CHAPTER 7

AN INTRODUCTION TO PORTFOLIO MANAGEMENT

Answers to Questions

1. Investors hold diversified portfolios in order to reduce risk, that is, to lower the variance of the portfolio, which is considered a measure of risk of the portfolio. A diversified portfolio should accomplish this because the returns for the alternative assets should not be correlated so the variance of the total portfolio will be reduced.

2. The covariance is equal to $\Sigma E[(R_i - E(R_i))(R_j - E(R_j))]$ and shows the absolute amount of comovement between two series. If they constantly move in the same direction, it will be a large positive value and vice versa. Covariance is important in portfolio theory because the variance of a portfolio is a combination of individual variances and the covariances among all assets in the portfolio. It is also shown that in a portfolio with a large number of securities the variance of the portfolio becomes the average of all the covariances.

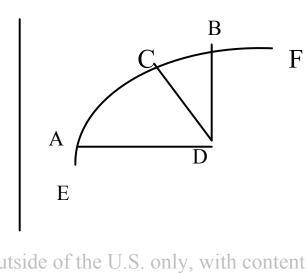
3. Similar assets like common stock or stock for companies in the same industry (e.g., auto industry) will have high positive covariances because the sales and profits for the firms are affected by common factors since their customers and suppliers are the same. Because their profits and risk factors move together you should expect the stock returns to also move together and have high covariance. The returns from different assets will not have as much covariance because the returns will not be as correlated. This is even more so for investments in different countries where the returns and risk factors are very unique.

4. The covariance between the returns of assets i and j is affected by the variability of these two returns. Therefore, it is difficult to interpret the covariance figures without taking into account the variability of each return series. In contrast, the correlation coefficient is obtained by standardizing the covariance for the individual variability of the two return series, that is: $r_{ij} = cov_{ij}/(\sigma_i\sigma_j)$

   Thus, the correlation coefficient can only vary in the range of -1 to +1. A value of +1 would indicate a perfect linear positive relationship between R_i and R_j.

5. The efficient frontier has a curvilinear shape because if the set of possible portfolios of assets is not perfectly correlated the set of relations will not be a straight line, but is curved depending on the correlation. The lower the correlation the more curved.

6. Expected Rate Of Return



Expected Risk (σ of Return)

A portfolio dominates another portfolio if: 1) it has a higher expected return than another portfolio with the same level of risk, 2) a lower level of expected risk than another portfolio with equal expected return, or 3) a higher expected return and lower expected risk than another portfolio. For example, portfolio B dominates D by the first criterion. A dominates D by the second, and C dominates D by the third.

The Markowitz efficient frontier is simply a set of portfolios that is not dominated by any other portfolio, namely those lying along the segment E-F.

7. The necessary information for the program would be:
   1) the expected rate of return of each asset
   2) the expected variance of return of each asset
   3) the expected covariance of return of all pairs of assets under consideration.

8. Investors' utility curves are important because they indicate the desired tradeoff by investors between risk and return. Given the efficient frontier, they indicate which portfolio is preferable for the given investor. Notably, because utility curves differ one should expect different investors to select different portfolios on the efficient frontier.

9. The optimal portfolio for a given investor is the point of tangency between his set of utility curves and the efficient frontier. This will most likely be a diversified portfolio because almost all the portfolios on the frontier are diversified except for the two end points - the minimum variance portfolio and the maximum return portfolio. These two could be significant.

10. The utility curves for an individual specify the trade-offs she is willing to make between expected return and risk. These utility curves are used in conjunction with the efficient frontier to determine which particular efficient portfolio is the best for a particular investor. Two investors will not choose the same portfolio from the efficient set unless their utility curves are identical.

11. The hypothetical graph of an efficient frontier of U.S. common stocks will have a curved shape (see the graph in the answer to question 6, above). Adding U.S. bonds to the portfolio will likely generate a new efficient frontier that is shifted up (or to the left) of the original stock-only frontier. The reason for the shift is that we expect bonds to be less correlated with stocks, thereby creating additional diversification potential.
    The third frontier (which includes international securities) will likely display another shift upward or to the left for similar reasons—lower correlation potential resulting in additional diversification potential.

12. The portfolio constructed containing stocks L and M would have the lowest standard deviation. As demonstrated in the chapter, combining assets with equal risk and return but with low positive or negative correlations will reduce the risk level of the portfolio.

13. CFA Examination Level II

Answer is C as expected return = (.15)(.50) + (.10).40) + (.06)(.10) = 12.1%

14. Answer is A as adding an investment that has a correlation of -1.0 will achieve maximum risk diversification.

CHAPTER 7

Answers to Problems

1.

$$r_{i,j} = \frac{Cov_{i,j}}{\sigma_i \sigma_j} = \frac{100}{19 \text{ x } 14} = \frac{100}{266} = 0.3759$$

2. $E(R_1) = .15 \quad E(\sigma_1) = .10 \quad w_1 = .5$

$E(R_2) = .20 \quad E(\sigma_2) = .20 \quad w_2 = .5$

$E(R_{port}) = .5(.15) + .5(.20) = .175$

If $r_{1,2} = .40$

$$\sigma_p = \sqrt{(.5)^2(.10)^2 + (.5)^2(.20)^2 + 2(.5)(.5)(.10)(.20)(.40)}$$

$$= \sqrt{.0025 + .01 + .004}$$

$$= \sqrt{.0165}$$

$$= 0.12845$$

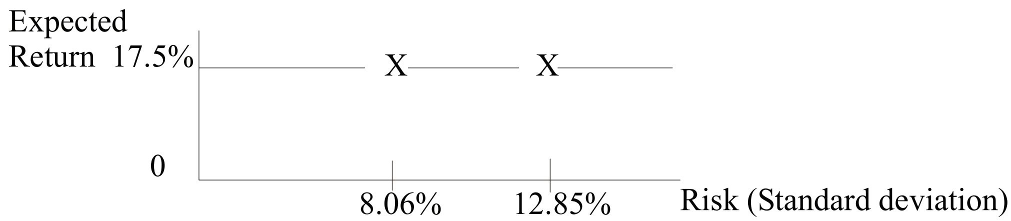
If $r_{1,2} = -.60$

$$\sigma_p = \sqrt{(.5)^2(.10)^2 + (.5)^2(.20)^2 + 2(.5)(.5)(.10)(.20)(-.60)}$$

$$= \sqrt{.0025 + .01 + (-.006)}$$

$$= \sqrt{.0065}$$

$$= .08062$$



The negative correlation coefficient reduces risk without sacrificing return.

3.

Month	DJIA (R_1)	S&P (R_2)	Russell (R_3)	Nikkei (R_4)	R_1-E(R_1)	R_2-E(R_2)	R_3-E(R_3)	R_4-E(R_4)
1	.03	.02	.04	.04	.01667	.00333	.01333	.00833
2	.07	.06	.10	-.02	.05667	.04333	.07333	-.05167
3	-.02	-.01	-.04	.07	-.03333	-.02667	-.06667	.03883
4	.01	.03	.03	.02	-.00333	.01333	.00333	-.01167
5	.05	.04	.11	.02	.03667	.02333	.08333	-.01167
6	-.06	-.04	-.08	.06	-.07333	-.05667	-.10667	.02833
Sum	.08	.10	.16	.19				

3(a).

$$E(R_1) = \frac{.08}{6} = .01333 \qquad E(R_2) = \frac{.10}{6} = .01667$$

$$E(R_3) = \frac{.16}{6} = .02667 \qquad E(R_4) = \frac{.19}{6} = .03167$$

3(b).

$$\Sigma_1 = (.01667)^2 + (.05667)^2 + (-.03333)^2 + (-.00333)^2 + (.03667)^2 + (-.07333)^2$$
$$= .00028 + .00321 + .00111 + .00001 + .00134 + .00538 = .01133$$

$$\sigma_1^2 = .01133/5 = .00226$$

$$\sigma_1 = (.00226)^{1/2} = .0476$$

$$\Sigma_2 = (-.00333)^2 + (.04333)^2 + (-.02667)^2 + (.01333)^2 + (.02333)^2 + (-.05667)^2$$
$$= .00001 + .00188 + .00071 + .00018 + .00054 + .00321 = .00653$$

$$\sigma_2^2 = .00653/5 = .01306$$

$$\sigma_2 = (.01306)^{1/2} = .0361$$

$$\Sigma_3 = (.01333)^2 + (.07333)^2 + (-.06667)^2 + (.00333)^2 + (.08333)^2 + (-.106672)^2$$
$$= .00018 + .00538 + .00444 + .00001 + .00694 + .01138 = .02833$$

$$\sigma_3^2 = .02833/5 = .00567$$

$$\sigma_3 = (.00567)^{1/2} = .0753$$

$\Sigma_4 = (.00833)^2+(-.05167)^2+ (.03833)^2+ (-.01167)^2+(-.01167)^2 + (.02833)^2$
$= .00007 + .00267 + .00147 + .00014 + .00014\ .00080 = .00529$

$\sigma_4^2 = .00529/5 \quad = .001058$

$\sigma_4 = (.001058)^{1/2} = .0325$

3(c).

$$COV_{1,2} = \frac{.00006 + .00246 + .00089 - .00004 + .00086 + .00416}{5}$$

$$= .00839/5 = .001678$$

$$COV_{2,3} = \frac{.00004 + .00318 + .00178 + .00004 + .00194 + .00604}{5}$$

$$= .01302/5 = .002604$$

$$COV_{2,4} = \frac{.00003 - .00224 - .00102 - .00016 - .00027 - .00161}{5}$$

$$= -.00527/5 = -.001054$$

$$COV_{3,4} = \frac{.00011 - .00379 - .00256 - .00004 - .00097 - .00302}{5}$$

$$= -.01027/5 = -.002054$$

3(d). Correlation equals the covariance divided by each standard deviation.
Correlation (DJIA, S&P) = 0.001678/ [(0.0476)(0.0361)] = .9765
Correlation (S&P, R2000) = 0.002604/ [(0.0361)(0.0753)] = .9579
Correlation (S&P, Nikkei) = -0.001054/ [(0.0361)(0.0325)] = -0.8984
Correlation (R2000, Nikkei) = -0.002054/ [(0.0753)(0.0325)] =-0.8393

3(e).

$$\sigma_{2,3} = \sqrt{(.5)^2(.0361)^2 + (.5)^2(.0753)^2 + 2(.5)(.5)(.002604)}$$

$$= .05518$$

$$E(R)_{2,3} = (.5)(.01667) + (.5)(.02667) = .02167$$

$$\sigma_{2,4} = \sqrt{(.5)^2(.0361)^2 + (.5)^2(.0325)^2 + 2(.5)(.5)(-.001054)}$$

$$= .009875$$

$$E(R)_{2,4} = (.5)(.01667) + (.5)(.03167) = .02417$$

The resulting correlation coefficients suggest a strong positive correlation in returns for the S&P 500 and the Russell 2000 combinations (.96), preventing any meaningful reduction in risk (.05518) when they are combined. Since the S&P 500 and Nikkei have a negative correlation (-.90), their combination results in a lower standard deviation (.009875).

4.

Month	Madison(R_i)	General Electric(R_j)	R_i-E(R_i)	R_j-E(R_j)	[R_i-E(R_i)] x [R_j-E(R_j)]
1	-.04	.07	-.057	.06	-.0034
2	.06	-.02	.043	-.03	-.0013
3	-.07	-.10	-.087	-.11	.0096
4	.12	.15	.103	.14	.0144
5	-.02	-.06	-.037	-.07	.0026
6	.05	.02	.033	.01	.0003
Sum	.10	.06			.0222

4(a). $E(R_{Madison}) = .10/6 = .0167$ $E(R_{GE}) = .06/6 = .01$

4(b).

$$\sigma_{Madison} = \sqrt{.0257/5} = \sqrt{.0051} = .0717$$

$$\sigma_{GE} = \sqrt{.04120/5} = \sqrt{.0082} = .0908$$

4(c). $COV_{ij} = 1/5\ (.0222) = .0044$

4(d).

$$r_{ij} = \frac{.0044}{(.0717)\ (.0908)}$$

$$= \frac{.0044}{.006510}$$

$$= .6758$$

Without rounding (using more significant digits are a spreadsheet) the correlation is computed as 0.682. One should have expected a positive correlation between the two stocks, since they tend to move in the same direction(s). Risk can be reduced by combining assets that have low positive or negative correlations, which is not the case for Madison and General Electric.

5(a). $E(R_p) = (1.00 \text{ x } .12) + (.00 \text{ x } .16) = .12$

$$\sigma_p = \sqrt{(1.00)^2(.04)^2 + (.00)^2(.06)^2 + 2(1.00)(.00)(.04)(.06)(.70)}$$

$$= \sqrt{.0016 + 0 + 0} = \sqrt{.0016} = .04$$

5(b). $E(R_p) = (.75 \text{ x } .12) + (.25 \text{ x } .16) = .13$

$$\sigma_p = \sqrt{(.75)^2(.04)^2 + (.25)^2(.06)^2 + 2(.75)(.25)(.04)(.06)(.70)}$$

$$= \sqrt{.0009 + .000225 + .00063} = \sqrt{.001755} = .0419$$

5(c). $E(R_p) = (.50 \times .12) + (.50 \times .16) = .14$

$$\sigma_p = \sqrt{(.50)^2(.04)^2 + (.50)^2(.06)^2 + 2(.50)(.50)(.04)(.06)(.70)}$$

$$= \sqrt{.0004 + .0009 + .00084} = \sqrt{.00214} = .0463$$

5(d). $E(R_p) = (.25 \times .12) + (.75 \times .16) = .15$

$$\sigma_p = \sqrt{(.25)^2(.04)^2 + (.75)^2(.06)^2 + 2(.25)(.75)(.04)(.06)(.70)}$$

$$= \sqrt{.0001 + .002025 + .00063} = \sqrt{.002755} = .0525$$

5(e). $E(R_p) = (.05 \times .12) + (.95 \times .16) = .158$

$$\sigma_p = \sqrt{(.50)^2(.04)^2 + (.95)^2(.06)^2 + 2(.05)(.95)(.04)(.06)(.70)}$$

$$= \sqrt{.000004 + .003249 + .00015960} = \sqrt{.0034126} = .0584$$

6.

Stock	Market Value	Weight	Security Return (R_i)	Portfolio Return $W_i \times R_i$
Morgan	$15,000	0.160	0.14	0.022
Starbucks	17,000	0.181	-0.04	-0.007
GE	32,000	0.340	0.18	0.061
Intel	23,000	0.245	0.16	0.039
Walgreens	7,000	0.074	0.05	0.004
TOTAL	94,000	1.0000		0.119

7. For all values of $r_{1,2}$:

$E(R_{port}) = (.6 \times .10) + (.4 \times .15) = .12$

$$\sigma_{port} = \sqrt{(.6)^2(.03)^2 + (.4)^2(.05)^2 + 2(.6)(.4)(.03)(.05)(r_{1,2})}$$

$$= \sqrt{.000324 + .0004 + .00072(r_{1,2})}$$

$$= \sqrt{.000724 + .00072(r_{1,2})}$$

7(a).

$$\sqrt{.000724+.00072(1.0)}=\sqrt{.001444}=.0380$$

7(b).

$$\sqrt{.000724+.00072(.75)}=\sqrt{.001264}=.0356$$

7(c).

$$\sqrt{.000724+.00072(.25)}=\sqrt{.000904}=.0301$$

7(d).

$$\sqrt{.000724+.00072(.00)}=\sqrt{.000724}=.0269$$

7(e).

$$\sqrt{.000724+.00072(-.25)}=\sqrt{.000544}=.0233$$

7(f).

$$\sqrt{.000724+.00072(-.75)}=\sqrt{.000184}=.0136$$

7(g).

$$\sqrt{.000724+.00072(-1.0)}=\sqrt{.000004}=.0020$$

8. $[E(R_i)]$ for Lauren Labs

Probability	Possible Returns		Expected Return
0.10	-0.20		-0.0200
0.15	-0.05		-0.0075
0.20	0.10		0.0200
0.25	0.15		0.0375
0.20	0.20		0.0400
0.10	0.40		0.0400
		$E(R_i)=$	0.1100

$$r_{i,j}=\frac{Cov_{i,j}}{\sigma_i\sigma_j}=\frac{100}{19 \text{ x } 14}=\frac{100}{266}=0.3759$$

APPENDIX 7

Answers to Problems

Appendix A

1(a). When $E(\sigma_1) = E(\sigma_2)$, the problem can be solved by substitution,

$$W_1 = \frac{E(\sigma_1)^2 - r_{1,2}\ E(\sigma_1)\ E(\sigma_1)}{E(\sigma_1)^2 + E(\sigma_1)^2 - 2\ r_{1,2}\ E(\sigma_1)\ E(\sigma_1)}$$

so that

$$W_1 = \frac{E(\sigma_1)^2\ [1 - r_{1,2}]}{2\ E(\sigma_1)^2 - 2\ r_{1,2}\ E(\sigma_1)^2}$$

$$= \frac{E(\sigma_1)^2\ [1 - r_{1,2}]}{2\ E(\sigma_1)^2\ [1 - r_{1,2}]}$$

$$= 1/2 = .5$$

1(b).

$$W_1 = \frac{(.06)^2 - (.5)(.04)(.06)}{(.04)^2 + (.06)^2 - 2(.5)(.04)(.06)}$$

$$= \frac{.0036 - .0012}{.0016 + .0036 - .0024}$$

$$= \frac{.0024}{.0028} = .8571\ \text{(or 6/7)}$$

Appendix B

Variance of the portfolio is zero when:

$$w_1 = \frac{E(\sigma_2)}{E(\sigma_1) + E(\sigma_2)} = \frac{.06}{.04 + .06} = .6 \qquad w_2 = 1 - w_1 = 1 - .6 = .4$$