Multiple Regression Analysis (Inference)

Objectives

1. Students know how to test hypotheses about the parameter (β)
2. Students can test for the validity of the proposed population model.

Econometrics Analysis - The Steps

1 Sampling Distribution of the OLS estimators ($\hat{\beta}_{OLS}$)

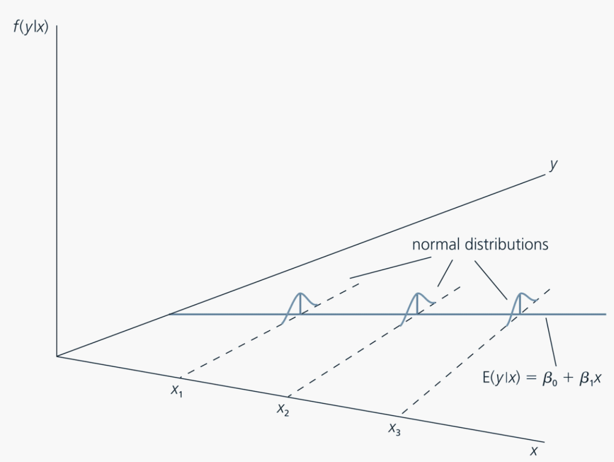
To be able to test hypotheses about the parameter ($\hat{\beta}$), we need an assumption about the distribution of u.

Assumption MLR 6 - Normality

The population error u is independent of the explanatory variables $X_1, X_2, ...X_k$ and is **normally distributed** with zero mean and variable σ^2.

FIGURE 4.1

The homoskedastic normal distribution with a single explanatory variable.



- By normality of the error term (u), we have nomality of $\hat{\beta}$

Or, using the standardization $\frac{Z-\mu}{\sigma} \sim$ normal (0,1), we have

2 Testing Hypotheses about an individual regression coefficient "the t-test"

- But we do not have $sd.(\hat{\beta}_j)$, we can only calculate the estimator of it :

- for degree of freedom > 30, $t \approx z$.We can use the $Z-score$ table to find the critical value(s).

2.1 Testing Against Two-Sided Alternatives

Consider the wage equation :

$$\log(wage) = \beta_0 + \beta_1 educ + \beta_2 \exp er + \beta_3 tenure + u.$$

Suppose we want to test whether experience has a partial effect on wage:

$$\begin{aligned} H_0 &: \beta_2 = 0 \quad \text{(experience has no partial effect)} \\ H_a &: \beta_2 \neq 0. \text{ (experience has a partial effect)} \end{aligned}$$

** Note that

Suppose $n = 34$, degree of freedom $= 34 - 3 - 1 = 30$.
From the table t-table, the 5% critical value for a 2-tailed test with 30 d.f. is _ _ _ _ _ _ _ _ _

- If we change the significance level to 10%, then the 2-tailed critical value becomes _ _ _ _ _ _ _ _ _
- If we cannot reject $H_0 : \beta_2 = 0$ (at a given significance level – e.g. 5%), we say that X_2 is statistically significant at the 5% level.
- In other words X_2 has a partial effect on the expected value of Y.

```
. regress log_wage educ exper tenure

      Source |       SS       df       MS              Number of obs =     526
-------------+------------------------------           F(  3,   522) =   80.39
       Model |  46.8741776     3  15.6247259           Prob > F      =  0.0000
    Residual |  101.455574   522  .194359337           R-squared     =  0.3160
-------------+------------------------------           Adj R-squared =  0.3121
       Total |  148.329751   525   .28253286           Root MSE      =  .44086

------------------------------------------------------------------------------
    log_wage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        educ |    .092029   .0073299    12.56   0.000     .0776292    .1064288
       exper |   .0041211   .0017233     2.39   0.017     .0007357    .0075065
      tenure |   .0220672   .0030936     7.13   0.000     .0159897    .0281448
       _cons |   .2843595   .1041904     2.73   0.007     .0796756    .4890435
------------------------------------------------------------------------------
```

2.2 Testing Against One-Sided Alternatives

Suppose the rule for rejecting H_0 becomes

$$H_a : \beta_j > 0.$$

We could have the H_0 as

$$\begin{aligned} H_0 &: \beta_j \leq 0 \qquad \text{or} \\ H_0 &: \beta_j = 0 \end{aligned}$$

depending on the context. For $H_0 : \beta_j = 0$, it implies that we are ruling out population values of β_j less than zero. For $H_0 : \beta_j \leq 0$,it implies that we are not ruling out population values of β_j less than zero. To given an example of a test against the one-sided alternative hypothesis, consider the wage equation again :

$$\log(wage) = \beta_0 + \beta_1 educ + \beta_2 \exp er + \beta_3 tenure + u$$

If we want to test whether experience has a positive partial effect on wage

$$\begin{aligned} H_0 &: \beta_2 = 0 \\ H_a &: \beta_2 > 0. \end{aligned}$$

We implicitly rule out the possibility that $\beta_2 < 0$ here.

3 Testing other hypotheses about β_j

- Most common H_0 is $H_0 : \beta_j = 0$.

- However, we can test other types of hypotheses. For example, $H_0 : \beta_j = a_j$ where a_j is a constant number.

If we want to test the H_0 using a 5% significance level, then we reject H_0 if

$$t_{wife\_income} > \_\_\_\_\_\_\_ \quad or$$
$$t_{wife\_income} < \_\_\_\_\_\_\_$$

4 Testing Hypotheses about a Single Linear Combination of the Parameter

Consider

$$\log(wage) = \beta_0 + \beta_1 jc + \beta_2 univ + \beta_3 \exp er + u$$

where jc = number of years attending a two-year college
$univ$ = number of years at a four-year college
$\exp er$ = months in the workforce.
We want to test whether $\beta_1 = \beta_2$.

another possible hypothesis test (one-tailed alternative)

5 Computing p-Values for t-Tests

- What is the significance level given the computed t-statistics?

- p-value : $P(|T| > |t|)$

Example 1: $H_0 : \beta_j \geq 0, \quad H_a : \beta_j < 0, \quad$ d.f.= 140.

suppose the calculated $t_{\hat{\beta}_j} = -2.75$

- From the z-table, the value -2.75 corresponds to area = _ _ _ _ _ _ _ _ _ _ _ _ _.
- Thus, p-value = _ _ _ _ _ _ _ _ _ _.
- Would we reject H_0 if we use the significance level = 5%?

Example 2: $H_0 : \beta_j = a_j, \quad H_a : \beta_j \neq a_j, \quad$ d.f.= 18.

suppose the calculated $t_{\hat{\beta}_j} = -2.18$

- From the t-table, the value -2.18 corresponds to area = _ _ _ _ _ _ _ _ _ _ _ _ _.
- Thus, p-value = _ _ _ _ _ _ _ _ _ _.
- Would we reject H_0 if we use the significance level = 5%?

6 Confidence Intervals (CI)

- Confidence Intervales for the POPULATION PARAMETER (β_j)
- A 95% CI of β_j is given by

Example 1: **95% CI**

Example 2: **99% CI**

7 Testing Multiple Linear Restrictions: The F-test

Suppose the model is specified by

$$\begin{aligned} y &= \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + u \\ H_0 &: \beta_1 = 0 \text{ and } \beta_2 = 0 \\ H_1 &: H_0 \text{ is not true} \end{aligned}$$

We can use the F-test to test this type of "multiple hypotheses".

1. Our full model is called the "unrestricted" model (ur). Suppose it can be expressed as:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + ... + \beta_k x_k + u$$

2. The model which takes out x (which we think its associated $\beta = 0$) is called the restricted model (r).

3. Some useful facts

4. Other ways to calculate the F-statistics:

Example: Suppose we are interested in understanding the determinant of a baseball player's salary.

$salary$	= season salary
$years$	= years in major leagues
$gamesyr$	= games per year in the league
$bavg$	= career batting average
$hrunsyr$	= homeruns per year
$rbisyr$	= runs batted in per year

- the unrestricted model (ur) is defined by

```
. regress log_salary years gamesyr bavg hrunsyr rbisyr

      Source |       SS       df       MS              Number of obs =     353
-------------+------------------------------           F(  5,   347) =  117.06
       Model |  308.989208     5  61.7978416           Prob > F      =  0.0000
    Residual |  183.186327   347  .527914487           R-squared     =  0.6278
-------------+------------------------------           Adj R-squared =  0.6224
       Total |  492.175535   352  1.39822595           Root MSE      =  .72658

------------------------------------------------------------------------------
  log_salary |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       years |   .0688626   .0121145     5.68   0.000     .0450355    .0926898
     gamesyr |   .0125521   .0026468     4.74   0.000     .0073464    .0177578
        bavg |   .0009786   .0011035     0.89   0.376    -.0011918     .003149
     hrunsyr |   .0144295    .016057     0.90   0.369    -.0171518    .0460107
      rbisyr |   .0107657    .007175     1.50   0.134    -.0033462    .0248776
       _cons |   11.19242   .2888229    38.75   0.000     10.62435    11.76048
------------------------------------------------------------------------------
```

- the restricted model (r) is defined by

```
. regress log_salary years gamesyr

      Source |       SS       df       MS              Number of obs =     353
-------------+------------------------------           F(  2,   350) =  259.32
       Model |  293.864058     2  146.932029           Prob > F      =  0.0000
    Residual |  198.311477   350  .566604221           R-squared     =  0.5971
-------------+------------------------------           Adj R-squared =  0.5948
       Total |  492.175535   352  1.39822595           Root MSE      =  .75273

------------------------------------------------------------------------------
  log_salary |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       years |    .071318    .012505     5.70   0.000     .0467236    .0959124
     gamesyr |   .0201745   .0013429    15.02   0.000     .0175334    .0228156
       _cons |    11.2238    .108312   103.62   0.000     11.01078    11.43683
------------------------------------------------------------------------------
```

Now, our H_0 and H_a becomes

8 How the Hypothesis Testing is done in Practice

1. Check the values of $t - statistic$ reported by the statistical software (i.e. STATA, SPSS, SAS)

   $\Rightarrow$ These $t - statistics$ are to test $H_0 : \beta_i = 0$

   $\Rightarrow$ If the d.f. > 30, then when $t \quad > 1.96$, we can reject H_0

   $\Rightarrow$ **When** $t \quad >$ **1.96**, we can say that β_i is **statistically significant** at 5% level. (value of $\beta_i \neq 0$)

   $\Rightarrow$ **When** $t \quad <$ **1.96** we can say that β_i is **not statistically significant** at 5% level.

   $\Rightarrow$ If $t \quad < 1.96$ we can drop x_i from the model

   $\Rightarrow$ After we drop x_i, we estimate the new regression function and obtain a new set of $\hat{\beta}$.

2. We can also perform other hypothesis testings of interest.

   e.g. $H_0 : \beta_i = \beta_j$

   or $\quad H_0 : \beta_i = 5 \quad$ etc.

   or perform an F-test for testing multiple linear restrictions

3. Usually, in economics, the estimation results are reported using this form

Dependent Variable: log(*salary*)			
Independent Variables	**(1)**	**(2)**	**(3)**
log(*sales*)	.224 (.027)	.158 (.040)	.188 (.040)
log(*mktval*)	——	.112 (.050)	.100 (.049)
profmarg	——	−.0023 (.0022)	−.0022 (.0021)
ceoten	——	——	.0171 (.0055)
comten	——	——	−.0092 (.0033)
intercept	4.94 (0.20)	4.62 0.25)	4.57 (0.25)
Observations *R*-squared	177 .281	177 .304	177 .353