

### Classical Normal Linear Regression Model (CNLRM)

### Classical theory of statistical inference

- Estimation
- Hypothesis testing

### The Probability Distribution of Disturbances

$$\hat{\beta}_2 = \sum k_i Y_i$$

where  $k_i = \frac{x_i}{\sum x_i^2} = \frac{(X_i - \bar{X})}{\sum (X_i - \bar{X})^2}$

$$Y_i = \beta_1 + \beta_2 X_i + u_i$$

$$\hat{\beta}_2 = \sum k_i (\beta_1 + \beta_2 X_i + u_i)$$

### The Normality Assumption for

The classical normal linear regression model assumes that each  $u_i$  is distributed normally with

$$E(u_i) = 0$$

$$E[u_i - E(u_i)]^2 = E(u_i^2) = \sigma^2$$

$$E\{(u_i - E(u_i))[u_j - E(u_j)]\} = E(u_i u_j) = 0 \quad i \neq j$$

$$u_i \sim N(0, \sigma^2)$$

Where **N** stands for normal distribution

$$u_i \sim NID(0, \sigma^2)$$

Where **NID** stands for normally and independent distributed

### Properties of OLS Estimators under the Normality Assumption

- Unbiased
- Minimum variance unbiased or efficient estimators
- Consistency
  - Sample size increases  $\rightarrow$  the estimators converge to their true population values

### Properties of OLS Estimators under the Normality Assumption

- $\hat{\beta}_1$  is normally distributed with

$$\text{Mean: } E(\hat{\beta}_1) = \beta_1$$

$$\text{var}(\hat{\beta}_1): \sigma_{\hat{\beta}_1}^2 = \frac{\sum X_i^2}{n \sum x_i^2} \sigma^2$$

$$\hat{\beta}_1 \sim N(\beta_1, \sigma_{\hat{\beta}_1}^2)$$

### Properties of OLS Estimators under the Normality Assumption

By the properties of the normal distribution, the variable Z,

$$Z = \frac{\hat{\beta}_1 - \beta_1}{\sigma_{\hat{\beta}_1}}$$

follows the standard normal distribution, that is, a normal distribution with zero mean and unit variance

$$Z \sim N(0,1)$$

### Properties of OLS Estimators under the Normality Assumption

- $\hat{\beta}_2$  is normally distributed with

$$\text{Mean: } E(\hat{\beta}_2) = \beta_2$$

$$\text{var}(\hat{\beta}_2): \sigma_{\hat{\beta}_2}^2 = \frac{\sigma^2}{\sum x_i^2}$$

$$\hat{\beta}_2 \sim N(\beta_2, \sigma_{\hat{\beta}_2}^2)$$

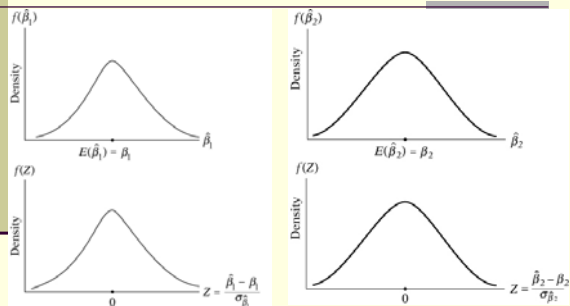
### Properties of OLS Estimators under the Normality Assumption

By the properties of the normal distribution, the variable Z,

$$Z = \frac{\hat{\beta}_2 - \beta_2}{\sigma_{\hat{\beta}_2}}$$

follows the standard normal distribution

### The probability distributions of $\hat{\beta}_1$ and $\hat{\beta}_2$



### Properties of OLS Estimators under the Normality Assumption

- $(n-2)(\hat{\sigma}^2 / \sigma^2)$  is distributed as the  $\chi^2$  (chi square) distribution with (n-2) degree of freedom
- $(\hat{\beta}_1, \hat{\beta}_2)$  are distributed independently of  $\hat{\sigma}^2$
- $\hat{\beta}_1$  and  $\hat{\beta}_2$  have minimum variance in the entire class of unbiased estimators, whether linear or not