Group 8 HW 2

1. Endo: Q, P

$$Q + 10P = 14000$$
$$Q - 20P = 2000$$

$Ax = d$

$x = \begin{bmatrix} Q \\ P \end{bmatrix}$ $\rightarrow$ $\underbrace{\begin{bmatrix} 1 & 10 \\ 1 & -20 \end{bmatrix}}_{A} \begin{bmatrix} Q \\ P \end{bmatrix} = \begin{bmatrix} 14000 \\ 2000 \end{bmatrix}$

$$\det(A) = (1)(-20) - (10)(1) = -30$$

$$\text{cof}(A) = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

$C_{11} = (-1)^2 \cdot -20 = -20$
$C_{12} = (-1)^3 \cdot 1 = -1$
$C_{21} = (-1)^3 \cdot 10 = -10$
$C_{22} = (-1)^4 \cdot = 1$

$$= \begin{bmatrix} -20 & -1 \\ -10 & 1 \end{bmatrix}$$

$$\text{adj}(A) = [\text{cof}(A)]^T = \begin{bmatrix} -20 & -10 \\ -1 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det(A)} \cdot \begin{bmatrix} -20 & -10 \\ -1 & 1 \end{bmatrix}$$

$$= \frac{1}{-30} \cdot \begin{bmatrix} -20 & -10 \\ -1 & 1 \end{bmatrix}$$

$$d = \begin{bmatrix} 14000 \\ 2000 \end{bmatrix} \rightarrow x = \begin{bmatrix} Q^* \\ P^* \end{bmatrix} = A^{-1} \cdot d$$

$$\begin{bmatrix} Q^* \\ P^* \end{bmatrix} = \begin{bmatrix} \frac{20}{30} & \frac{10}{30} \\ \frac{1}{30} & -\frac{1}{30} \end{bmatrix} \cdot \begin{bmatrix} 14000 \\ 2000 \end{bmatrix}$$

$$Q^* = \left(\frac{20}{30}\right)(14000) + \left(\frac{10}{30}\right)(2000)$$

$$= 10000$$

$$P^* = \left(\frac{1}{30}\right)(14000) + \left(-\frac{1}{30}\right)(2000)$$

$$= 400$$

2. Consider a modified version of the IS-LM model where government spending is tied to the level of GDP (Y).

$C = C_0 + C_1 Y_d - C_2 r, \quad 0 < C_1 < 1$

$I = I_0 + I_1 Y - I_2 r, \quad 0 < I_1 < 1$

$G = G_0 - G_1 Y, \quad 0 < G_1 < 1$

$T = T_0$

$M^s = M_0$

$L^d = L_0 + L_1 Y - L_2 r$

where C is the private consumption, I is the private investment, G is the government spending, T is tax, M^s is the level of money supply, L^d is the level of real money demand, and r is the level of nominal interest rate. Suppose that price is fixed equal to 1. All the coefficients are non-negative. Additionally, we assume that $I_1 + C_1 - G_1 < 1$.

a. Discuss about the nature of government behavior. Does the assumption over the behavior of government make sense in practice?

b. What does C_2 represent? What does it imply about the behavior of private consumption? Does the assumption make sense?

c. Derive the IS equation. Interpret the meaning of the IS equation. Discuss about the key relation derived from IS equation.

a.) Due to the fact that the equation of the government is $G = G_0 - G_1 Y$, This means that this government spending will decrease as the GDP(Y) increases. However, in the real world, this doesn't make sense because practically the government spending will expand simultaneously as the GDP increases.

B.) $C = C_0 + C_1 Y_d - C_2 r$

In this case the C_2 is the coefficient of r (interest rate) which is reversely related to consumption. Therefore, when interest rate (r) increases, the consumption will decreases due to the substitution effect. (People will save more money.)

C.) IS Equation

$$Y = C + I + G$$

$$Y = C_0 + C_1 Y_d - C_2 r + I_0 + I_1 Y - I_2 r + G_0 - G_1 Y$$

$Y_d = Y - T_0$;

$$Y = C_0 + C_1(Y - T_0) - C_2 r + I_0 + I_1 Y - I_2 r + G_0 - G_1 Y$$

$$Y = C_0 + C_1 Y - C_1 T_0 - C_2 r + I_0 + I_1 Y - I_2 r + G_0 - G_1 Y$$

$$Y - C_1 Y - I_1 Y + G_1 Y = C_0 - C_1 T_0 - C_2 r + I_0 - I_2 r + G_0$$

$$Y(1 - C_1 - I_1 + G_1) = C_0 - C_1 T_0 - C_2 r + I_0 - I_2 r + G_0$$

$$Y = \frac{C_0 - C_1 T_0 - C_2 r + I_0 - I_2 r + G_0}{(1 - C_1 - I_1 + G_1)}$$

- Slope of IS: $\frac{\Delta Y}{\Delta r} = \frac{-C_2 - I_2}{(1 - C_1 - I_1 + G_1)}$ $\Rightarrow$ Interest rate is negatively correlated to the level of GDP(Y)

D.) When IS is flat, income (Y) is highly sensitive to changes in interest rate(r)

$$\text{Slope of IS} = \frac{\Delta r}{\Delta Y} = \frac{(1 - C_1 - I_1 + G_1)}{-C_2 - I_2}$$

$$= \frac{\cancel{(-1)}(-1 + C_1 + I_1 - G_1)}{\cancel{(-1)}(C_2 + I_2)}$$

$$= \frac{-1 + C_1 + I_1 - G_1}{C_2 + I_2}$$

∴ For IS to be flat, $C_2 + I_2$ has to be large e.) Tax multiplier: $\frac{\Delta Y}{\Delta T_0} = \frac{-c_1}{(1 - c_1 - I_1 + G_1)}$

∴ When the IS slope is flat, tax multiplier will be larger, and inversely
(MPC↑) (-MPC)

$$\frac{\Delta r^*}{\Delta M_0} \qquad \frac{-1 + C_1 + I_1 - G_1}{[(1 - C_1 - I_1 + G_1)\cdot(L_2 P)] - [(-L_1 P)\cdot(C_2 + I_2)]}$$

given that $I_1 + C_1 - G_1 < 1$;

$$I_1 + C_1 - 0 < 1$$

$$I_1 + C_1 < 1$$

$$I_1 + C_1 - 1 < 0$$

$\times(-1)$; $1 - I_1 - C_1 > 0$

$\therefore$ Suppose that $G_1 = 0$, the multiplier is larger than the government case being purely exogenous because $0 < G < 1$.

f) $\frac{M^s}{P} = L^d$

$\frac{M_0}{P} = L_0 + L_1 Y - L_2 r$

$r = \frac{1}{L_2}\left(L_0 + L_1 Y - \frac{M_0^s}{P}\right)$

g.) slope of LM: $\frac{\Delta r}{\Delta Y} = \frac{L_1}{L_2}$

$\frac{L_1 \to \text{small}}{L_2 \to \text{large}}$ then LM curve flat

$\therefore$ The change in GDP has small impact on interest rate

h.) $Y = C_0 + C_1 Y - C_1 T_0 - C_2 r + I_0 + I_1 Y - I_2 r + G_0 - G_1 Y$

$Y - C_1 Y + C_2 r - I_1 Y + I_2 r + G_1 Y = C_0 - C_1 T_0 + I_0 + G_0$

$Y(1 - C_1 - I_1 + G_1) + r(C_2 + I_2) = C_0 - C_1 T_0 + I_0 + G_0$

$r = \frac{1}{L_2}\left(L_0 + L_1 Y - \frac{M_0^s}{P}\right)$

$-L_1 P Y + L_2 P r = L_0 P - M_0^s$

$$\begin{bmatrix} 1 - C_1 - I_1 + G_1 & C_2 + I_2 \\ -L_1 P & L_2 P \end{bmatrix} \begin{bmatrix} Y \\ r \end{bmatrix} = \begin{bmatrix} C_0 - C_1 T_0 + I_0 + G_0 \\ L_0 P - M_0^s \end{bmatrix}$$

i.) $$Y^* = \frac{\det(A_1)}{\det(A)} = \frac{\begin{vmatrix} C_0 - C_1 T_0 + I_0 + G_0 & C_2 + I_2 \\ L_0 P - M^S_0 & L_2 P \end{vmatrix}}{\begin{vmatrix} 1 - C_1 - I_1 + G_1 & C_2 + I_2 \\ -L_1 P & L_2 P \end{vmatrix}}$$

$$= \frac{[(C_0 - C_1 T_0 + I_0 + G_0) \cdot (L_2 P)] - [(L_0 P - M^S_0) \cdot (C_2 + I_2)]}{[(1 - C_1 - I_1 + G_1) \cdot (L_2 P)] - [(-L_1 P) \cdot (C_2 + I_2)]}$$

$$r^* = \frac{\det(A_1)}{\det(A)} = \frac{\begin{vmatrix} 1 - C_1 - I_1 + G_1 & C_0 - C_1 T_0 + I_0 + G_0 \\ -L_1 P & L_0 P - M^S_0 \end{vmatrix}}{\begin{vmatrix} 1 - C_1 - I_1 + G_1 & C_2 + I_2 \\ -L_1 P & L_2 P \end{vmatrix}}$$

$$= \frac{[(1 - C_1 - I_1 + G_1) \cdot (L_0 P - M^S 0)] - [(C_0 - C_1 T_0 + I_0 + G_0) \cdot (-L_1 P)]}{[(1 - C_1 - I_1 + G_1) \cdot (L_2 P)] - [(1 - C_1 - I_1 + G_1) \cdot (L_2 P) - (-L_1 P) \cdot (C_2 + I_2)]}$$

j.) $$Y^* = \frac{[(C_0 - C_1T_0 + I_0 + G_0)\cdot(L_2P)] - [(L_0P - M^s_0)\cdot(C_2+I_2)]}{[(1 - C_1 - I_1 + G_1)\cdot(L_2P)] - [(-L_1P)\cdot(C_2+I_2)]}$$

$$r^* = \frac{[(1 - C_1 - I_1 + G_1)\cdot(L_0P - M^s_0)] - [(C_0 - C_1T_0 + I_0 + G_0)\cdot(-L_1P)]}{[(1 - C_1 - I_1 + G_1)\cdot(L_2P)] - [(1 - C_1 - I_1 + G_1)\cdot(L_2P) - (-L_1P)\cdot(C_2+I_2)]}$$

$$\frac{\Delta Y^*}{\Delta G_0} = \frac{L_2P}{[(1 - C_1 - I_1 + G_1)\cdot(L_2P)] - [(-L_1P)\cdot(C_2+I_2)]}$$

$$= \frac{L_2P}{L_2P - L_2PC_1 - L_2PI_1 + L_2PG_1 + L_1PC_2 + L_1PI_2}$$

$$= \frac{L_2\cancel{P}}{\cancel{P}(L_1 - L_2C_1 - L_2I_1 + L_2G_1 + L_1C_2 + L_1I_2)}$$

$$= \frac{L_2}{L_1 - L_2C_1 - L_2I_1 + L_2G_1 + L_1C_2 + L_1I_2}$$

$$\frac{\Delta Y^*}{\Delta M_0} = \frac{C_2 + I_2}{[(1 - C_1 - I_1 + G_1)\cdot(L_2P)] - [(-L_1P)\cdot(C_2+I_2)]}$$

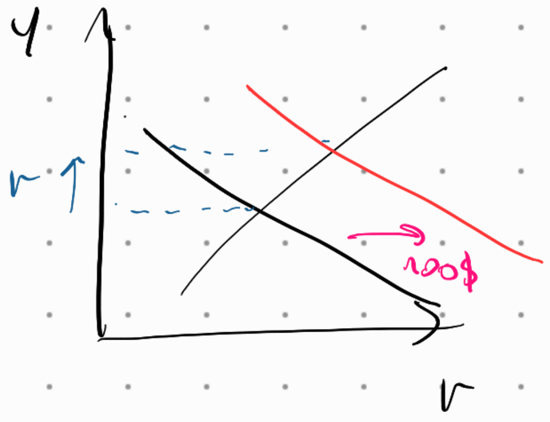
$$\frac{\Delta r^*}{\Delta G_0} = \frac{L_1P}{[(1 - C_1 - I_1 + G_1)\cdot(L_2P)] - [(-L_1P)\cdot(C_2+I_2)]}$$

$$= \frac{L_1P}{L_2P - L_2PC_1 - L_2PI_1 + L_2PG_1 + L_1PC_2 + L_1PI_2}$$

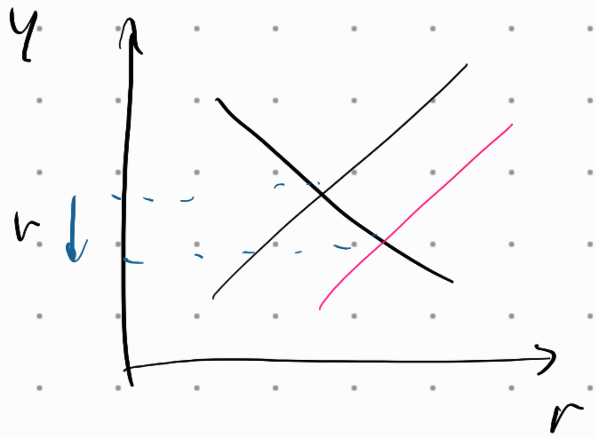
$$= \frac{L_1\cancel{P}}{\cancel{P}(L_1 - L_2C_1 - L_2I_1 + L_2G_1 + L_1C_2 + L_1I_2)}$$

$$= \frac{L_1}{L_1 - L_2C_1 - L_2I_1 + L_2G_1 + L_1C_2 + L_1I_2}$$

K.) for fiscal Policy



for Monetary Policy



both fiscal and monetary policy will drives the interest rate away from the optimal point

but suppose a case in which g) is presented where $\frac{L_1}{L_2}$ is small ≈ 0

the LM curve will be flat and the interest rate will change just a little or no change if slope ≈ 0

k cont.)

$\Delta Y = 100$

$$\frac{\Delta Y}{L_2} = \frac{100}{L_2} = \frac{\Delta G_0}{(1 - C_1 - I_1 + G_1)L_2 + (C_2 + L_2) L_1}$$

$$\frac{100}{\cancel{L_2}} \left((1 - C_1 - I_1 + G_1)\cancel{L_2} + (C_2 + L_2) \frac{L_1}{L_2} \right) = \Delta G_0$$

let $\frac{L1}{L2} = 0 \rightarrow$ LM flat

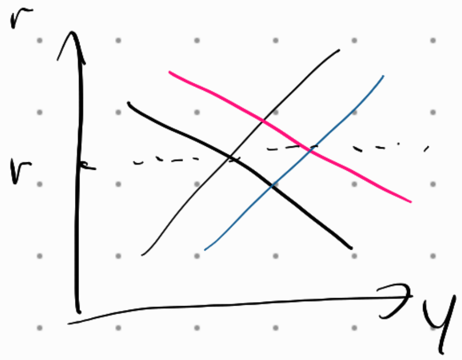
$\therefore \; 100 (1 - C_1 - I_1 + G_1) = \Delta G$

$$\frac{\Delta Y}{\Delta G} = \frac{\cancel{100}\ 1}{\cancel{100}(1 - C_1 - I_1 + G_1)}$$

$$\frac{\Delta Y}{\Delta M_0} = \frac{100}{\Delta M_0} = \frac{C_2 + I_2}{((1 - C_1 - I_1 + G_1)(L_2 P) (-L_1 P)(C_2 + I_2))}$$

$$\frac{100}{C_2 + I_2} \left((1 - C_1 - I_1 + G_1)(L_2 P) - (-L_1 P) \cdot [C_2 + L_2] \right] = \Delta M_0$$

l.)



To keep the interest rate at the same optimal level. Government must use expansionary fiscal policy and expansionary monetary policy simultaneously