Quiz 1: Date: April 19, 2022 from 11.00-12.30

Question 1 (10 Points) Score.....................

Consider the one-period model of consumption and portfolio choice. Let an individual in this economy has the utility function as follow:

$$U(C) = ln(C)$$

Also, let $\frac{C_1}{C_0}$ is distributed as log-normal with mean equals μ_c and its variance is σ_c.
Please read and answer the following questions carefully and completely.

Score.....................

Question 1.1 (10 marks) Calculate the risk free rate R_f in terms of the individual's consumption, C_0 and C_1. Then,explain the relationship between the level of consumption and the risk free rate in this economy.

$$\frac{1}{R_f} = E[m_{01}]$$

$$\frac{1}{R_f} = E\left[\frac{\delta U'(C_1)}{U'(C_0)}\right]$$

$$\frac{1}{R_f} = E\left[\delta \frac{C_0}{C_1}\right]$$

$$\frac{1}{R_f} = \delta \frac{C_0}{C_1}$$

$$R_f = \frac{1}{\delta}\frac{C_1}{C_0}$$

when R_f is high, so the high expected growth in consumption.

Score.....................

Question 1.2 (10 marks) Calculate the elasticity of intertemporal substitution in this setting. If in the next year, the interest rate is falling, Will the individual's consumption level increase or decrease? Why? To support your answer, use the concepts of income effect and substitution effect.

$$\varepsilon = \frac{R_f}{\frac{C_1}{C_0}} \frac{\partial \frac{C_1}{C_0}}{\partial R_f}$$

FOC:
$$1 = \delta\left(\frac{C_1}{C_0}\right)^{-1} \pi_s R_f$$

$$0 = -\delta\left(\frac{C_1}{C_0}\right)^{-2} \pi_s R_f d\left(\frac{C_1}{C_0}\right) + \delta\left(\frac{C_1}{C_0}\right)^{-1} \pi_s \, dR_f$$

$$0 = \delta\pi_s\left(-\left(\frac{C_1}{C_0}\right)^{-2} R_f d\frac{C_1}{C_0} + \left(\frac{C_1}{C_0}\right)^{-1} dR_f\right)$$

$$\cancel{-}\left(\frac{C_1}{C_0}\right)^{-1} dR_f = \cancel{-}\left(\frac{C_1}{C_0}\right)^{-2} R_f \, d\frac{C_1}{C_0}$$

$$\left(\frac{C_1}{C_0}\right)^{-1+2} = R_f \frac{d\frac{C_1}{C_0}}{dR_f}$$

$$1 = \frac{R_f}{\frac{C_1}{C_0}} \frac{d\frac{C_1}{C_0}}{dR_f}$$

$$\varepsilon^I = 1$$

$\therefore$ $\varepsilon^I = 1$: when R_f increases 1% that cause increasing in future consumption growth of 1% and vice versa.

In this case, SE = IE due to $\varepsilon = 1$

SE : When interest rate is falling, people tend to save less

IE : When having lower return, people would decrease consumption in both periods.

Score.....................

Question 1.3 (10 marks) Solve for the pricing kernel P_i of any risky asset i in this economy. Then explain the meaning of this pricing kernel.

$$P_i = E[m_{01} X_i]$$

$$P_i = E\left[\frac{\delta C_0}{C_1} X_i\right]$$

∴ At good state where C_1 is high, the marginal utility of C_1 or $U'(C_1)$ will be low so that the asset's payoffs are not highly valued that means P_i will be small and investors require high rate of return.

However, at bad state where C_1 is low, the marginal utility of C_1 or $U'(C_1)$ will be high so that the asset's payoffs are much desired that means P_i will be high and investors require low rate of return.

Score.....................

Question 1.4 (10 marks) Calculate Hansen-Jaganathan Bound and explain the meaning.

$$\frac{\sigma_{m01}}{E[m_{01}]} = \frac{\sqrt{Var\ e^{(-1)\ln\frac{c_1}{c_0}}}}{E\left[e^{(-1)\ln\frac{c_1}{c_0}}\right]}$$

$$= \frac{\sqrt{E\left[e^{2(-1)\ln\frac{c_1}{c_0}}\right] - E\left[e^{(-1)\ln\frac{c_1}{c_0}}\right]^2}}{E\left[e^{(-1)\ln\frac{c_1}{c_0}}\right]}$$

$$= \sqrt{E\left[e^{2(-1)\ln\frac{c_1}{c_0}}\right] / E\left[e^{(-1)\ln\frac{c_1}{c_0}}\right]^2 - 1}$$

$$= \sqrt{e^{2(-1)\mu_c + 2(-1)^2\sigma_c^2} / e^{2(-1)\mu_c + (-1)^2\sigma_c^2} - 1}$$

$$= \sqrt{e^{\sigma_c^2} - 1}$$

$$= \sigma_c$$

$$\therefore \left|\frac{E[R_i] - R_f}{\sigma_{R_i}}\right| \leq \sigma_c$$

It means that the sharpe ratio of any assets will be less than or equal to the standard deviation of consumption growth (σ_c).