GAME THEORY (A QUICK REVIEW)

P → PLAYERS $N \geq 2$

A → ACTIONS OR STRATEGIES

P → PAYOFFS OR OUTCOMES RECEIVED BY PLAYERS

I → INFORMATION → ORDERS OF THE PLAY < SIMULTANEOUS MOVE / SEQUENTIAL MOVE

→ RULES OF THE GAME

→ RATIONALITY OF PLAYERS

→ HISTORY OF THE GAME

- COURNOT (Q) → SIMULTANEOUS MOVE GAME
- STACKELBERG (Q) → SEQUENTIAL MOVE GAME
- BERTRAND (P) → SIMULTANEOUS MOVE GAME

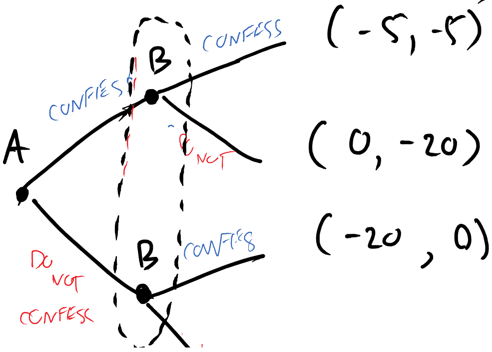
		B: CONFESS	B: DO NOT CONFESS
A	CONFESS	$(-5^{*}, -5^{*})$	$(0^{*}, -20)$
A	DO NOT CONFESS	$(-20, 0^{*})$	$(-1, -1)$

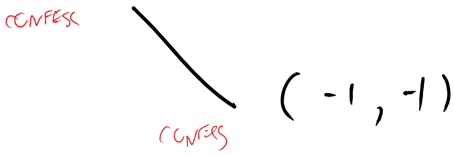
PARETO INFERIOR OUTCOME

SO N-E : $(s_A^*, s_B^*) =$ (CONFESS, CONFESS)

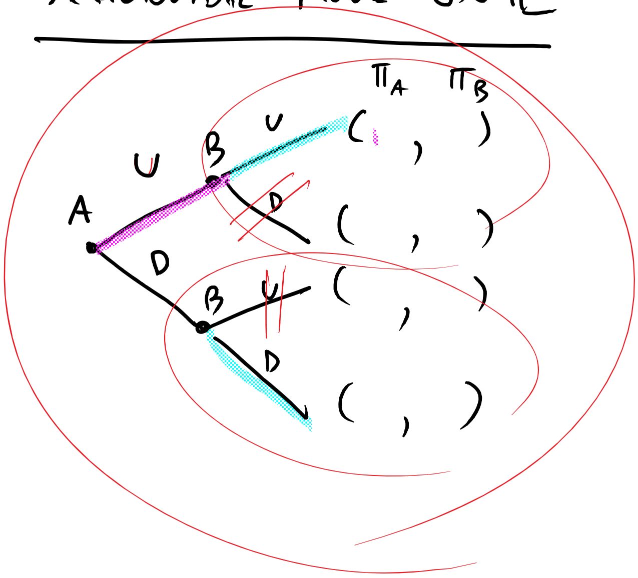
$$\left(\pi_A^*(s_A^*, s_B^*),\ \pi_B^*(s_A^*, s_B^*)\right) = (-5, -5)$$

$\pi_A \quad \pi_B$





SEQUENTIAL-MOVE GAME



A : FIRST MOVER

B : FOLLOWER

WE SOLVE THIS GAME FOR A NASH EQUILIBRIUM BY USING "BACKWARD INDUCTION"

: THINKING AHEAD AND REASONING BACKWARD.

STATIC OLIGOPOLY MODELS

- COURNOT MODEL (1838)
- BERTRAND MODEL OF PRICE COMPETITION (1883)
- STACKELBERG MODEL

COURNOT (1838)

ASSUMPTIONS

1. CONSUMERS ARE PRICE TAKERS
2. HOMOGENEOUS PRODUCTS
3. NO ENTRY

CHOICE : LEVEL OF OUTPUT / QUANTITY / SALE VOLUMES

SETUP

1. 2 IDENTICAL FIRMS W/ NO ENTRY
2. ONE-SHOT GAME
3. HOMOGENEOUS GOODS WHERE $q_1 + q_2 = Q$.
4. $P = A - BQ$: MARKET DEMAND FUNCTION
   $P = A - B(q_1 + q_2)$
   $P = A - Bq_1 - Bq_2$.
5. EACH FIRM HAS A CONSTANT AND EQUAL $MC = c$.
6. EACH FIRM CHOOSES ITS OWN OUTPUT SIMULTANEOUSLY.

COURNOT-NASH EQUILIBRIUM : FOR q_1^c AND q_2^c TO BE A NASH EQUILIBRIUM QUANTITY, 2 FOLLOWING CONDITIONS MUST BE TRUE :

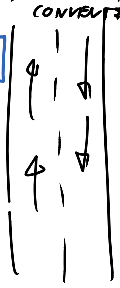
$$\pi_1(q_1^c, q_2^c) \geq \pi_1(q_1, q_2^c) \quad \text{FOR ANY } q_1 \quad (1)$$

$$\pi_2(q_1^c, q_2^c) \geq \pi_2(q_1^c, q_2) \quad \text{FOR ANY } q_2 \quad (2)$$

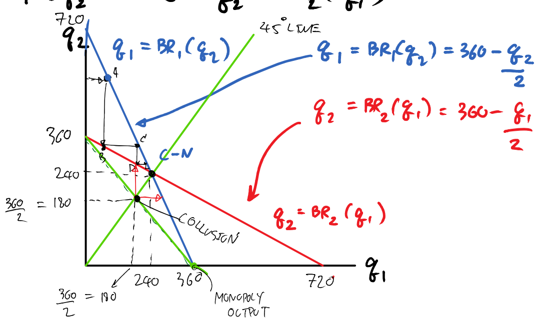
C-N EQUILIBRIUM CAN BE DERIVED BY USING "BEST-RESPONSE FUNCTIONS."

SOCIAL STABLE CONVENTION.

$\boxed{Q = 1000 - 1000P}$



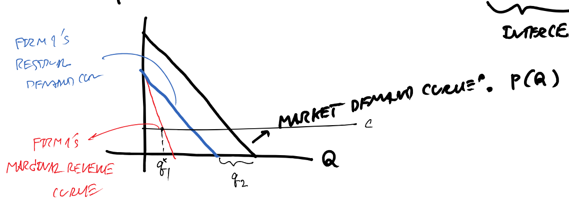
$$q_1^c = BR_1(q_2^c) \quad \& \quad q_2^c = BR_2(q_1^c)$$



CONSIDER A MARKET DEMAND CURVE : $P(Q) = A - BQ$
WHERE $Q = q_1 + q_2$

COST OF PRODUCTION : $C = c \cdot q_i$ WHERE $i = 1, 2$

- FIRM 1's RESIDUAL DEMAND CURVE : $P(q_1, q_2) = \underbrace{(A - Bq_2)}_{\text{INTERCEPT}} - Bq_1$



- FIRM 1's MR CURVE : $MR(q_1, q_2) = (A - Bq_2) - 2Bq_1$.
- PROFIT MAXIMIZING OUTPUT FOR FIRM 1 : $A - Bq_2 - 2Bq_1 = c$

$$q_1 = \frac{A - Bq_2 - c}{2B}$$

BY SYMMETRY, $\boxed{q_2 = \frac{A - Bq_1 - c}{2B}}$

2-EQUATION & 2-UNKNOWN, WE CAN SOLVE FOR q_1^c, q_2^c

$$\boxed{q_1^c = q_2^c = \frac{A-c}{3B}}$$ C-N EQUILIBRIUM QUANTITIES

$$\boxed{Q = q_1^c + q_2^c = 2 \cdot \left(\frac{A-c}{3B}\right)}$$

$$P = A - BQ = A - \cancel{B} \cdot 2\left(\frac{A-c}{3\cancel{B}}\right)$$

$$\boxed{P^c = \frac{A+2c}{3}}$$

$$\pi_i = P \cdot q_i - c \cdot q_i$$

$$\boxed{\pi_i^c = \frac{(A-c)^2}{9B}}$$

PROPERTIES OF C-N EQUILIBRIUM

RECALL THAT $TR = P(Q) \cdot Q$

$$\boxed{MR = \frac{\partial TR}{\partial Q} = P(Q) + Q \cdot \frac{\partial P(Q)}{\partial Q}}$$ OR $P(Q) + Q \frac{dP(Q)}{dQ}$

GIVEN THIS KNOWLEDGE, $MR(q_1, q_2) = P(q_1, q_2) + \frac{dP(q_1, q_2)}{dQ} \cdot q_1 = MC(q_1)$

LET'S DIVIDE BOTH SIDES BY $P(q_1^c, q_2^c)$:

$$\frac{P(q_i^c, q_j^c) - MC(q_i)}{P(q_i^c, q_j^c)} = -\frac{dP(q_i^c, q_j^c)}{dQ} \cdot q_i \cdot \frac{Q^c}{Q^c} \cdot \frac{1}{P(q_i, q_j)}$$

$$\frac{P - MC}{P} = -\frac{1}{\varepsilon} \cdot \frac{q_i}{Q^c}$$

$$\boxed{\frac{P - MC}{P} = -\frac{1}{\varepsilon} \cdot s_i}$$