EE312 Macroeconomic Theory

Chapter 8

A Real Intertemporal Model

with Investment (Part I)

State of your knowledge

- In the previous two chapters, we discussed about the two basic settings.

  Model 1 - Static **consumption-leisure** equilibrium: **static production economy**

  Model 2 - Intertemporal **consumption-saving** model: **Intertemporal pure endowment economy**

Real Intertemporal Model

- This chapter takes you another step closer to the reality.
- We **merge together** the two basic models (model 1 and model 2), and then newly introduce an **"investment" decision problem** as an extension.
- This basically gives rise to a **real intertemporal model.**
  - Model 1 + Model 2 + Investment problem

Real Intertemporal Model

- Why do we do this?
  - Our goal is to build **a short-term business cycle model** that can be used to understand the transmission mechanism of "real shocks" on aggregate fluctuations.
  - This class of business cycles model is often know as the **"real business cycles model"**
    - Kyland and Prescott (1981) / Long and Plaster (1982)

Real Intertemporal Model: agents

- The **real model** (*no money*) with **three sectors** as the basis for the analysis of short-term economic fluctuations.
  - ***Representative consumer*** (consumption, labor supply and savings)
  - ***Representative firm*** (production, labor demand and ***investment***)
  - **Government** (spending, taxes and borrowing)

Real Intertemporal Model: markets

- **The labor market**: the firm's demand and the consumer's supply of labor.
  - *The real wage rate*. (labor demand v.s. labor supply)
- **The output market**: the firm's supply and the consumer's demand for output.
  - *The real interest rate. (saving v.s. investment)*
- **Analysis of real exogenous shocks.**
  - Changes in government spending, capital stock, total factor productivity.

Investment

- Expenditure on plants, equipment and new housing.
  - Investment goods currently produced for future production of goods and services.
  - Increases in ***future productive capacity***.
  - **The consumer's tradeoff** between current and future consumptions (savings).
  - **The firm's tradeoff** between current profits and higher future capital stock (and future profits).

Real v.s. financial investment

- This chapter discusses only **"real investment"**.
- In general, we need to address the financial issue along with the issue on real investment.
  - This is known under the "firm financing problem".
  - Under perfect capital market, **financial issue is irrelevant**; all real investments get financed.
- Under capital market imperfection, financial issue matters to the feasible choice of real investment.
  - Some "profitable" project cannot be implemented if firms choose one mean of financing over the other.
  - Firms might be financially constrained (credit rationing) due to frictions in the financial system.

Consumer's optimal decisions

- **Work-leisure** in current and future periods.
- **Consumption-savings** in the current period.
  - h = total time available;
  - w and w' = current and future real wages;
  - r = the real interest rate;
  - T and T' = current and future lump-sum taxes;
  - C and C' = current and future consumptions;
  - L and L' = current and future leisure time;
  - Sp = private savings.

Current budget constraint

- The consumer is **a price-taker** (w, w', r and T are given).
  - w(h - L) = real-wage income;
  - π = dividend income from the firm;
  - T = lump-sum taxes paid to the government.
- Then, **disposable income** is:

$$C + S^p = w(h - l) + \pi - T$$

Future budget constraint

- The consumer still receives real wages, dividend income, and pays future taxes.
  - Also the principal and interest on savings.
  - No bequests; all wealth is consumed.

$$C' = w'(h - l') + \pi' - T' + (1+r)S^p$$

Lifetime budget constraint

$$C + \frac{C'}{1+r} = w(h-l) + \pi - T + \frac{w'(h-l') + \pi' - T'}{1+r}$$

- The PV of **lifetime consumption** equals the PV of **lifetime disposable income**.
- Decision on the optimal bundles of C, C', L, and L' subject to the lifetime budget constraint.

Current period optimal condition

$$MRS_{l,C} = w$$

- The consumer chooses the optimal bundle of current leisure and consumption:
  - **The marginal rate of substitution of current leisure for current consumption** is equal to the real wage.
  - **w** is the relative price of leisure in terms of consumption goods.

Future period optimal condition

$$MRS_{l',C'} = w'$$

- The consumer chooses the optimal bundle of future leisure and future consumption:
  - **The marginal rate of substitution of future leisure for future consumption** is equal to the future real wage.

Intertemporal optimal condition

$$MRS_{C,C'} = 1 + r$$

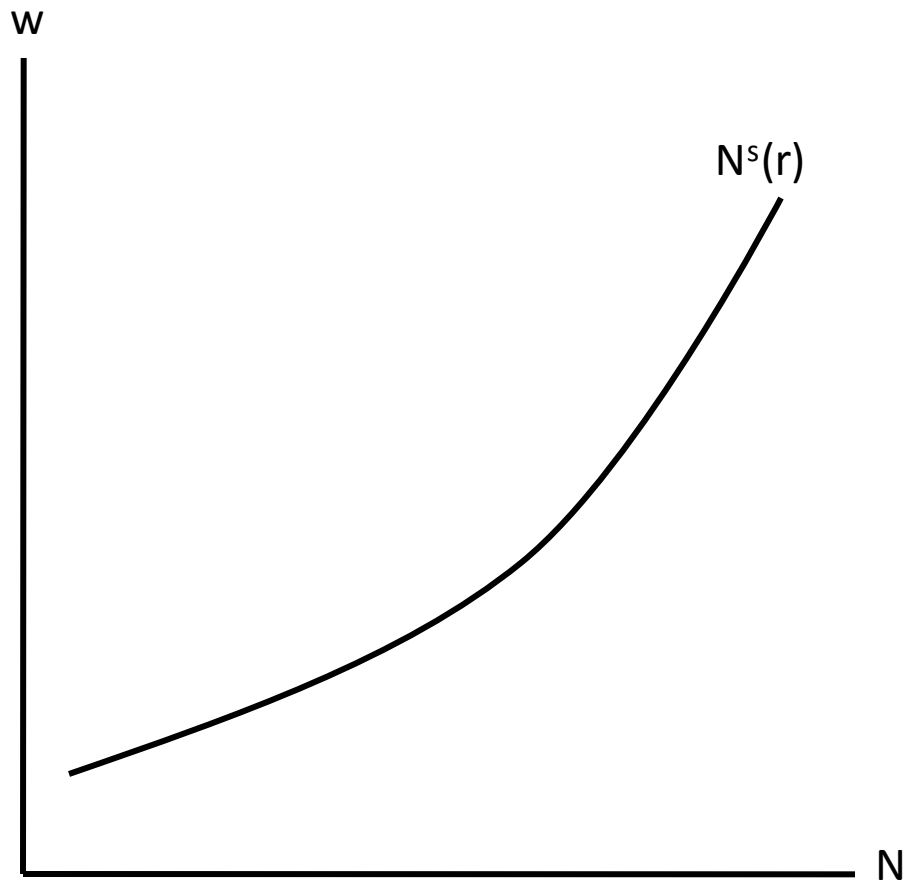
- The consumer chooses the optimal bundle of current and future consumption (savings):
  - **The marginal rate of substitution of current consumption for future consumption** is equal to the real interest rate.
  - **(1 + r)** is the relative price of current consumption in terms of future consumption.

Current labor supply

- The consumer provides labor supply to the firm through work-leisure decision.
- Factors which determine current labor supply:
  - The current real wage;
  - The real interest rate;
  - Lifetime wealth.

Current labor supply curve

- Current labor supply increases with the real wage, given r (assuming the dominant substitution effect).

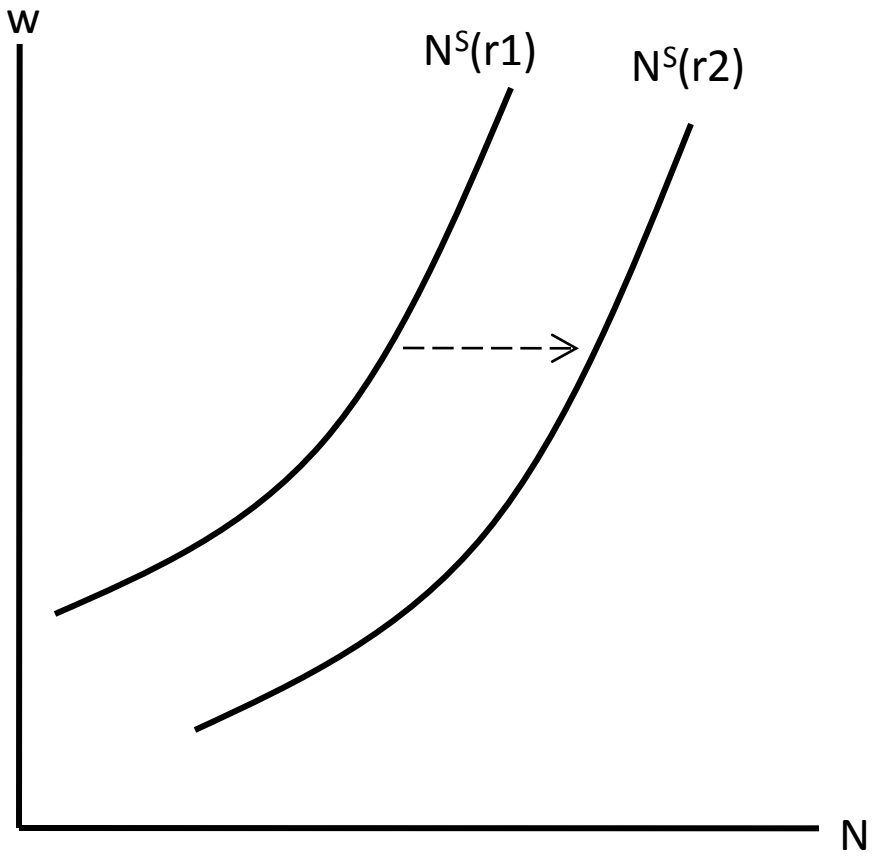


An increase in the real interest rate

- Current labor supply increases as the real interest rate increases.
  - **w(1+r)/w'** is the relative price of current leisure in terms of future leisure.
  - Given w and w', a higher r means the higher price of current leisure in terms of future leisure.
  - Less current leisure, and more current supply of labor, assuming the dominant substitution effect.

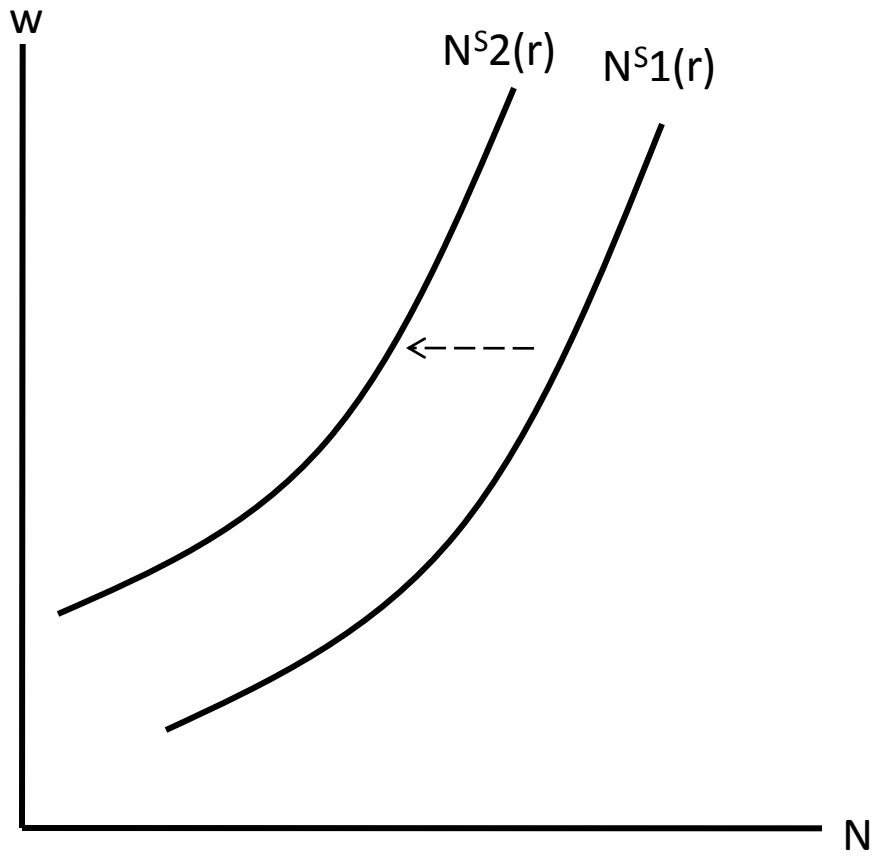
Labor supply increases with r

- Given w, labor supply increases with the rising real interest rate (r2 > r1), assuming the dominant substitution effect.



An increase in lifetime wealth

- Current leisure increases and current labor supply decreases with rising lifetime wealth.
  - Current and future consumptions also increase.



Demand for current consumption goods

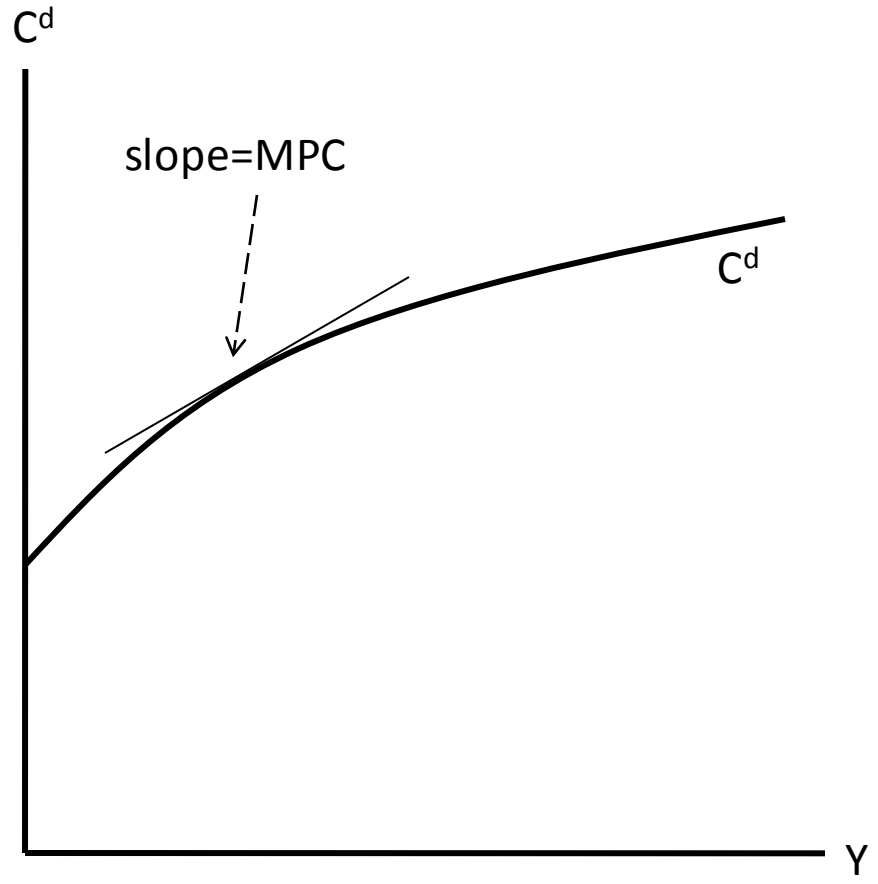
- **The individual demand for current consumption goods (C^d)** is a function of current income (Y), given r.
  - The marginal propensity to consume (MPC) < 1 .
- A higher real interest rate (r) causes the demand to fall, assuming:
  - The substitution effect dominates the income effect; the consumer is a lender.

Demand for current consumption

$$C^d = f(Y, r)$$

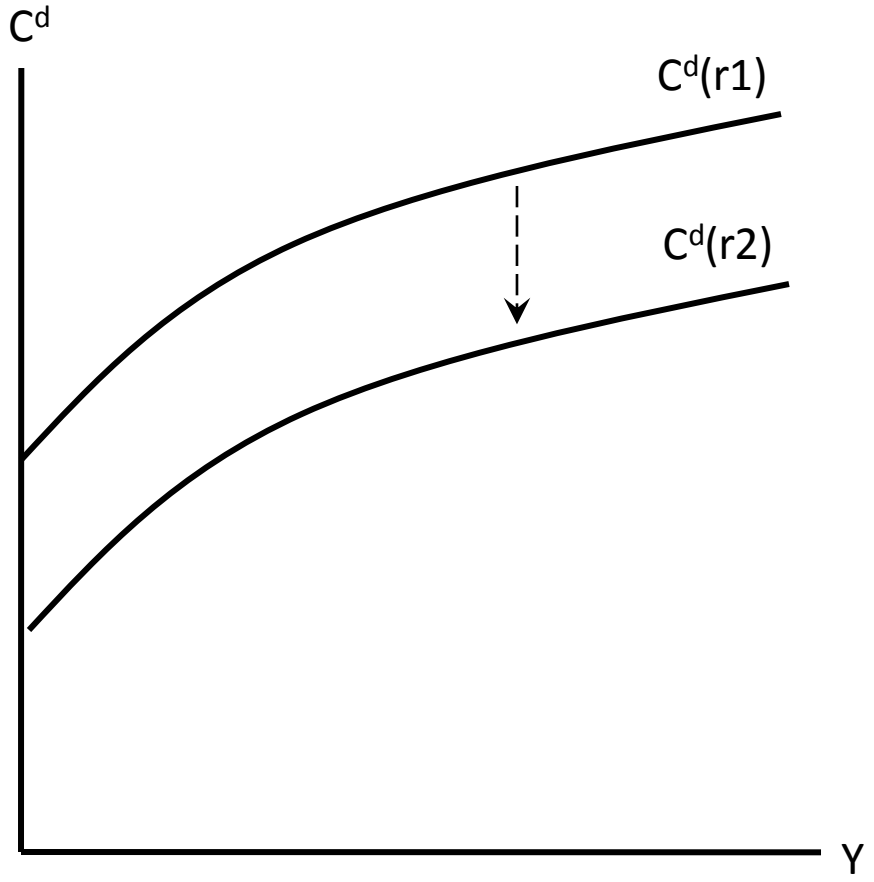
$$MPC = \frac{\partial C^d}{\partial Y} < 1;$$

$$\frac{\partial C^d}{\partial r} < 0; \frac{\partial C^d}{\partial we} > 0$$



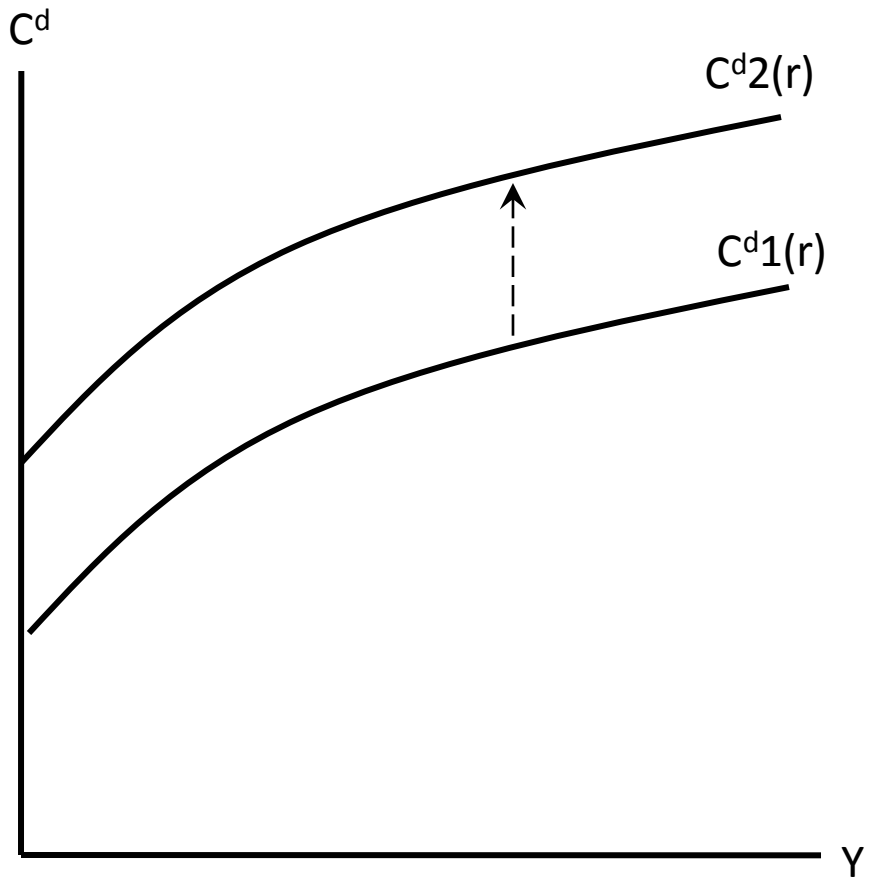
A higher r reduces C^d

- r2 > r1.
- The consumer reduces current consumption, assuming stronger substitution effect and a lender.



An increase in wealth raises C^d

- An increase in lifetime wealth raises current consumption.



Representative firm

- Optimal decisions on:
  - Maximized present value of profits;
  - The level of current labor inputs.
  - **Optimal investment level**: tradeoff between current profits and future capital stock (and future profits).

Current production function

$$Y = zF(K, N)$$

- Y = current output;
- z = current total factor productivity;
- K = current capital stock;
- N = current labor input.
- And the future production function:

$$Y' = z'F(K', N')$$

Change in capital stock

- The firm's investment is foregone current profits (consumption) for future profits:
  - d = the rate of depreciation;
  - I = current investment.
- Future capital stock is current capital stock net of depreciation plus investment.

$$K' = (1-d)K + I$$

The firm's current profits

- Maximization of the present value of current and future profits.
  - π = current profits;
  - I = current investment = foregone current consumption goods.

$$\pi = Y - wN - I$$

The firm's future profits

- The leftover capital stock in the future period can be sold off as junk value.
  - (1 - d)K' = capital stock remaining as junk at the end of the future period.

$$\pi' = Y' - w'N' + (1-d)K'$$

Maximized PV of profits

- The firm maximizes the PV of profits.
  - The same as maximized PV of dividend income for the consumer.
  - V = max. present value of profits through optimal choice on N, N’ and I:

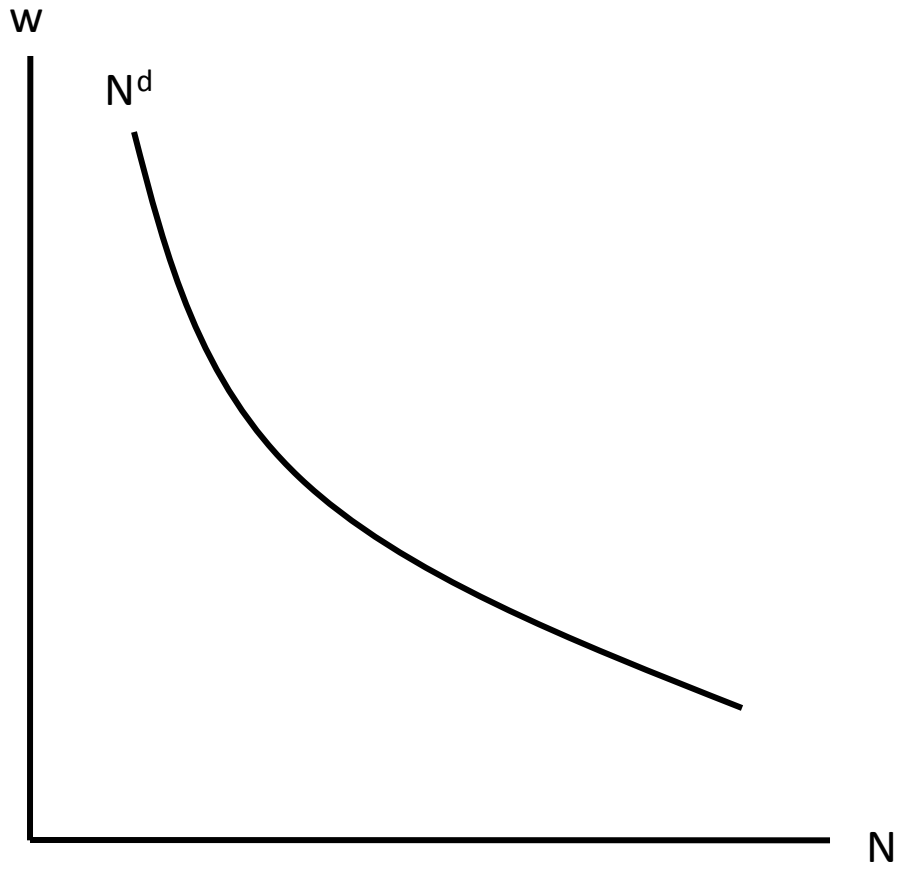
$$V = \pi + \frac{\pi'}{1+r}$$

Current labor demand

- The firm's choice of current labor demand (N^d) affects only current profits (π).
  - The firm hires current labor until the current marginal product of labor equals the current real wage ($MP_N = w$).
  - Thus the firm's MP_N curve is also **the firm's current labor demand curve**.
  - An increase in current z or K raises MP_N and current labor demand.

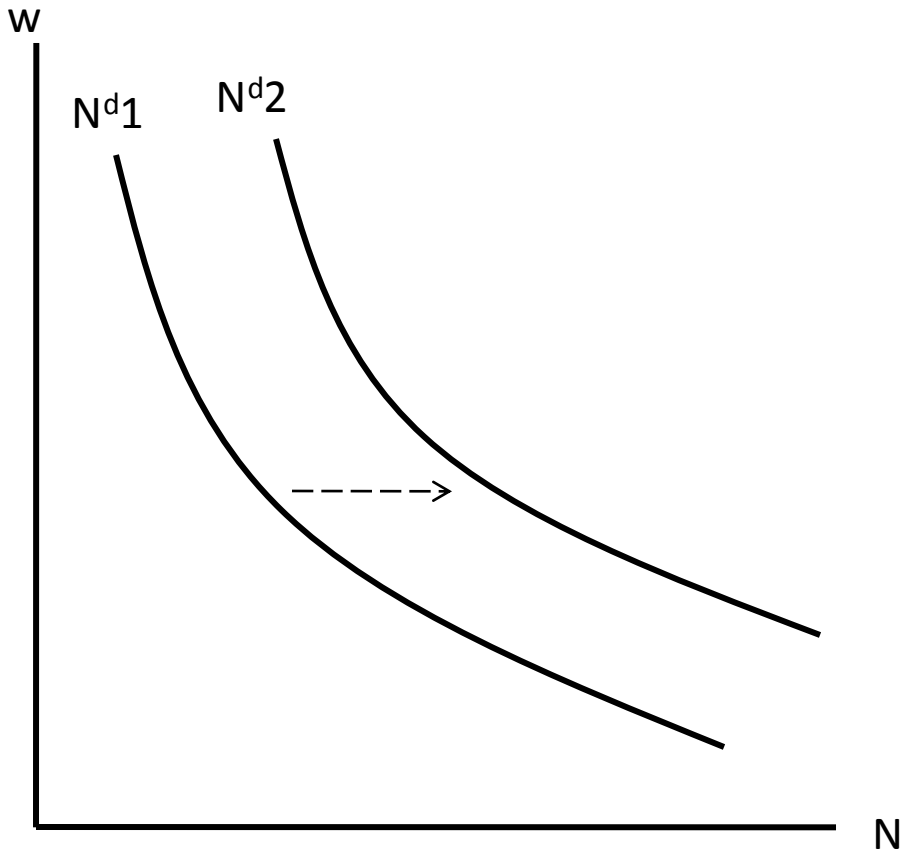
Current labor demand curve

- The current labor demand: $MP_N = w$.
- MP_N is falling as the labor input increases.



Labor demand with rising z or K

- An increase in current z or K shifts the current labor demand curve to the right.



The firm's investment decision

- The firm invests to the point where the marginal benefit from investment equals marginal cost.
- **MC(I) = marginal cost of investment** = PV of profits (V) given up for one unit of capital.
  - One unit of investment reduces current π (and V) by one unit.

$$MC(I) = 1$$

- **MB(I) = marginal benefit of investment** = additional units of V (PV of profits) received from one extra unit of current investment.
  - MP_K' = additional output from one extra unit of K'.
  - Quantity of capital left from depreciation at the end of the future period (1 - d) for liquidation.
  - I's future profits is $(MP'_K + 1 - d) = \pi'$.

$$MB(I) = \frac{MP_K' + 1 - d}{1 + r}$$

Optimal investment decision

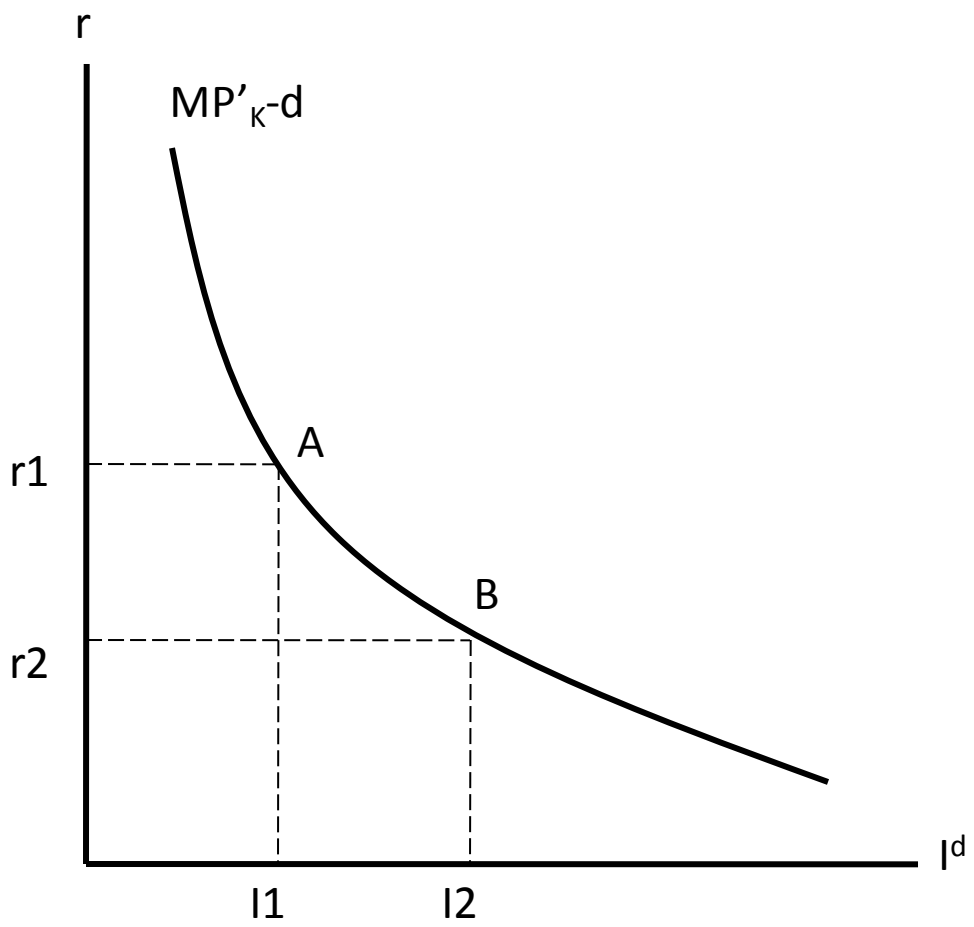
$$\frac{MP'_K + 1 - d}{1 + r} = 1$$

$$MP'_K - d = r$$

- The firm invests until the net future marginal product of capital equals the real interest rate.
- **r = the opportunity cost of more capital** = the rate of return on the alternative asset (bonds) otherwise earned by the consumer who owns the firm.

Optimal investment curve

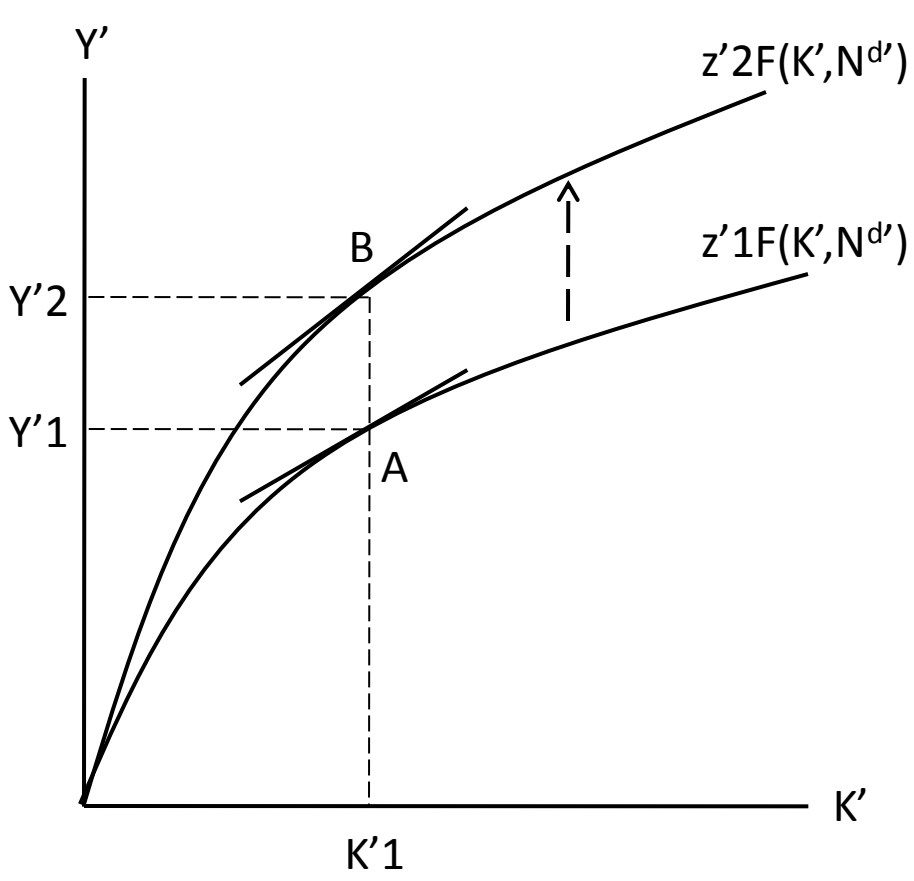
- $I^d = (MP'_K - d)$ gives the level of I required for the net MP'_K to be equal to the real interest rate, given K.

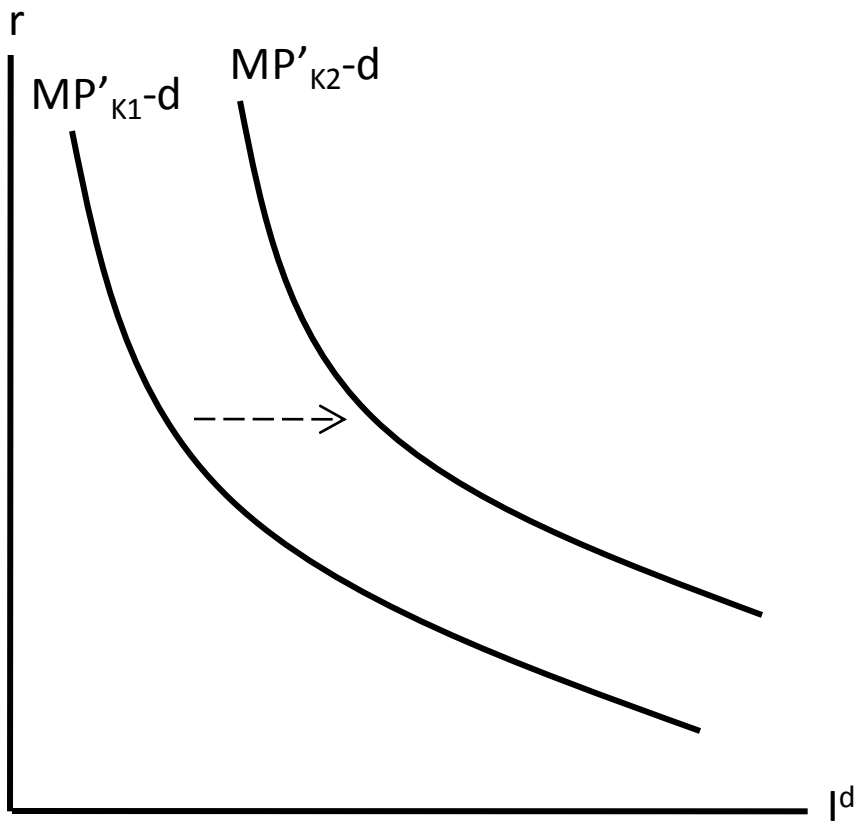


Changes in z’ and K

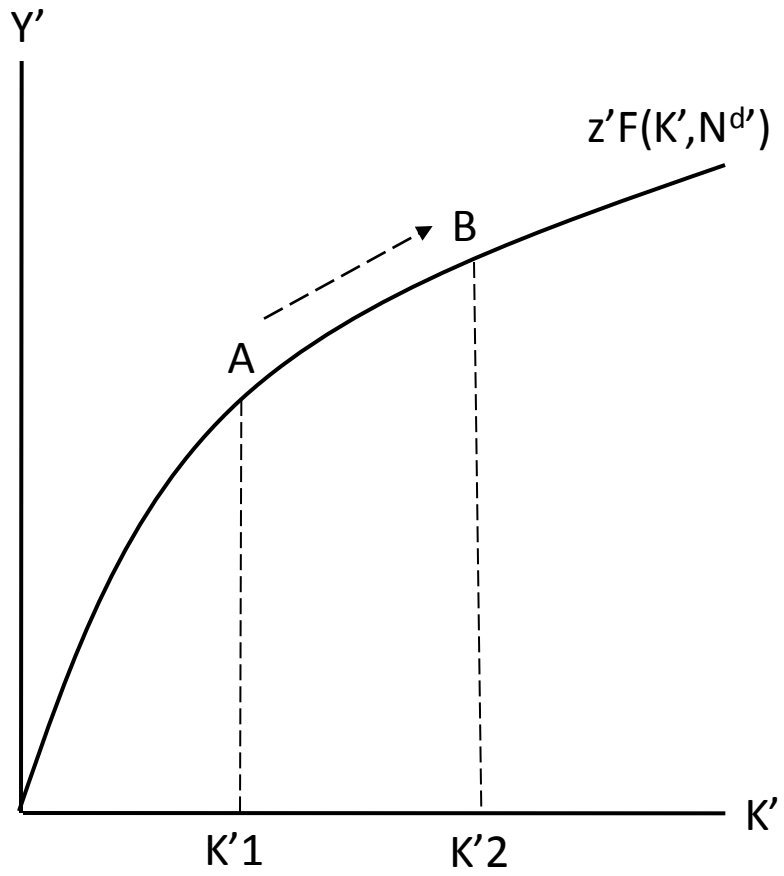
- Factors affecting future marginal product of capital shift the optimal investment curve.
- Higher **future total factor productivity** (z’) increases **future MP'_K** and current optimal investment.
  - The optimal investment curve shifts to the right.
- Higher **current capital stock** results in larger future net capital stock and lower MP'_K.
  - The optimal investment curve shift to the left.

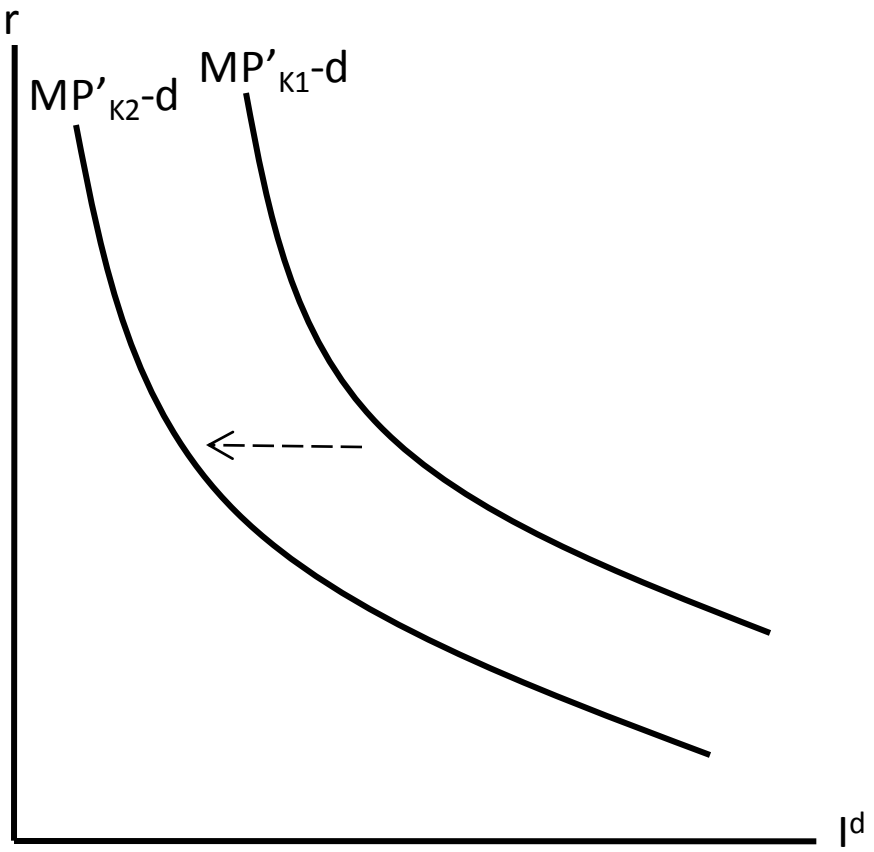
Higher z’ and MP'_K





Higher current K and lower MP'_K

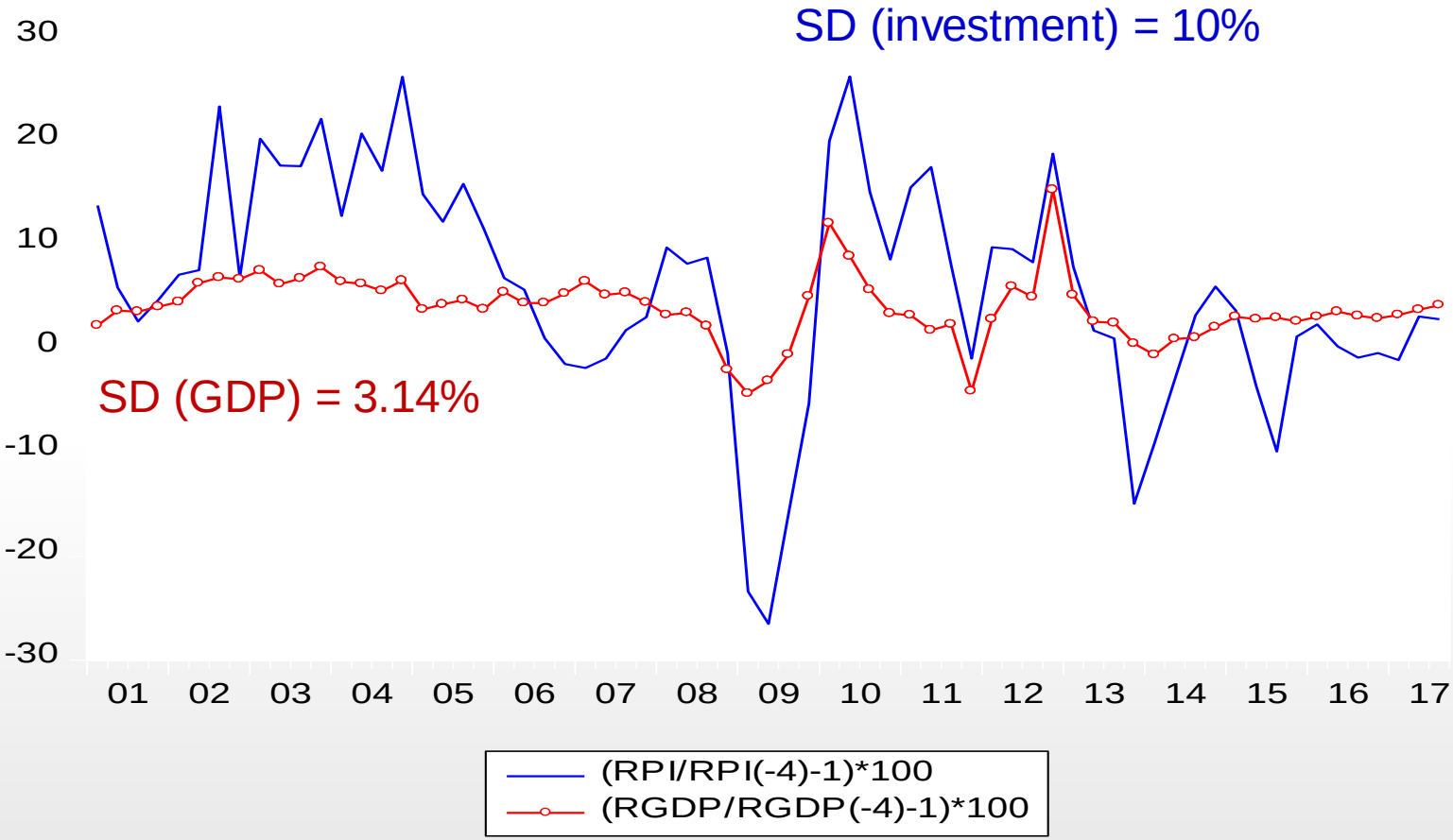




Volatile investment and GDP

- Aggregate consumption is less variable than income due to consumption smoothing.
- Investment is **much more volatile** --- short-run economic fluctuations.
  - Investment responds to perceived marginal rates of return to investment.
  - Changes in the real interest rate cause movements along the investment curve.
  - Changes in future total factor productivity shift the investment curve.

Investment and GDP growth



Government sector

- Government purchases of consumption goods (G and G') are exogenously determined.
- Government financing:
  - Current lump-sum taxes and bond sale;
  - Future lump-sum taxes and payments of the principal and interest.

$$G + \frac{G'}{1+r} = T + \frac{T'}{1+r}$$

Competitive equilibrium

- **The labor market**:
  - The consumer supplies labor service.
  - The firm demands labor service.
  - The real wage and the level of employment.
- **The output market**:
  - The consumer, the firm and government purchase output.
  - The firm supplies the goods.
  - The real interest rate and the level of aggregate output.