

# Chapter 6 : A Real Intertemporal Model with Investment (Part 1)

EE312

Macroeconomics, Stephen Williamson, Chapter 11

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# 1. Real Intertemporal Model

- **The real model (no money)** with three sectors as the basis for the analysis of short-term economic fluctuations.
  - **Representative consumers** (consumption, labor supply and saving);
  - **Representative firms** (production, labor demand and investment);
  - **Government** (spending, taxes and borrowing).

- **The labor market:** the firm's demand and the consumer's supply of labor.  
⇒ The real wage rate.
- **The output market:** the firm's supply and the consumer's demand for output.  
⇒ The real interest rate. Analysis of real macroeconomic shocks.
- **Changes in government spending, capital stock, total factor productivity.**

- Expenditure on plants, equipment and new housing.
  - Goods currently produced for future production of goods and services.
  - Increases in future productive capacity.
  - **The consumer's tradeoff** between current and future consumptions (saving).
  - **The firm's tradeoff** between current profits and higher future capital stock (and future profits).

## 2. Consumer's optimal decisions

- Work-leisure in current and future periods.
- Consumption-savings in the current period.
  - $h$  = total time available;
  - $w$  and  $w'$  = current and future real wages;
  - $r$  = the real interest rate;
  - $T$  and  $T'$  = current and future lump-sum taxes;
  - $C$  and  $C'$  = current and future consumptions;
  - $\ell$  and  $\ell'$  = current and future leisure time;
  - $S^p$  = private savings.

## Current budget constraint

- The consumer is a price-taker ( $w$ ,  $w'$ ,  $r$  and  $T$  are given).
- $w(h - \ell)$  = real-wage income;
- $\pi$  = dividend income from the firm;
- $T$  = lump-sum taxes paid to the government.
- Then, disposable income is:

$$C + S^P = w(h - \ell) + \pi - T$$

## Future budget constraint

- The consumer still receives real wages, dividend income, and pays future taxes.
- Receives the principal and interest on savings.
- No bequests; all wealth is consumed.

$$C' = w'(h - \ell') + \pi' - T' + (1 + r)S^p$$

## Lifetime budget constraint

$$C + \frac{C'}{1+r} = w(h - \ell) + \pi - T + \frac{w'(h - \ell') + \pi' - T'}{1+r}$$

- The PV of lifetime consumption equals the PV of lifetime disposable income.
- Decision on the optimal bundles of  $C$ ,  $C'$ ,  $\ell$  and  $\ell'$  subject to the lifetime budget constraint.

$$MRS_{\ell,C} = w$$

- The consumer chooses the optimal bundle of current leisure and consumption:
  - The marginal rate of substitution of current leisure for current consumption is equal to the real wage.
  - $w$  is the relative price of leisure in terms of consumption goods.

$$MRS_{\ell', C'} = w'$$

- The consumer chooses the optimal bundle of future leisure and future consumption:
- The marginal rate of substitution of future leisure for future consumption is equal to the future real wage.

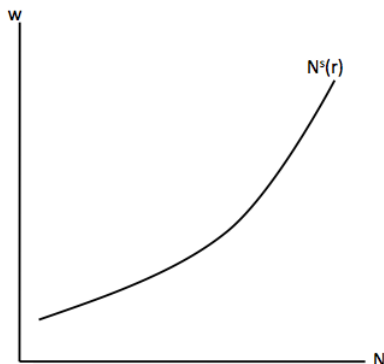
$$MRS_{C,C'} = 1 + r$$

- The consumer chooses the optimal bundle of current and future consumption (savings):
- The marginal rate of substitution of current consumption for future consumption is equal to the real interest rate.
- $(1 + r)$  is the relative price of current consumption in terms of future consumption.

- The consumer provides labor supply to the firm through work-leisure decision.
- Factors which determine current labor supply:
  - The current real wage;
  - The real interest rate;
  - Lifetime wealth.

## Current labor supply curve

- Current labor supply increases with the real wage, given  $r$  (assuming the dominant substitution effect).

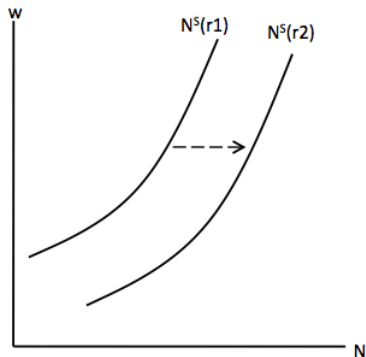


## An increase in the real interest rate

- Current labor supply increases as the real interest rate increases.
- $\frac{w(1+r)}{w'}$  is the relative price of current leisure in terms of future leisure.
- Given  $w$  and  $w'$ , a higher  $r$  means the higher price of current leisure in terms of future leisure.
- Less current leisure, and more current supply of labor, assuming the dominant substitution effect.

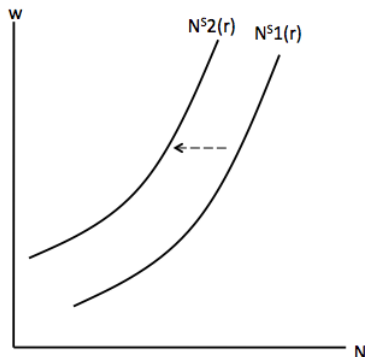
## Labor supply increases with $r$

- Given  $w$ , labor supply increases with the rising real interest rate ( $r_2 > r_1$ ), assuming the dominant substitution effect.



## An increase in lifetime wealth

- Current leisure increases and current labor supply decreases with rising lifetime wealth.
  - Current and future consumption also increase.

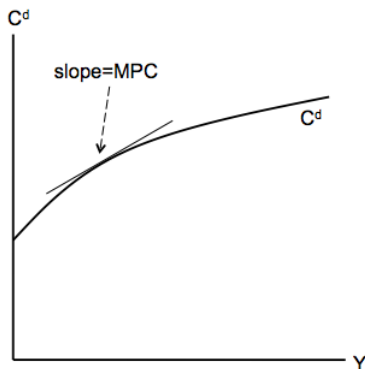


## Demand for current consumption goods

- The individual demand for current consumption goods ( $C^d$ ) is a function of current income ( $Y$ ), given  $r$ .
- The marginal propensity to consume ( $MPC$ )  $< 1$  .
- A higher real interest rate ( $r$ ) causes the demand to fall, assuming:
- The substitution effect dominates the income effect; the consumer is a lender.

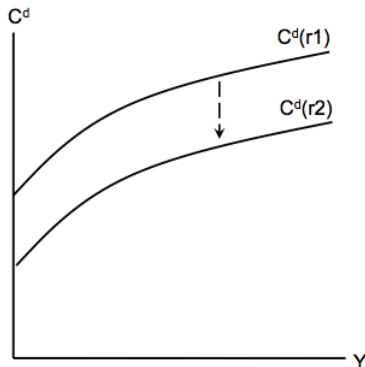
## Demand for current consumption

- $C^d = f(Y, r)$
- $MPC = \frac{\partial C^d}{\partial Y} < 1$  ;
- $\frac{\partial C^d}{\partial r} < 0$ ;  $\frac{\partial C^d}{\partial we} > 0$



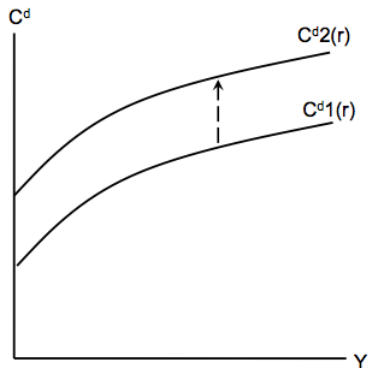
## A higher $r$ reduces $C^d$

- $r_2 > r_1$ .
- The consumer reduces current consumption, assuming stronger substitution effect and a lender.



## An increase in we raises $C^d$

- An increase in lifetime wealth raises current consumption.



### 3. The Representative firm

- Optimal decisions on:
  - Maximized present value of profits;
  - The level of current labor inputs.
  - **Optimal investment level:** tradeoff between current profits and future capital stock (and future profits).

$$Y = zF(K, N)$$

- $Y$  = current output;
- $z$  = current total factor productivity;
- $K$  = current capital stock;
- $N$  = current labour input.
- And the future production function:

$$Y' = z'F(K', N')$$

- The firm's investment is foregone current profits (consumption) for future profits:
  - $d$  = the rate of depreciation;
  - $I$  = current investment.
- Future capital stock is current capital stock net of depreciation plus investment.

$$K' = (1 - d)K + I$$

## The firm's current profits

- Maximization of the present value of current and future profits.
- $\pi$  = current profits;
- $I$  = current investment = foregone current consumption goods.

$$\pi = Y - wN - I$$

## The firm's future profits

- The leftover capital stock in the future period can be sold off as junk value.
- $(1 - d)K'$  = capital stock remaining as junk at the end of the future period.

$$\pi' = Y' - w'N' + (1 - d)K'$$

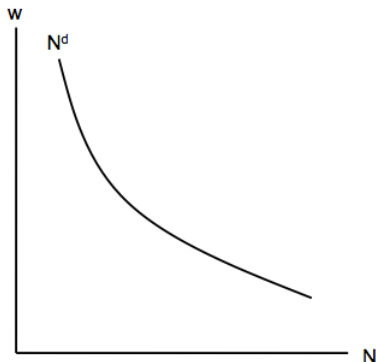
- The firm maximizes the PV of profits.
  - The same as maximized PV of dividend income for the consumer.
  - $V = \max.$  present value of profits through optimal choice on  $N$ ,  $N'$  and  $I$ :

$$V = \pi + \frac{\pi'}{1+r}$$

- The firm's choice of current labor demand ( $N^d$ ) affects only current profits ( $\pi$ ).
- The firm hires current labor until the current marginal product of labor equals the current real wage ( $MP_N = w$ ).
- Thus the firm's  $MP_N$  curve is also the firm's current labor demand curve.
- An increase in current  $z$  or  $K$  raises  $MP_N$  and current labor demand.

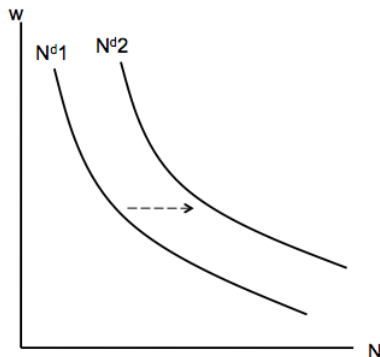
## Current labor demand curve

- The current labor demand:  $MP_N = w$ .
- $MP_N$  is falling as the labor input increases.



## Labor demand with rising $z$ or $K$

- An increase in current  $z$  or  $K$  shifts the current labor demand curve to the right.



## The firm's investment decision

- The firm invests to the point where the marginal benefit from investment equals marginal cost.
- $MC(I)$  = marginal cost of investment = PV of profits ( $V$ ) given up for one unit of capital.
- One unit of investment reduces current  $\pi$  (and  $V$ ) by one unit.

$$MC(I) = 1$$

- $MB(I)$  = marginal benefit of investment = additional units of  $V$  (PV of profits) received from one extra unit of current investment.
- $MPK'$  = additional output from one extra unit of  $K'$  .
- Quantity of capital left from depreciation at the end of the future period  $(1 - d)$  for liquidation.
- $I$ 's future profits is  $(MP'_K + 1 - d) = \pi'$ .

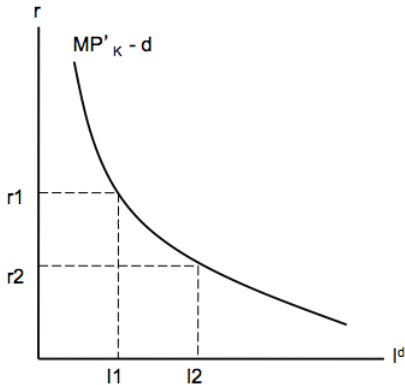
$$MB(I) = \frac{MP'_K + 1 - d}{1 + r}.$$

$$\frac{MP'_K + 1 - d}{1 + r} = 1.$$

$$MP'_K - d = r.$$

- The firm invests until the net future marginal product of capital equals the real interest rate.
- **r = the opportunity cost of more capital** = the rate of return on the alternative asset (bonds) otherwise earned by the consumer who owns the firm.

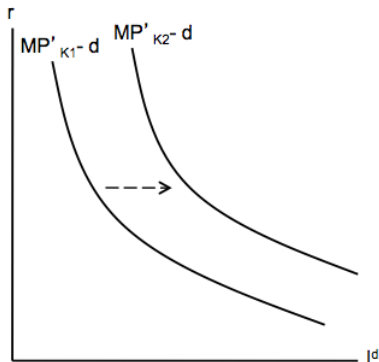
- $I^d = (MP'_K - d)$  gives the level of  $I$  required for the net  $MP'_K$  to be equal to the real interest rate, given  $K$ .



## Changes in $z'$ and $K$

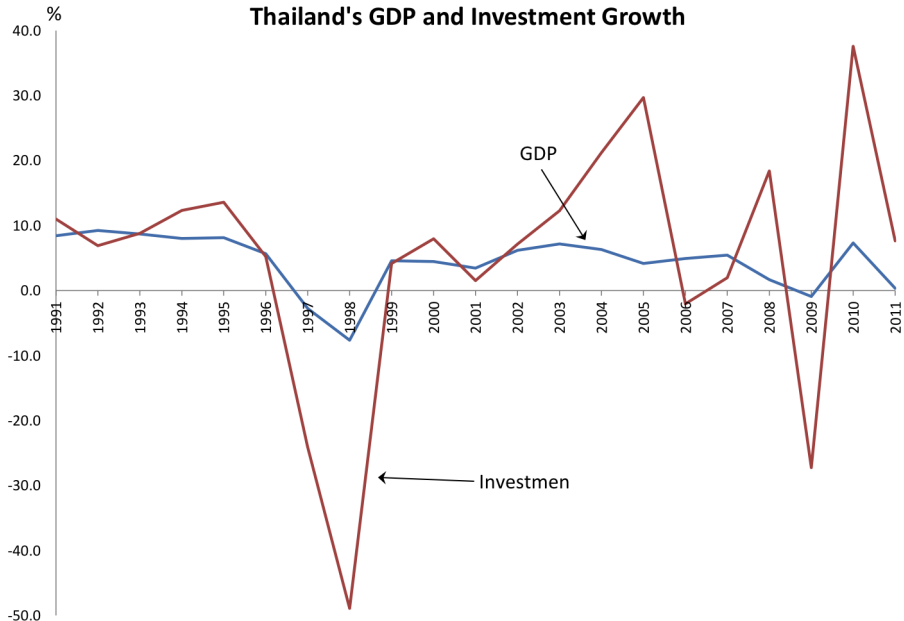
- Factors affecting future marginal product of capital shift the optimal investment curve.
- Higher **future total factor productivity** ( $z'$ ) increases future  $MP'_K$  and current optimal investment.
- The optimal investment curve shifts to the right.
- Higher **current capital stock** results in larger future net capital stock and lower  $MP'_K$ .
- The optimal investment curve shift to the left.

- A higher  $z'$  or a lower  $K$  increases  $MP'_K$ . The optimal investment curve shifts to the right.



- Aggregate consumption is less variable than income due to consumption smoothing.
- Investment is much more volatile — short-run economic fluctuations.
  - Investment responds to perceived marginal rates of return to investment.
  - Changes in the real interest rate cause movements along the investment curve.
  - Changes in future total factor productivity shift the investment curve.

## Thailand's GDP and Investment Growth



- Government purchases of consumption goods ( $G$  and  $G'$ ) are exogenously determined.
- Government financing:
  - Current lump-sum taxes and bond sale;
  - Future lump-sum taxes and payments of the principal and interest.

$$G + \frac{G'}{1+r} = T + \frac{T'}{1+r}.$$

# Competitive equilibrium

- The labor market:
  - The consumer supplies labor service.
  - The firm demands labor service.
  - The real wage and the level of employment.
- The output market:
  - The consumer, the firm and government purchase output.
  - The firm supplies the goods.
  - The real interest rate and the level of aggregate output.