in the capital stock during the interval from  $t = 0$  to  $t = 5$ ?

(b) If  $K(t_0) = K_0$ , find an expression for the total increase in the capital stock from time  $t = t_0$  to  $t = T$  when the investment function  $I(t)$  is as in part (a).

7. Suppose that the demand and supply curves are  $P = f(Q) = 200 - 0.2Q$  and  $P = g(Q) = 20 + 0.1Q$ , respectively. Find the equilibrium quantity and compute the consumer and producer surplus.

8. Suppose that the demand and supply curves are  $P = f(Q) = \frac{6000}{Q+50}$  and  $P = g(Q) = Q + 10$ . Find the equilibrium price, and compute the consumer and producer surplus.

9. Evaluate the following integrals by using integrations by parts ( $r \neq 0$ ).

(a)  $\int_0^T bte^{-rt} dt$

(b)  $\int_0^T (a + bt)e^{-rt} dt$

10. Evaluate the following integrals by using integrations by substitution:

(a)  $\int_0^1 x\sqrt{1+x^2} dx$

(b)  $\int_1^e \frac{\ln y}{y} dy$