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1.

S_i : the price of the underlying asset at date i

D_0 : date 0 present value of dividends pay 0-1

F_{02} : forward price

$f_2 = S_2 - F_{02}$

SDF ; $E_0[m_{02} f_2] = E_0[m_{02}(S_2 - F_{02})] = E[m_{02} S_2] - E[m_{02} F_{02}]$

$$S_0 = E_0[m_{01} D_1] + E_0[m_{02} S_2] = D_0 + E_0[m_{02} S_2]$$

$$E[m_{02} F_{02}] = E[m_{02}] F_{02} = R_f^{-2} F_{02}$$

Then we have $E_0[m_{02} f_2] = E[m_{02} S_2] - E[m_{02} F_{02}]$

$$= S_0 - D_0 - R_f^{-2} F_{02}$$

The absence of abitrage implies that the forward price satisfies

$F_{02} = R_f^2 (S_0 - D_0) \rightarrow E_0[m_{02} f_{02}] = 0$

2.

$$P_0 = E_0\left[\sum_{t=1}^{\infty} \frac{U_c(C_t, t)}{U_c(C_0, 0)} d_c\right] = E_0\left[\sum_{t=1}^{\infty} \delta^t e^{-a(c_t - c_0)} d_t\right] \#$$

3.
$$P_t = E_t\left[\sum_{t=1}^{\infty} \delta^j \left(\frac{C^*_{t+j}}{C^*_t}\right)^{\gamma-1} d_{t+j}\right]$$

$$P_t/d_t = E_t\left[\sum_{j=1}^{\infty} \delta^j \left(\frac{C^*_{t+j}}{C^*_t}\right)^{\gamma-1}\left(\frac{d_{t+j}}{d_t}\right)\right]$$

$$= E_t\left[\sum_{j=1}^{\infty} \delta^j e^{(\gamma-1)\ln(C_{t+j}/C_t) + \ln(d_{t+j}/d_t)}\right]$$

$$\ln(C_{t+j}/C_t) = j\cdot\mu_c + \sigma_c \sum_{j=1}^{j} \eta_{t+j}$$

$$\ln(D_{t+j}/D_t) = j\cdot\mu_d + \sigma \sum_{j=1}^{j} \varepsilon_{t+j}$$

so that

$$P_t/d_t = E_t\left[\sum_{j=1}^{\infty} \delta^j e^{(\gamma-1)\left(j\mu_c + \sigma_c \sum_{i=1}^{j} \eta_{t+j}\right) + j\mu_d + \sigma_d \sum_{i=1}^{j} \varepsilon_{t+j}}\right]$$

$$= E_t\left[\sum_{j=1}^{\infty} \delta^j e^{\{(\gamma-1)\mu_c + \mu_c\} + \sum_{i=1}^{j}\left[(\gamma-1)\sigma_c \eta_{t+j} + \sigma_d \varepsilon_{t+j}\right]}\right]$$

$$= \sum_{j=1}^{\infty} \delta^j e^{\{(\gamma-1)\mu_c + g_d\}} e^{\frac{j}{2}\left[(1-\gamma)^2\sigma_c^2 + \sigma_d^2 - 2(1-\gamma)\sigma_c\sigma_d\rho\right]}$$

$$= \sum_{j=1}^{\infty} e^{j\left[\ln\delta - (1-\gamma)\mu_c + \mu_d + \frac{1}{2}\left((1-\gamma)^2\sigma_c^2 + \sigma_d^2\right) - (1-\gamma)\sigma_c\sigma_d\rho\right]}$$

$$= \frac{1}{1 - \delta e^{-(1-\gamma)\mu_c + \mu_d + \frac{1}{2}\left((1-\gamma)^2\sigma_c^2 + \sigma_d^2\right) - (1-\gamma)\sigma_c\sigma_d\rho}} - 1$$

so
$$P_t = d_t \frac{\delta e^{\alpha}}{1 - \delta e^{\alpha}}$$

where

$$\alpha \equiv \mu_d - (1-\gamma)\mu_c + \frac{1}{2}\left[(1-\gamma)^2\sigma_c^2 + \sigma_d^2\right] - (1-\gamma)\rho\,\sigma_c\sigma_d \quad \#$$

a. check that (2) satisfies $E_t[b_{t+1}] = R_f b_t$

$$E_t[b_{t+1}] = \frac{R_f}{q_t} b_t q_t + E_t[e_{t+1}] q_t + (1-q_t) E_t[z_{t+1}] = R_f b_t$$

So it is a valid solution #

b. Since $E_t[b_{t+1}] = R_f b_t$

$$\lim_{i \to \infty} E_t[b_{t+i}] = \begin{cases} +\infty & \text{if } b_t > 0 \\ -\infty & \text{if } b_t < 0 \end{cases}$$

For limited liability assets, such as oil, we cannot have a bubble path with a price becoming negative, so we need to consider only bubbles with $b_t > 0$. In this case, we see from the above equation (*) that for a bubble solution to exist the bubble component must be expected to increase infinitely. But this cannot be a rational expectation if there is an upperbound on the price of oil, as would be the case if there was a perfect substitute in perfectly elastic supply. Thus, since p_t cannot rise above p_{solar}, b_t cannot rise above $p_{solar} - p_t^*$. Thus, a bubble path where b_t must be expected to increase to infinity cannot possibly occur. #

c. For similar reasons, a rational speculative bubble cannot exist for the price of a bond. Since, at maturity, the bond's price must be $p_T = d_T$ and zero date T its price cannot rationally be expected to satisfy equation (*) and increase infinitely. Thus, a bubble path is invalid, and the only rational price is $p_t = p_t^*$ #

5.

With the economy, and therefore assets (or asset markets), having a finite horizon, asset price could not have the form $p_t = f_t + b$ with $b_t \neq 0$ because at date T $p_T = f_T = d_T$ which is an asset's final dividend payment, since $b_T = 0$ with certainty. Then the bubble process $E_t[b_{t+1}] = \delta^{-1} b_t$ implies $E_{T-1}[b_T] = E_{T-1}[0] = \delta^{-1} b_{T-1}$ or $b_{T-1} = 0$. A similar argument implies $b_t = 0$ for all previous dates, $t < T-1$ #